Carrier Concentrations

Presented by Chanam Lee

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Carrier Concentration

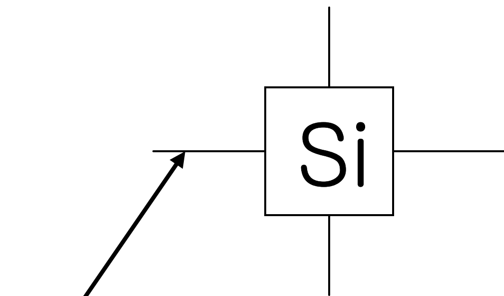
- Carrier Properties
- State and Carrier Distributions
- Equilibrium Carrier Concentration
- Carrier Concentration for the Quantum Well Devices

Carrier Properties

- Carrier Movement in Free Space
- Carrier Movement Within the Crystal
- Intrinsic Carrier Concentration
- Extrinsic n-Type Semiconductor
- Extrinsic p-Type Semiconductor

Electronic Materials

- Two-dimensional representation of an Individual Si atom.

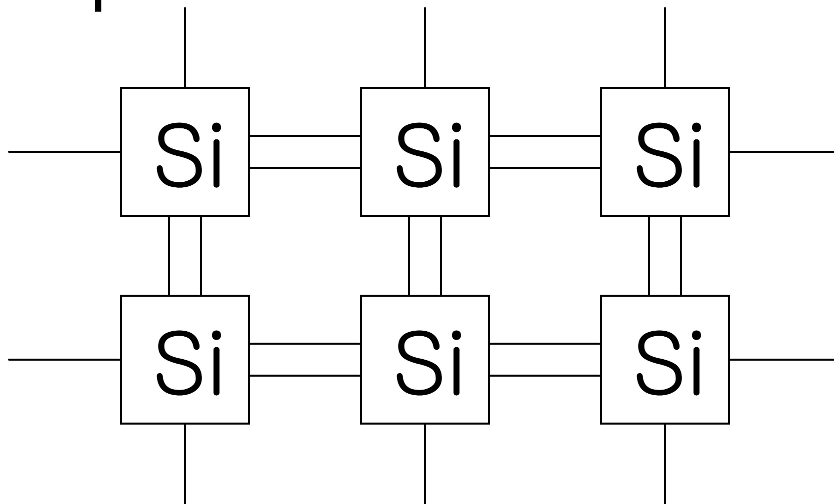


Elemental semiconductors			
Valence	III	IV	V
	B	C	
	Al	Si	P
	Ga	Ge	As

Represents each valence electron

Semiconductors

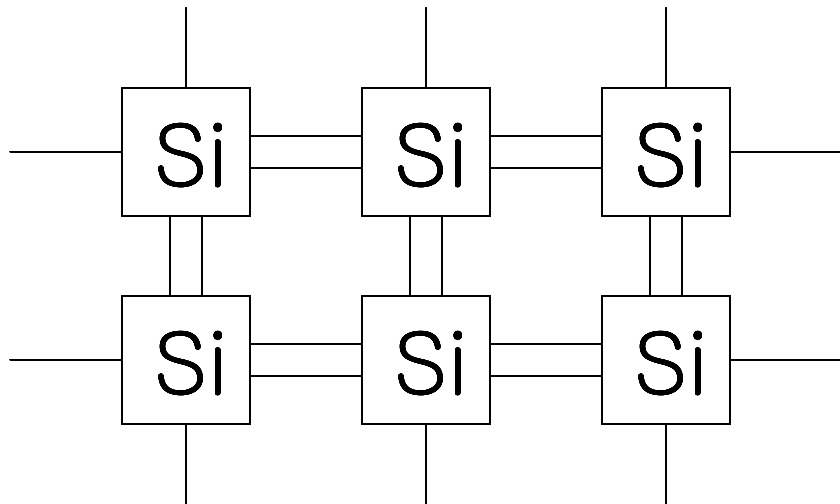
- When Si (or Ge & GaAs) atoms contact other Si atoms, they form a tetrahederal
- 2D representation of lattice structure:

