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Rate Equations

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Overview

- Introduction to rate equations
- Derivation of rate equations
  - The phenomenological approach for the carrier density
  - The theoretical approach for the photon density
- Simulation & Application
  - Static Properties
  - Dynamic properties
- Summary & Discussion

Introduction

- Two coupled-differential equations for the carrier density (N) and the photon density (Np)
- Describing the variation of N and Np with the external force
- In the active region
- Charge neutrality

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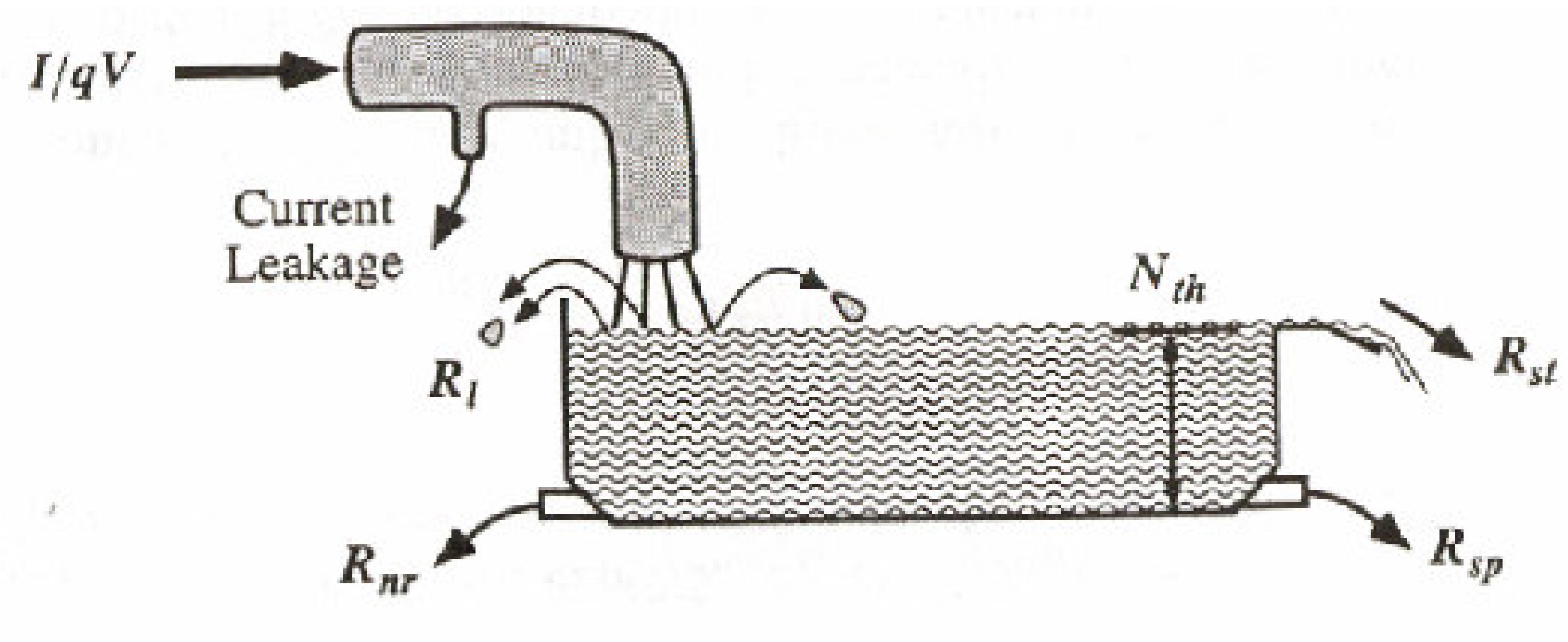
The carrier density



$$\frac{dN}{dt} = \frac{\eta_i I}{qV} - \frac{N}{\tau} - R_{st}$$

From reference [1]