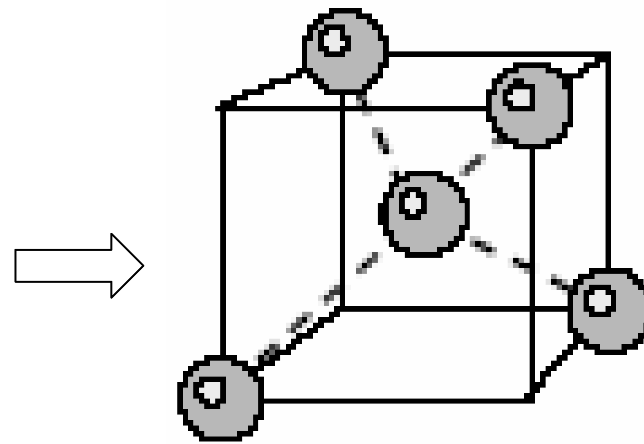


Electronic Materials

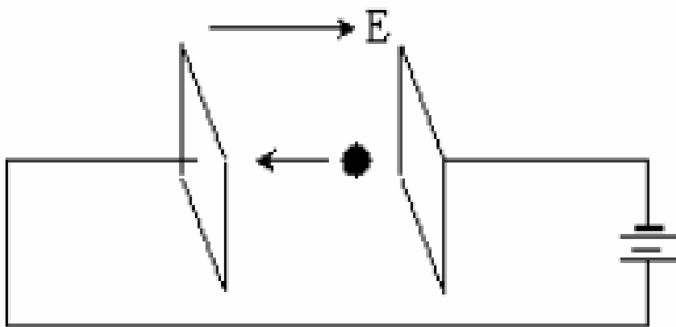
Two dimensions

Three dimensions





Carrier Movement in Free Space

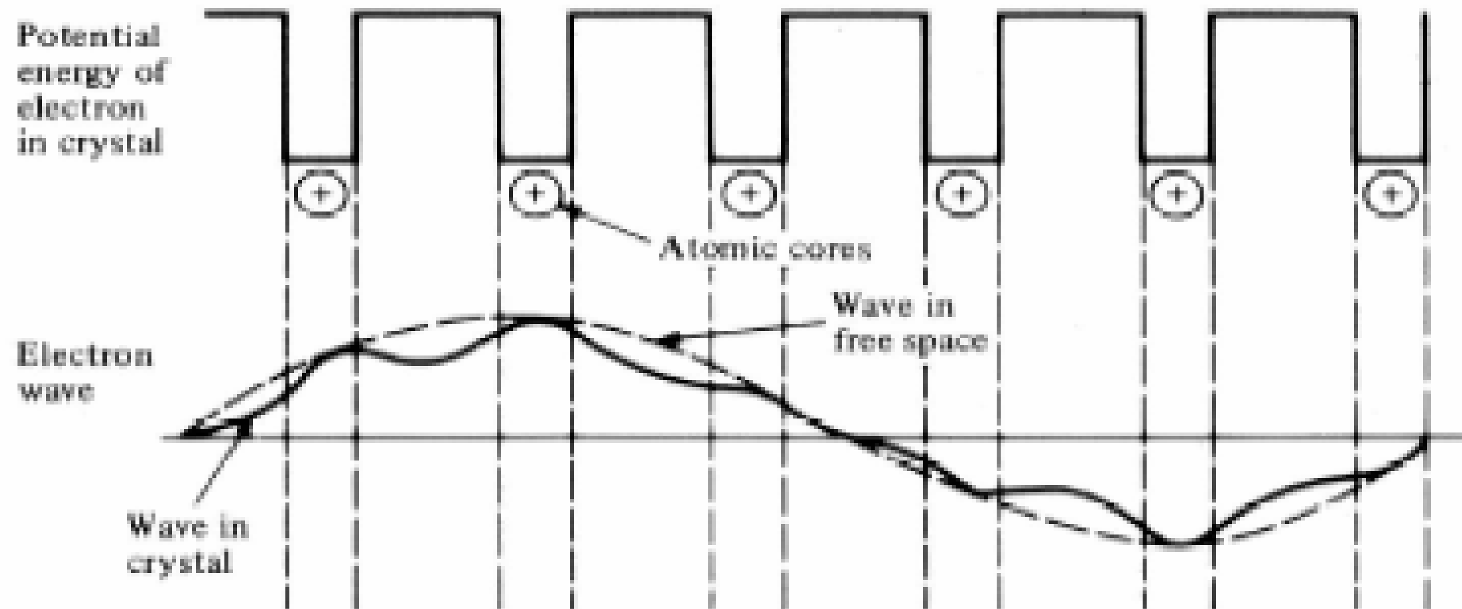


Newton's second law

$$F = -qE = m_0 \frac{dv}{dt}$$

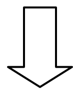
Carrier Movement Within the Crystal

- Electrons moving inside a semiconductor crystal will collide with semiconductor atoms ==> behaves as a “wave” due to the quantum mechanical effects
- The electron “wavelength” is perturbed by the crystals periodic potential



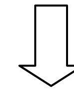
Carrier Movement Within the Crystal

m_n^* (electron effective mass)



$$F = -qE = m_n^* \frac{dv}{dt}$$

m_p^* (hole effective mass)