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The photon density

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2(\varepsilon E)}{\partial t^2} = 0 \qquad (P = \varepsilon_0 \chi(\omega) E)$$

$$E(x,y,z,t) = \frac{1}{2}\varphi(x)\phi(y)\sin(kz)E(t)e^{-i\omega t} + c.c, k = \frac{n\Omega}{c}$$

$$\frac{dE}{dt} = \frac{i\bar{n}}{n_g}(\omega - \Omega)E + \frac{i\omega}{n_g}\left(\Gamma\Delta n_p + i\bar{\alpha}/(2k_0)\right)E$$

$$E = A\exp(-i\phi) \qquad \dot{A} = \frac{1}{2}v_g[\Gamma g - (\alpha_{\text{int}} + \alpha_m)]A$$

$$\dot{N}_p = \left(\Gamma v_g g - \frac{1}{\tau_p}\right)N_p + \Gamma\beta R_{sp}$$

From reference [2]

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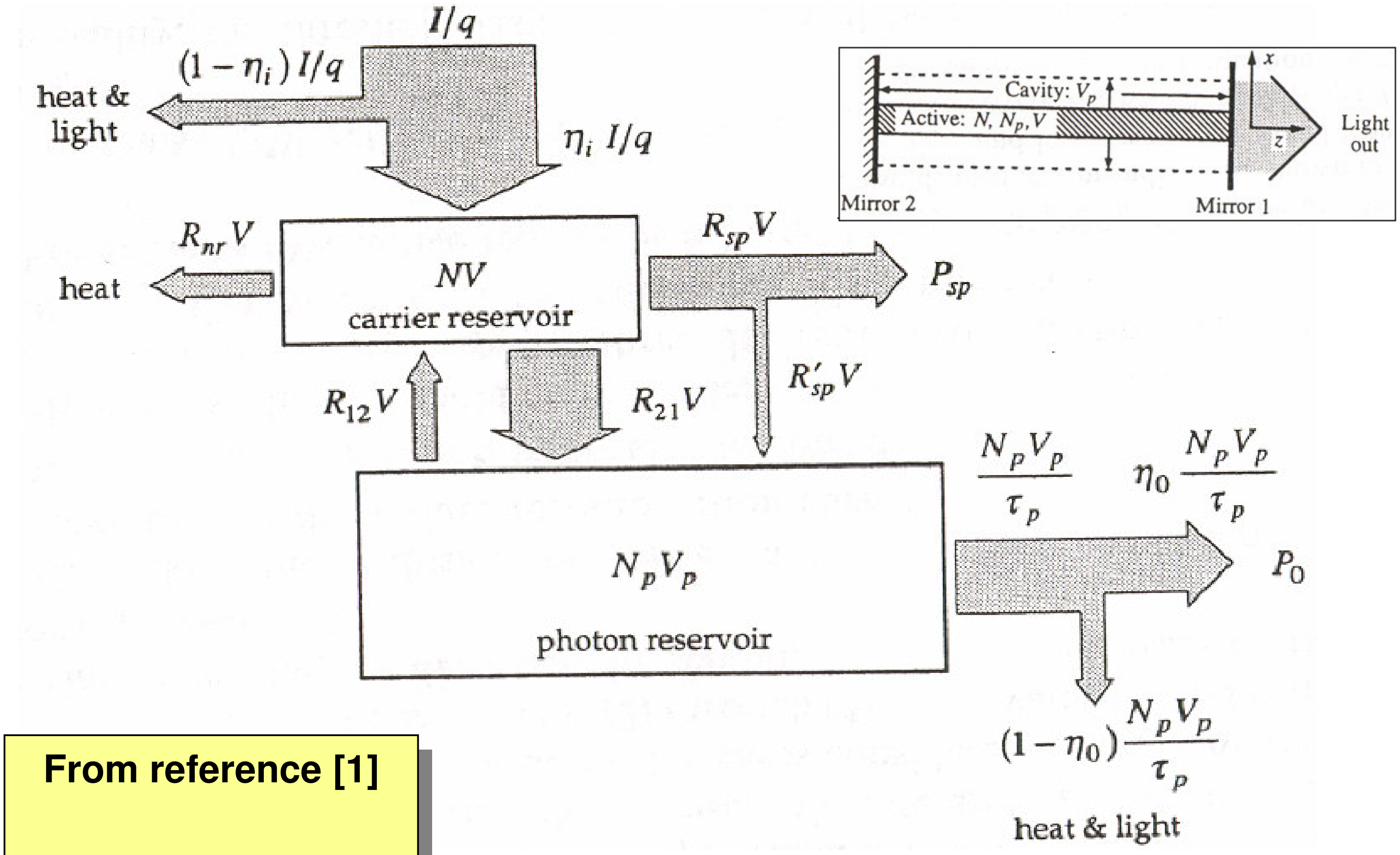
Reservoir Model