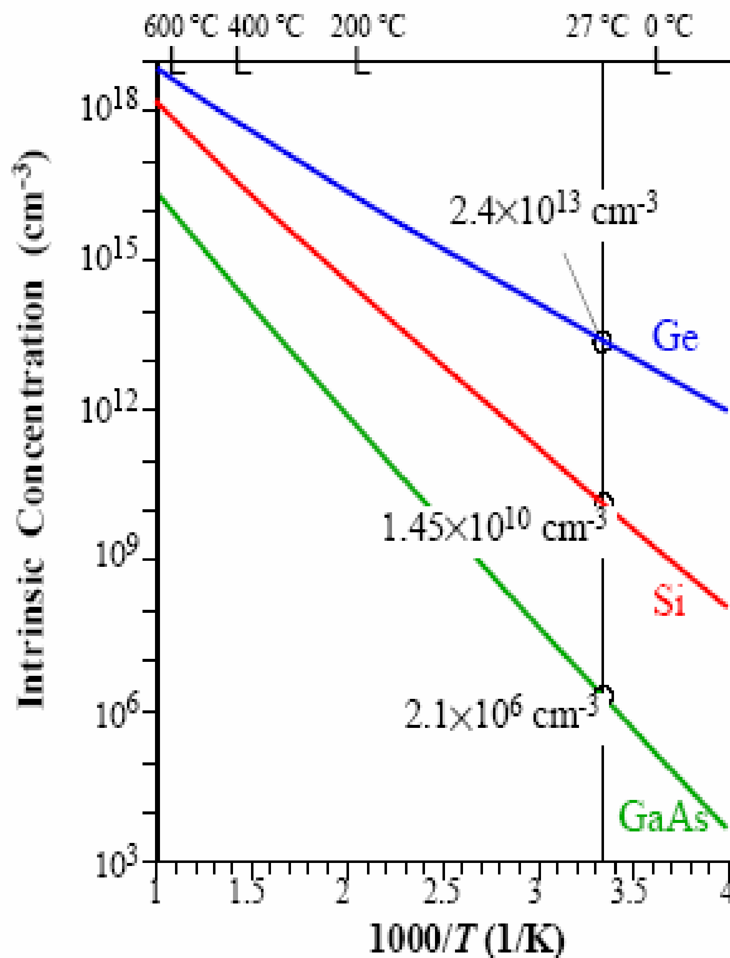


$$F = -qE = m_p^* \frac{dv}{dt}$$

Material	m_n^* / m_0	m_p^* / m_0
Si	1.18	0.81
Ge	0.55	0.36
GaAs	0.066	0.52

Density of States Effective Masses at 300 K

Intrinsic Carrier Concentration



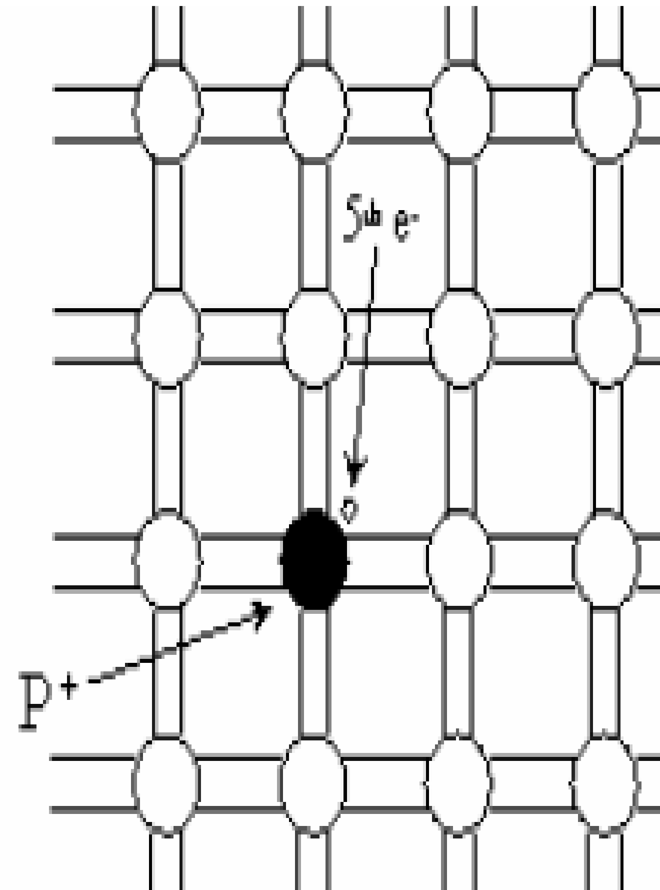
- Contains an insignificant concentration of impurity atoms
- Under the equilibrium conditions, for every electron is created, a hole is created also

  $$n = p = n_i$$

- As temperature is increased, the number of broken bonds (carriers) increases
- As the temperature is decreased, electrons do not receive enough energy to break a bond and remain in the valence band.

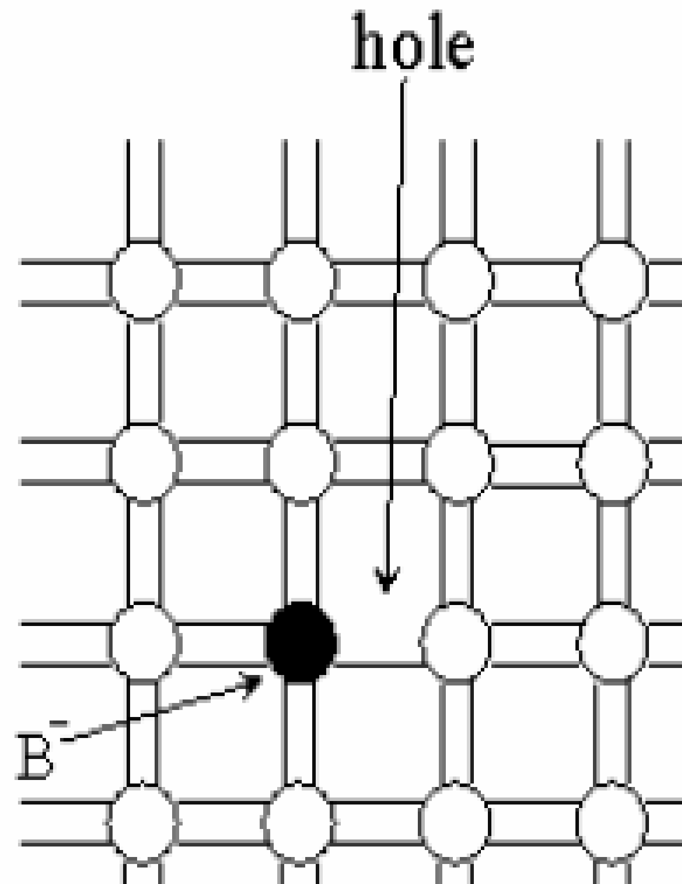
Extrinsic n-Type Semiconductor

- Donors (Group V): The 5th in a five valence electrons is readily freed to wander about the lattice at room temperature
- There is no room in the valence band so the extra electron becomes a carrier in the conduction band
- Does NOT increase the number of hole concentration



Extrinsic p-Type Semiconductor

- Acceptors (Group III) : three valence electrons readily accept an electron from a nearby Si-Si bond
- Completing its own bonding creates a hole that can wander about the lattice
- Does NOT increase the number of electron concentration

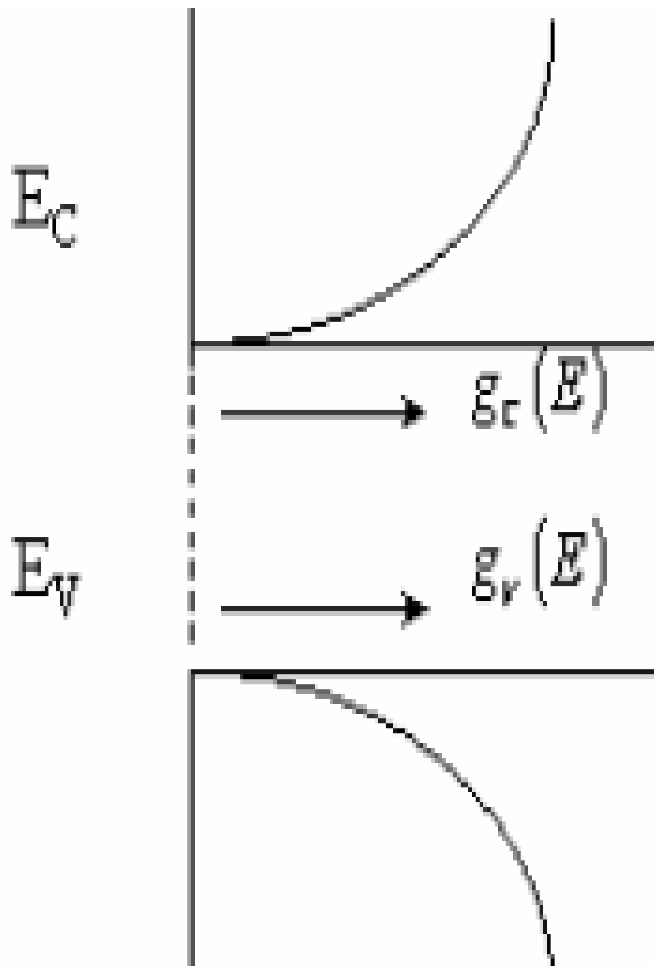


State and Carrier Distribution

- How the allowed energy states are distributed in energy
- How many allowable states were to be found at any given energy in the conduction and valence bands?
- Essential component in determining carrier distributions and concentration

- Density of States
- Fermi Function
- Dopant States

Density of States: $\left(\frac{Number \quad of \quad States}{cm^3}\right) / eV$



$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3}, \quad E \geq E_c$$

- $g_c(E)dE$ represents the number of conduction band states/ cm^3 lying in the energy range between E and E + dE

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}, \quad E \leq E_v$$

- $g_v(E)dE$ represents the number of valence band states/ cm^3 lying in the energy range between E and E + dE

$$g_c(E_c) = g_v(E_v) = 0$$

- How many of the states at the energy E will be filled with an electron

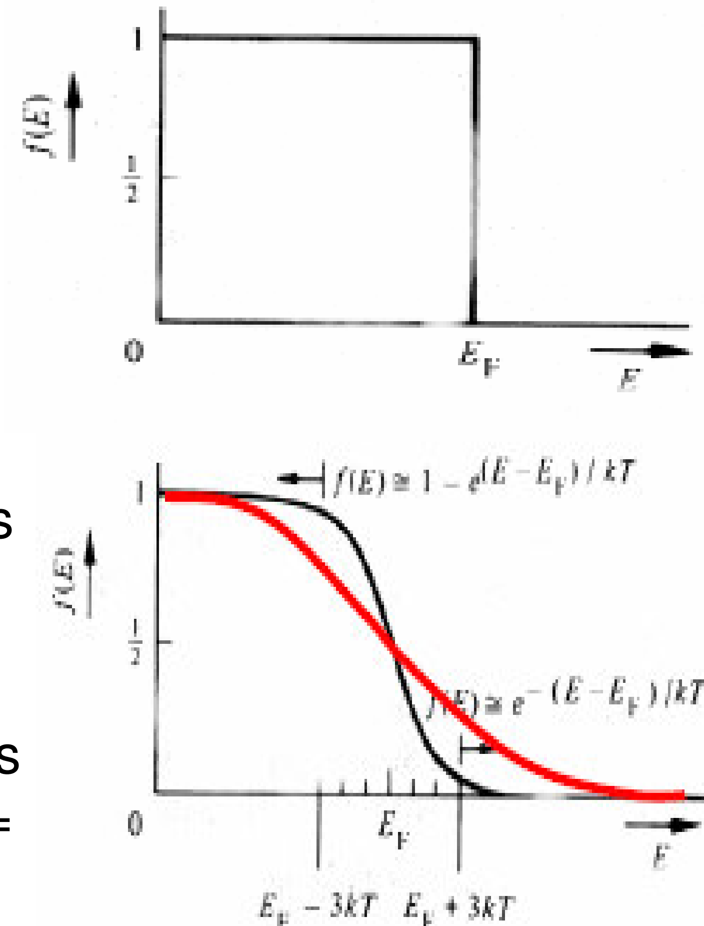
$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

- *f(E)*, under equilibrium conditions, the probability that an available state at an energy E will be occupied by an electron

- *1- f(E)*, under equilibrium conditions, the probability that an available state at an energy E will NOT be occupied by an electron