From reference [1]

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Rate equations

$$V\frac{dN}{dt}=\frac{\eta_i I}{q}-\left(R_{sp}+R_{nr}+R_l\right)V-\left(R_{21}-R_{12}\right)V$$

$$V\frac{dN_p}{dt}=\left(R_{21}-R_{12}\right)V-\frac{N_p V_p}{\tau_p}+\beta R_{sp}V$$

$$\frac{dN}{dt}=\frac{\eta_i I}{qV}-\left(R_{sp}+R_{nr}+R_l\right)-v_g g N_p$$

$$\frac{dN_p}{dt}=\left(\Gamma v_g g-\frac{1}{\tau_p}\right)N_p+\Gamma\beta R_{sp}$$

From reference [1]

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Static properties (I)

$$\frac{dN}{dt} = \frac{dN_p}{dt} = 0$$

$$N = \tau \, {}^{\eta_i I}\!/_{qV} \qquad N_p = \Gamma \beta R_{sp} \tau_p$$

$$P_{LD}(I < I_{th}) = \eta_i \eta_r \frac{\alpha_m}{\alpha_m + \alpha_{\text{int}}} \beta \frac{h\nu}{q} I$$

$$P_{LED} = \eta_i \eta_r \eta_c \frac{h\nu}{q} I$$

From reference [1]

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Static properties (II)

$$\frac{dN}{dt} = \frac{dN_p}{dt} = 0$$

$$N = N_{th} = \tau(AN_{th} + BN_{th}^2 + CN_{th}^3)$$

$$N_P = \frac{\eta_i (I - I_{th})}{q v_g g_{th} V}$$

$$g = \Gamma g_{th} = \alpha_{\text{int}} + \alpha_m = \frac{1}{v_g \tau_p}$$

$$P_{LD}(I > I_{th}) = N_p V_p h \nu \alpha_m v_g$$

$$= \eta_d \frac{h\nu}{q}(I - I_{th})$$

From reference [1]