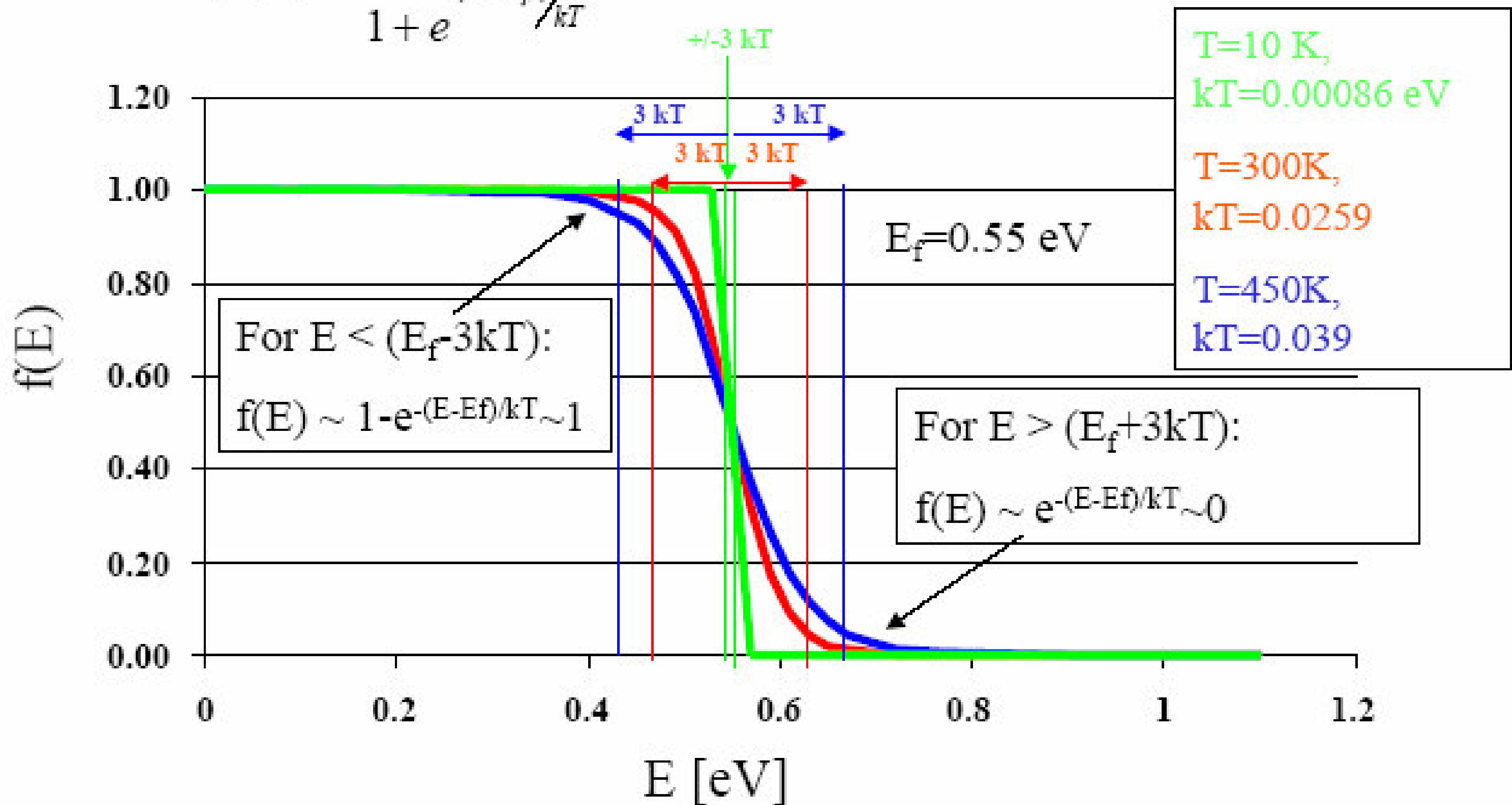
Fermi Function (II)

- If $E = E_F, f(E_F) = 1/2$
- If $E \geq E_F + 3k_BT$,
  $f(E) \approx \exp[(E_F - E)/k_BT]$
- If $E \leq E_F - 3k_BT$,
  $f(E) \approx 1 - \exp[(E - E_F)/k_BT]$
- At T=0K (above), No occupation of states above EF and complete occupation of states below E_F
- At T>0K (below), occupation probability is reduced with increasing energy $f(E=E_F)$ = 1/2 regardless of temperature.



- At higher temperatures, higher energy states can be occupied, leaving more lower energy states unoccupied (1-f(E))

Fermi Function (III)

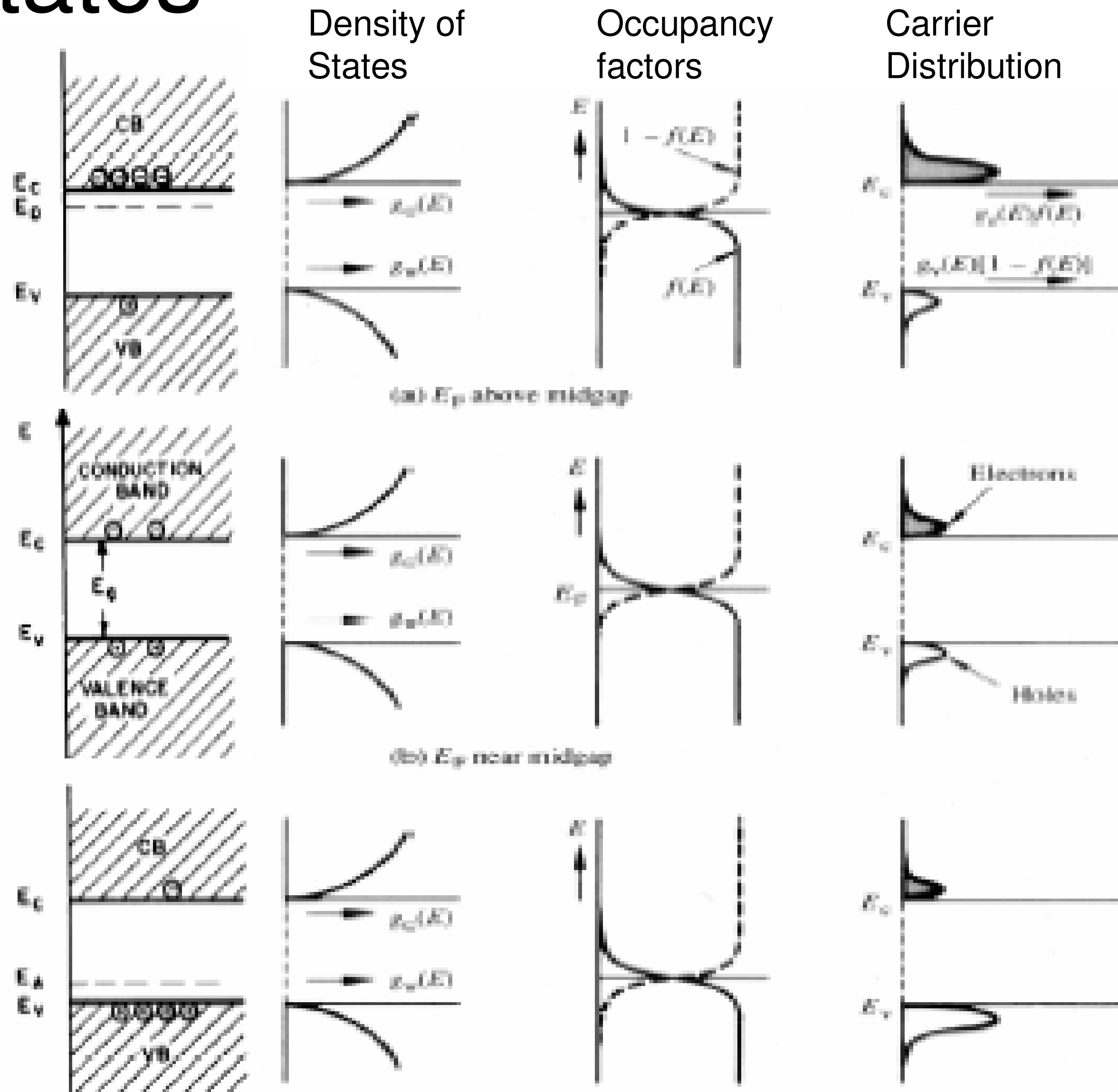


Dopant States

n-type: more electrons than Holes

Intrinsic: Equal number of electrons and holes

p-type: more holes than electrons



Equilibrium Carrier Concentration

- Formulas for n and p
- Degenerate vs. Non-degenerate Semiconductor
- Alternative Expressions for n and p
- n_i and the np Product
- Charge Neutrality Relationship
- Carrier Concentration Calculations
- Determination of E_F

Formulas for n and p

$$n = \int_{E_c}^{E_{top}} g_c(E) f(E) dE$$

$$p = \int_{E_{bottom}}^{E_v} g_v(E)[1 - f(E)] dE$$

$$n = \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3} \int_{E_c}^{E_{top}} \frac{\sqrt{(E - E_c)}}{1 + e^{(E - E_F)/kT}} dE$$

$$N_c = 2\left[\frac{2\pi m_n^* kT}{h^2}\right]^{3/2}, N_v = 2\left[\frac{2\pi m_p^* kT}{h^2}\right]^{3/2}$$

$$Letting \quad \eta = \frac{E - E_c}{kT} \quad and \quad \eta_c = \frac{E_F - E_c}{kT}$$

$$when \quad E = E_c \quad , \quad \eta = 0$$

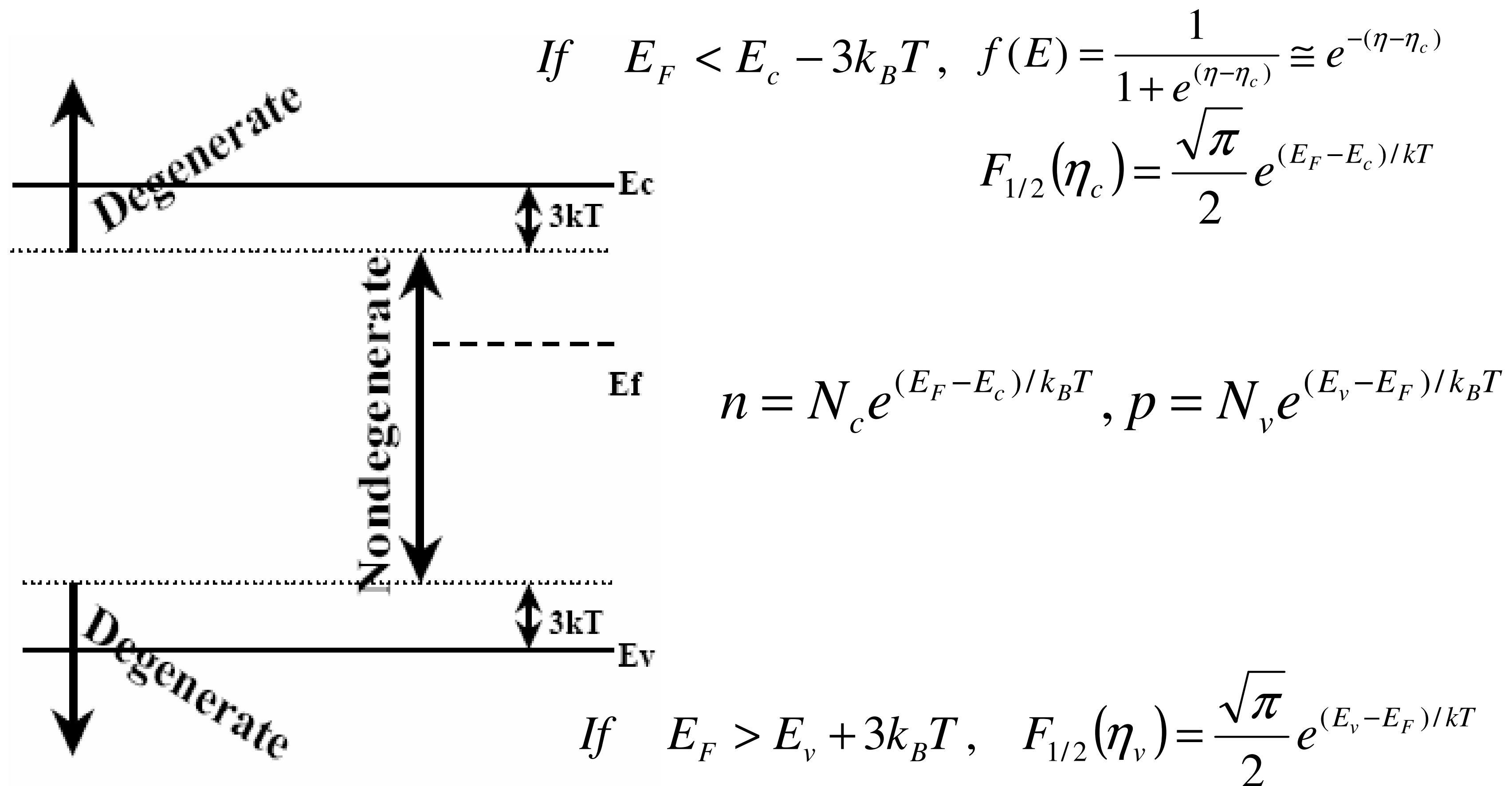
$$Let \quad E_{Top} \quad \to \quad \infty$$