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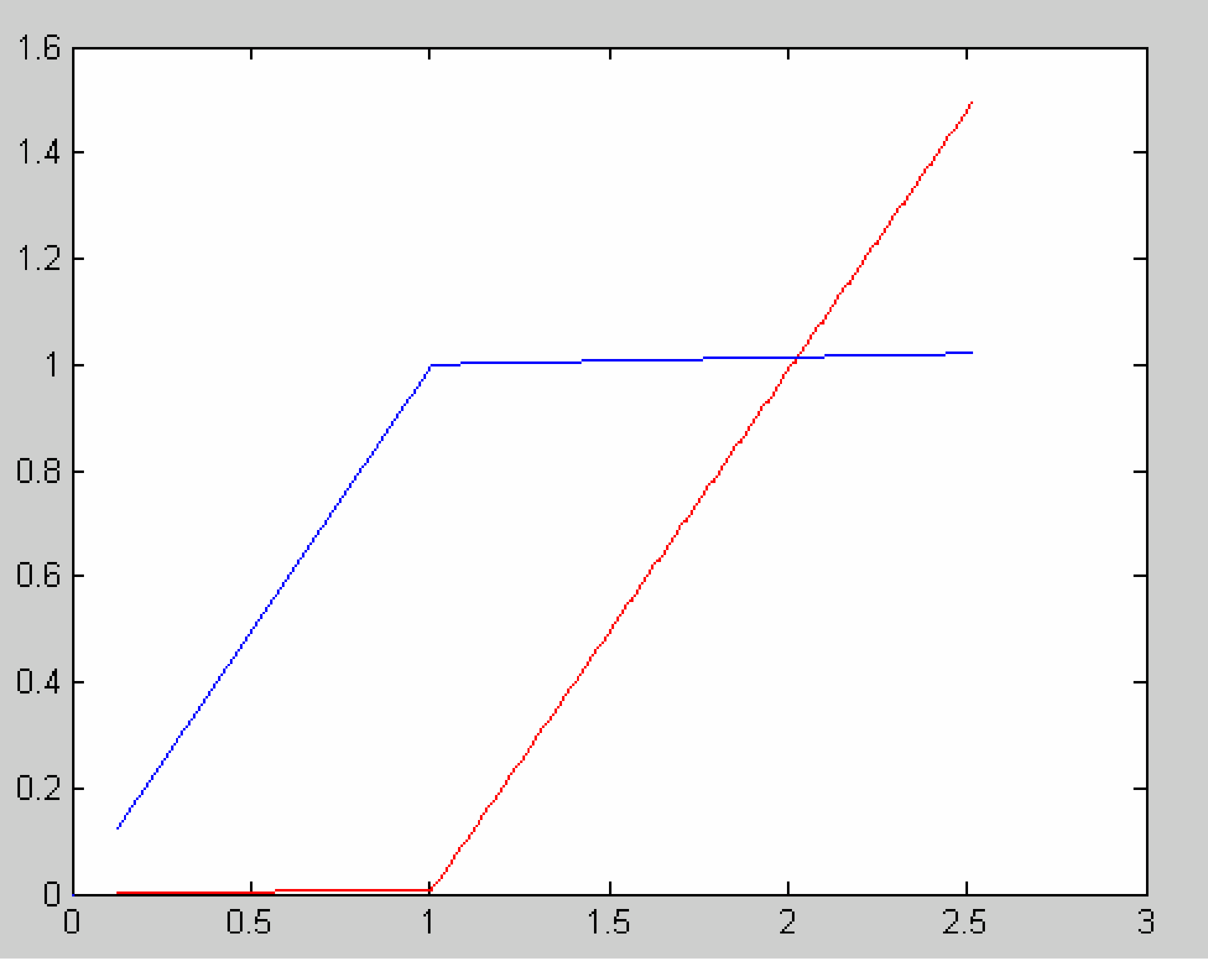
Static properties (III)



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Dynamic properties (I)

$$\frac{d}{dN}\left(\frac{dN}{dt}\right)=\frac{\eta_i}{qV}\frac{dI}{dN}-\left(\frac{dR_{sp}}{dN}+\frac{dR_{nr}}{dN}\right)-\left(v_g g\frac{dN_p}{dN}+v_g N_p\frac{dg}{dN}\right)$$

$$\frac{d}{dN}\left(\frac{dN_p}{dt}\right)=\left(\Gamma v_g g-\frac{1}{\tau_p}\right)\frac{dN_p}{dN}+N_p\Gamma v_g\frac{dg}{dN}+\Gamma\frac{d(\beta R_{sp})}{dN}$$

$$\frac{d}{dt}\begin{bmatrix} dN \\ dN_p \end{bmatrix}=\begin{bmatrix} -\gamma_{NN} & -\gamma_{NP} \\ \gamma_{PN} & -\gamma_{PP} \end{bmatrix}\begin{bmatrix} dN \\ dN_p \end{bmatrix}+\frac{\eta_i}{qV}\begin{bmatrix} dI \\ 0 \end{bmatrix}$$

From reference [1]

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Dynamic properties (II)

$$dN(t) = N_1 e^{j\omega t} \qquad dN_p(t) = N_{p1} e^{j\omega t}$$

$$dI(t) = I_1 e^{j\omega t} \qquad d/dt -> j\omega$$

$$\begin{bmatrix} \gamma_{NN} + j\omega & \gamma_{NP} \\ -\gamma_{PN} & \gamma_{PP} + j\omega \end{bmatrix} \begin{bmatrix} N_1 \\ N_{p1} \end{bmatrix} = \frac{\eta_i I_1}{qV} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H(\omega) = \frac{\omega_R^2}{\Delta} = \frac{\omega_R^2}{\omega_R^2 - \omega^2 - j\omega\gamma}$$

$$N_1 = \frac{\eta_i I_1}{qV} * \frac{\gamma_{PP} + j\omega}{\omega_R^2} H(\omega)$$

$$N_{p1} = \frac{\eta_i I_1}{qV} * \frac{\gamma_{PN}}{\omega_R^2} H(\omega)$$

From reference [1]