$$n = \frac{m_n^* \sqrt{2m_n^*} (kT)^{3/2}}{\pi^2 \hbar^3} \underbrace{\int_{E_0}^{E_\infty} \frac{\eta^{1/2}}{1 + e^{(\eta - \eta_c)}} d\eta}_{F_{1/2}(\eta_c)}$$

$$n \equiv N_c \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c)$$

$$p \equiv N_v \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_v)$$

Degenerate vs. Non-degenerate Semiconductor



$$If \quad E_F < E_c - 3k_BT\,, \quad f(E) = \frac{1}{1+e^{(\eta-\eta_c)}} \cong e^{-(\eta-\eta_c)}$$

$$F_{1/2}(\eta_c) = \frac{\sqrt{\pi}}{2} e^{(E_F - E_c)/kT}$$

$$n = N_c e^{(E_F - E_c)/k_BT}, \; p = N_v e^{(E_v - E_F)/k_BT}$$

$$If \quad E_F > E_v + 3k_BT\,, \quad F_{1/2}(\eta_v) = \frac{\sqrt{\pi}}{2} e^{(E_v - E_F)/kT}$$

Alternative Expressions for n and p

$$n_0 = N_c e^{(E_F - E_c)/k_B T} \qquad p_0 = N_v e^{(E_v - E_F)/k_B T}$$

When $n = n_i$, $E_F = E_i$, then

$$n_i = N_c e^{(E_i - E_c)/k_B T} = N_v e^{(E_v - E_i)/k_B T}$$

$$N_c = n_i e^{(E_c - E_i)/k_B T} \quad , \quad N_v = n_i e^{(E_i - E_v)/k_B T}$$

$$n_0 = n_i e^{(E_c - E_i + E_F - E_c)/k_B T} = n_i e^{(E_F - E_i)/k_B T}$$

$$p_0 = n_i e^{(E_i - E_v + E_v - E_F)/k_B T} = n_i e^{(E_i - E_F)/k_B T}$$

$$\boxed{n_0 p_0 = n_i^2}$$

n_i and the np Product

$$n_i = N_c e^{(E_i - E_c)/k_B T} = N_v e^{(E_v - E_i)/k_B T}$$

$$n_i^2 = N_c N_v e^{-(E_c - E_v)/k_B T} = N_c N_v e^{-E_g/k_B T}$$

$$n_i = \sqrt{N_c N_v} e^{-E_g/2k_B T}$$

Charge Neutrality Relationship

- For uniformly doped semiconductor:

Charge must be balanced under equilibrium conditions otherwise charge would flow

$$\underbrace{qp - qn}_{} - \underbrace{qN_A^- + qN_D^+}_{} = 0$$

Thermally generated + dopant addition

assume ionization of all dopant sites

Carrier Concentration Calculations

$$(p - N_A) + (N_D - n) = 0$$

$$(\frac{n_i^2}{n} - N_A) + (N_D - n) = 0$$

$$n^2 - n(N_D - N_A) - n_i^2 = 0$$

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2}, p = \frac{N_A - N_D}{2} + \left[\left(\frac{N_A - N_D}{2} \right)^2 + n_i^2 \right]^{1/2}$$

$$np = n_i^2$$

Relationship for N_D^+ and N_A^-

$$N_A^- = \frac{N_A}{1 + g_A{}^{(E_A - E_F)/kT}}, N_D^+ = \frac{N_D}{1 + g_D{}^{(E_F - E_D)/kT}}$$

- The degeneracy factors account for the possibility of electrons with different spin, occupying the same energy level (no electron with the same quantum numbers can occupy the same state)

- Most semiconductor g_D=2 to account for the spin degeneracy at the donor sites

- g_A is 4 due to the above reason combined with the fact that there are actually 2 valence bands in most semiconductors Thus, 2 spins x 2 valance bands makes g_A=4

Determination of E_F (Intrinsic Material)

$$n = N_c e^{(E_i - E_c)/k_B T} = N_v e^{(E_v - E_i)/k_B T} = p$$

$$N_c e^{(E_i - E_c)/k_B T} = N_v e^{(E_v - E_i)/k_B T}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln\left(\frac{N_v}{N_c}\right)$$

$$N_c = 2\left[\frac{2\pi m_n^* kT}{h^2}\right]^{3/2}, N_v = 2\left[\frac{2\pi m_p^* kT}{h^2}\right]^{3/2} \Rightarrow \frac{N_v}{N_c} = \left(\frac{m_p^*}{m_n^*}\right)^{1/2}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{3k_B T}{4} \ln\left(\frac{m_p^*}{m_n^*}\right)$$

Determination of E_F (Extrinsic Material)

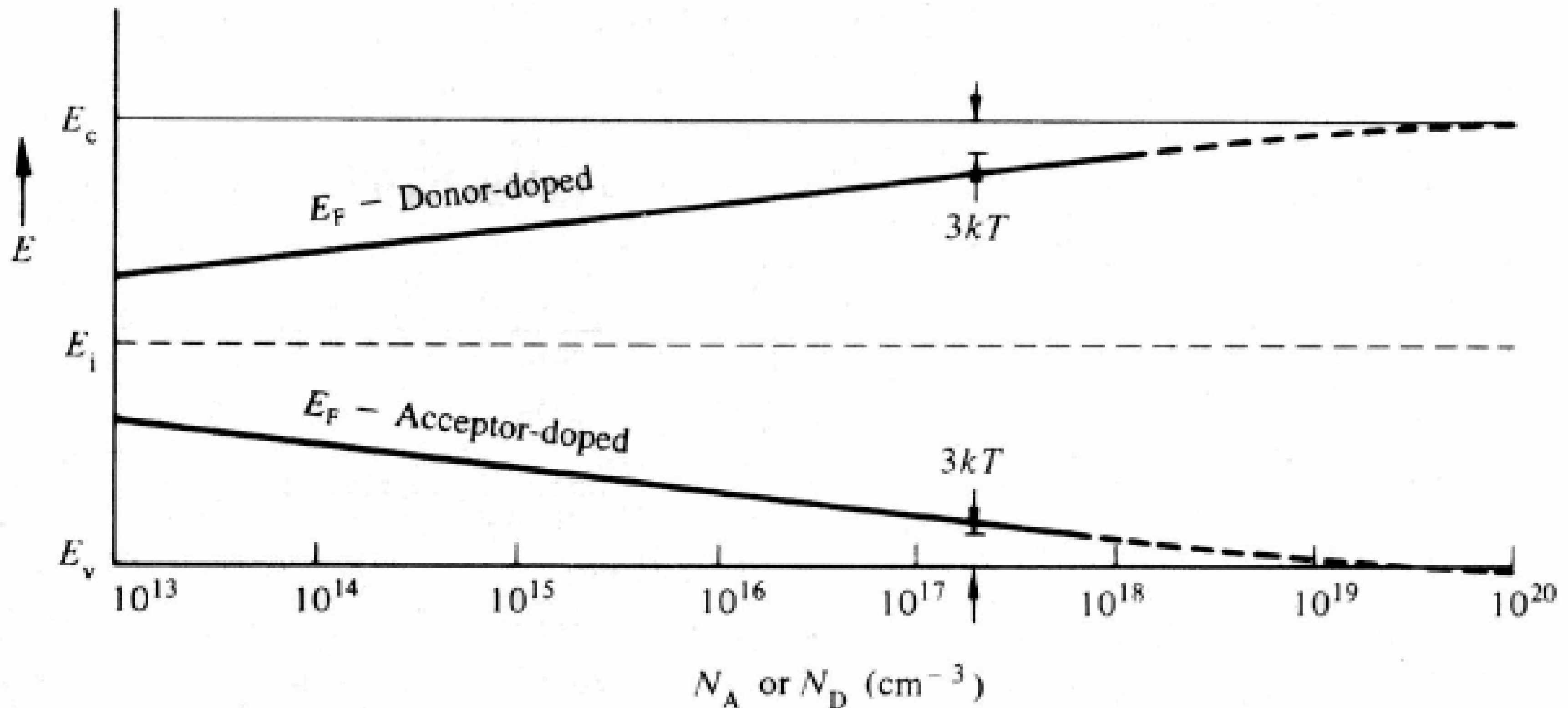
$$n = n_i e^{(E_F - E_i)/k_B T} \quad , \quad p = n_i e^{(E_i - E_F)/k_B T}$$

$$E_F - E_i = kT \ln\left(\frac{n}{n_i}\right) = -kT \ln\left(\frac{p}{n_i}\right)$$

$or \quad for \;\; N_D >> N_A \;\; and \;\; N_D >> n_i \qquad\qquad or \quad for \;\; N_A >> N_D \;\; and \;\; N_A >> n_i$

$$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right) \qquad\qquad E_F - E_i = -kT \ln\left(\frac{N_A}{n_i}\right)$$

Determination of E_F (Extrinsic Material)



Fermi level positioning in Si at room temperature as a function of the doping concentration. Solid E_F lines were established using Eq. (previous page)

Carrier Concentration for the Quantum Well Devices

- Density of States 3D vs. 2D
- Carrier Concentration – 2D
- Charge Neutrality

Density of States 3D vs. 2D

- 3D

$$g(E) = \frac{m\sqrt{2mE}}{\pi^2\hbar^3} \Rightarrow \text{Energy Dependent}$$

- 2D

$$g = \frac{m}{\pi^2\hbar^3 Lz} \Rightarrow \text{Energy Independent}$$

Carrier Concentration – 2D

$$n = \int_{E_c}^{E_{top}} g_c(E) f(E) dE$$

$$n = \frac{m_n^* kT}{\pi^2 \hbar^3 L_z} \sum_i \ln\left[1 + e^{(E_{FC} - E_i)/kT}\right]$$

$$p = \frac{m_p^* kT}{\pi^2 \hbar^3 L_z} \sum_i \ln\left[1 + e^{(E_{FV} - E_i)/kT}\right]$$

Charge Neutrality

$$P_{hh} + P_{lh} = N_{\Gamma} + N_{\mathrm{X}} + N_{L}$$

References

- Robert F. Pierret, *Semiconductor Fundamentals (VOLUME I),* Addison-Wesley Publishing Company, 1988, chapter 2
- Robert F. Pierret, *Advanced Semiconductor Fundamentals (VOLUME VI*), Addison-Wesley Publishing Company, 1987, chapter 4
- Ben G. Streetman and Sanjay Banerjee, *Solid State Electronic Devices*, Prentice Hall, Inc., 2000, chapter 3
- Peter S. Zory, JR., *Quantum Well Lasers*, Academic Press, 1993, chapter 1, 7

Effective Mass of Holes - 3D

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}$$

$$= \frac{m_{hh}^* \sqrt{2m_{hh}^*(E_v - E)}}{\pi^2 \hbar^3} + \frac{m_{lh}^* \sqrt{2m_{lh}^*(E_v - E)}}{\pi^2 \hbar^3}$$

$$(m_p^*)^{3/2} = (m_{hh}^*)^{3/2} + (m_{lh}^*)^{3/2}$$

$$\boxed{m_p^* = \left[(m_{hh}^*)^{3/2} + (m_{lh}^*)^{3/2}\right]^{2/3}}$$