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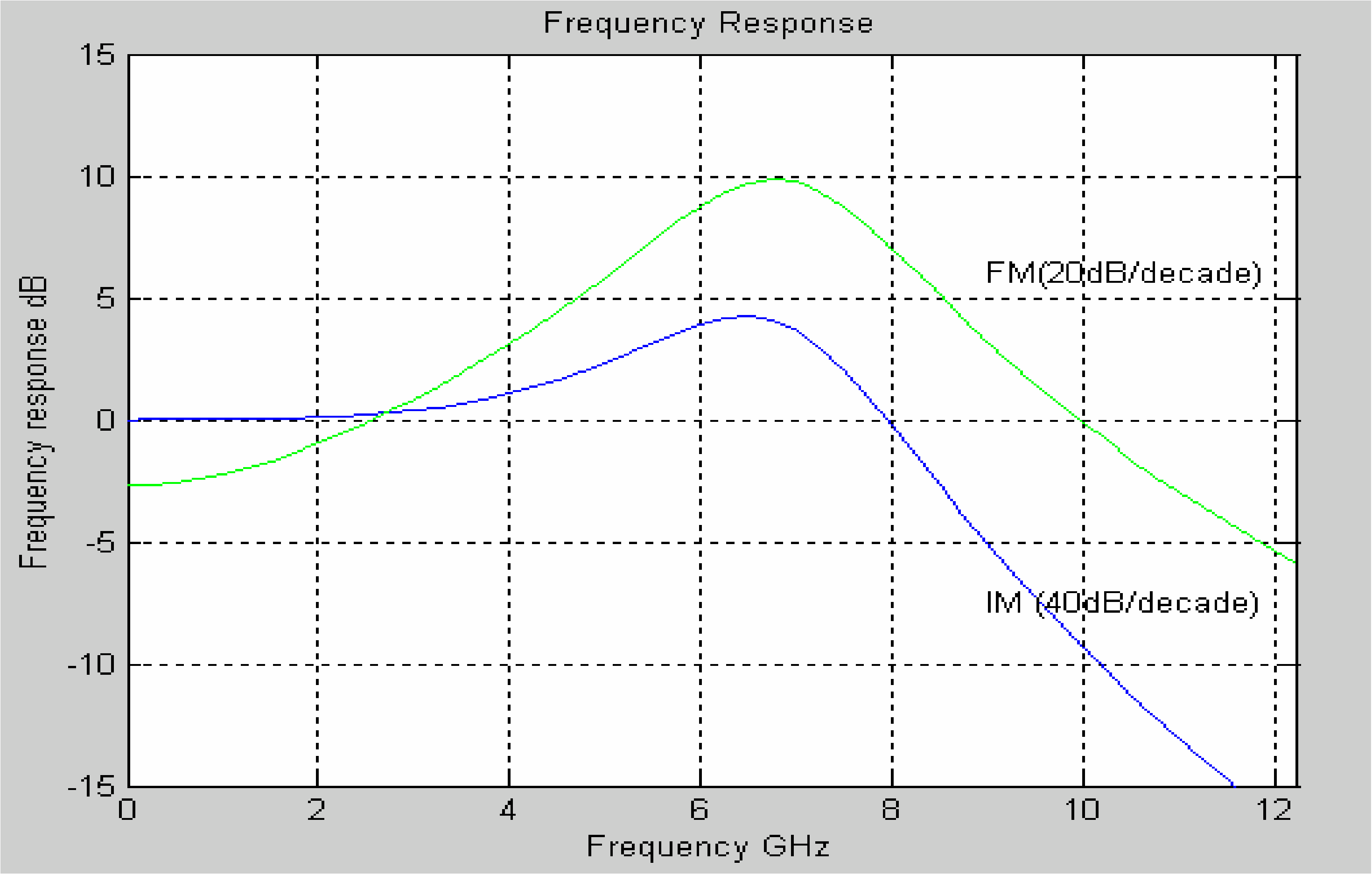
Small-signal (I)



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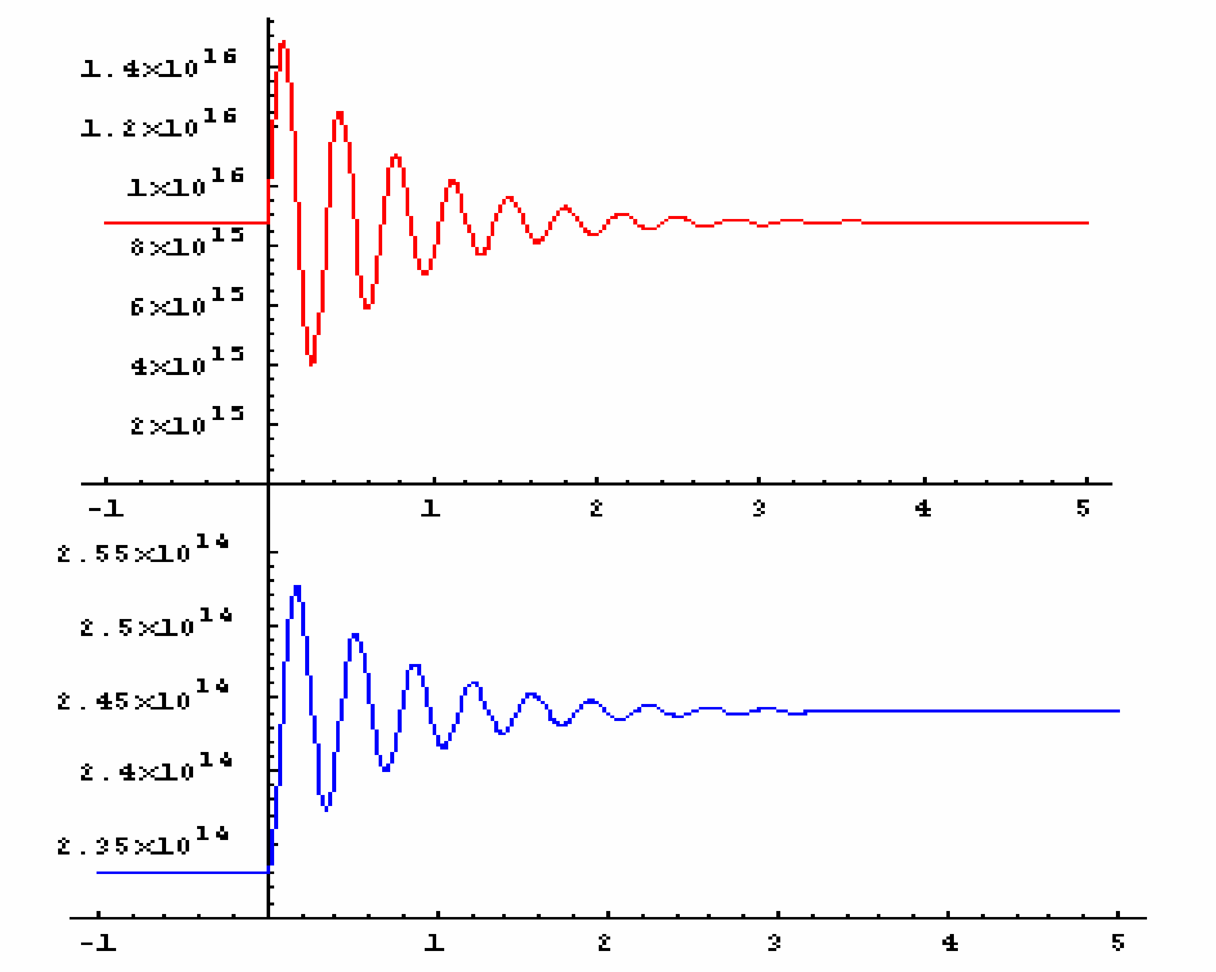
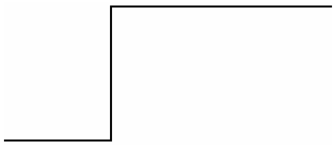
Small-signal (II)





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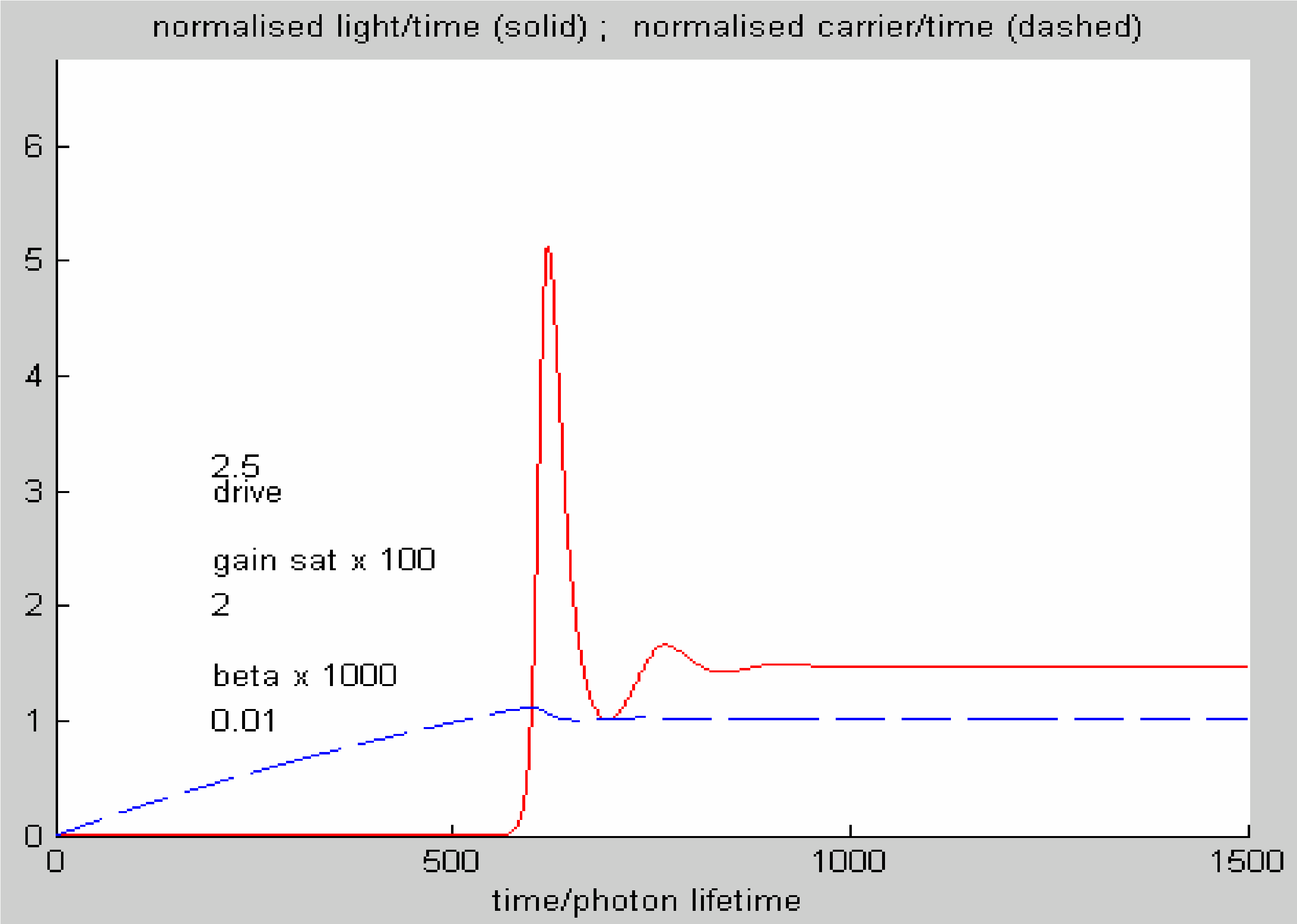
Large-signal response



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Summary & Discussion

- Physical meaning of each parameter.
- Study both the static and dynamic properties
- Analyze the noise characteristics
- Extend to include the phase, carriers in SCH regions, and the carrier energy.

References

1. L.A. Coldren, S.Q. Corzine, “Diode Lasers and Photonic Integrated Circuits”, John Wiley & Sons, Inc., 1995
2. G.P.Agrawal, N.K.Dutta, “Long-wavelength Semiconductor Lasers”, Van Nostrand Reinhold Company, New York, 1986