Relaxation Frequency

September 12, 2003

Joy Chuang

Today's Outline

- What is relaxation oscillation frequency (f_R)
- Derivation of f_R
- Numerical example of f_R
- Phenomena related to f_R under modulation response: frequency chirping, gain compression, linewidth, transport effects, noise
- Different structure (bulk, QW or strained QW)

Review

- Population inversion:
  - The excited atom population > ground state density; start at transparency
  - In semiconductor: we need to consider quasi-Fermi level for electrons and holes
  - $E_{fc} - E_{fv} > E_g$ to lase
- Recombination R(N) :
  - Radiative and nonradiative recombination which are usually dependent on carrier density (N)
  - will be considered together as a constant
- Carrier lifetime :
  - Recombination rate (R(N):#/s):

    $R(N)=N/\tau$ τ: average carrier lifetime
  - In fact, stimulate lifetime < spontaneous lifetime

Review 2

- Photon (cavity) lifetime:
  - For a Fabry-Perot laser, both end mirror has the same reflection factor
  - For the intensity in the cavity, it will decay as $dI/dt=-I/\tau_p \Rightarrow I(t)=I_0 e^{-t/\tau p}$
  - Combine with the intensity at threshold from round trip, we get the photon lifetimeτ_p

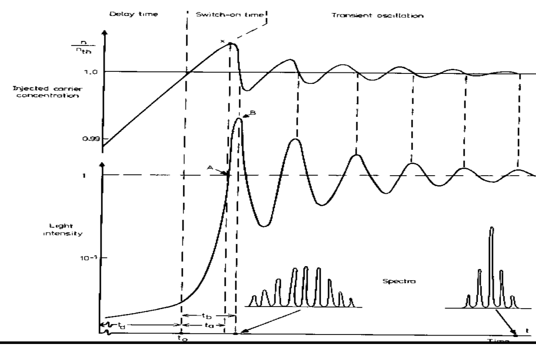
$$\boxed{\frac{1}{\tau_p} = \frac{c}{n}\left(\alpha + \frac{1}{L}\ln\frac{1}{R}\right)}$$

C: light velocity(cm/s) n: refractive index
α: loss of the material(1/cm) L: cavity length(cm)
R: power reflection fraction of mirror

- Usually, for semiconductor lasers, the cavity lifetime is about a few ps and the carrier lifetime at threshold is about a few ns.

Relaxation Oscillation Frequency

- Qualitatively description:
  - A desirable feature of a laser is the constant amplitude, shown in the following figure. Right after the laser turns on, the amplitude varies for a while and then gets stabilized to a constant. We call the frequency before the laser gets stabilized the relaxation oscillation frequency
  - The relaxation oscillation frequency is the frequency seen when the system relaxes close to its stable state.



Single Mode Rate Equations for Semiconductor Lasers

- Carrier rate equation:

$$\boxed{\frac{dN}{dt} = \frac{\eta_i I}{qV} - \frac{N}{\tau} - v_g g P} \qquad \text{-----------(1)}$$

N: carrier density(#/cm^3)
V: Volume of the active region (cm^3)
I: injected current (mA)
V_g: photon group velocity,c/n* (cm/s)

η_i: internal quantum efficiency
q: electron
τ: carrier lifetime(1/s)
g: optical gain (cm^{-1})

- Photon rate equation:

$$\boxed{\frac{dP}{dt} = \Gamma v_g g P + \Gamma \beta_{sp} R_{sp} - \frac{P}{\tau_p}} \qquad \text{-----------(2)}$$

P: photon density(#/cm^3)
Γ: confinement factor
β_{sp}: spontaneous emission factor
R_{sp}: spontaneous spontaneous emission rate (#/s-cm^3)
τ_P: cavity lifetime (1/s)

Derivation of f_R

- The optical gain (g) in (1) and (2) can be approximated by a straight line: $g \cong a(N - N_{tr})$, a:differential gain, N_{tr}: transparency carrier density
- Consider the application of an above-threshold dc current (I_0) with a small ac current (I_1) to a diode laser:

  $I = I_0 + I_1 e^{j\omega t}$, $N = N_0 + N_1 e^{j\omega t}$, $P = P_0 + P_1 e^{j\omega t}$ -----(3)
- Plugging (3) to (1) and (2), ignore second harmonic terms involve $e^{j2\omega t}$ and divide an $e^{j\omega t}$; the dc components can be set to 0

$$j\omega N_1 = \frac{\eta_i I_1}{qV} - \frac{N_1}{\tau} - \frac{P_1}{\Gamma \tau_p} - v_g a N_1 P_0 \qquad \text{-----------(4)}$$

$$j\omega P_1 = \Gamma v_g a N_1 P_0 \qquad \text{-----------(5)}$$

- N_1 and P_1 depends on each other

Derivation of f_R 2

- Resonance:
  - N_1 increase and become positive with time and P_1 increase from (5)
  - The increase of P_1 from stimulated emission decrease N_1 from (4)
  - N_1 decrease and become negative and P_1 decrease
  - Once P_1 become negative, it produces an increase in N_1
  - Then the cycle repeat
- Find f_R:
  - We mutiply (4) and (5), ignore all of the terms, but the stimulated term

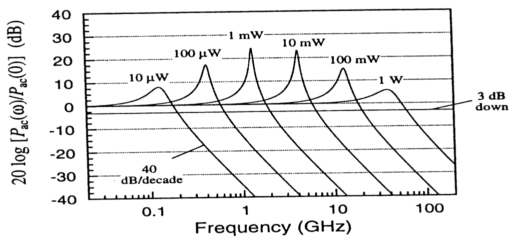
$$\omega_R^2 = \frac{v_g a P_0}{\tau_p} \qquad \text{-----------(6)} \qquad \Rightarrow \qquad f_R = \frac{\omega_R}{2\pi}$$

  - f_R is the relaxation resonance frequency
  - f_R related to square root of differential gain, average photon density in the cavity and photon lifetime

Relation of Output Power and f_R

- Express f_R in terms of current:
  - P_0 as steady state photon density: $P_0 = \frac{\eta_i (I - I_{th})}{q v_g g_{th} V}$
  - Threshold gain g_{th} : $g_{th} = \frac{1}{\Gamma v_g \tau_p}$
  - Use the above two equation and (6), we get $\omega_R = \left[\frac{\Gamma v_g a}{qV} \eta_i (I - I_{th}) \right]^{1/2}$
- Output power(OP) with relaxation frequency, the modulation response

$$\frac{OP_{ac}(\omega)}{I_1(\omega)} = \frac{\eta_d h\nu / q}{1-(\omega/\omega_R)^2 + j(\omega/\omega_R)[\omega_R \tau_P + 1/\omega_R \tau]}$$



Typical Parameter Values for a 1.3um InGaAsP Buried-heterostructure Laser

Parameter	Value	Parameter	Value
Cavity length (L)	250 um	Gain constant	2.5×10^{-6} cm^2
Active-region width (w)	2 um	Carrier density at transparency	1×10^{18} cm^{-3}
Active-layer thickness (d)	0.2 um	Nonradiative recombination rate(A_{nr})	1×10^{8} s^{-1}
Confinement factor(Γ)	0.3	Radiative redcombination coefficient (B)	1×10^{-10} cm^3/s
Effective mode index	3.4	Auger recombination coefficient (C)	3×10^{-29} cm^6/s
Line-width enhancement factor	5	Threshold carrier population	2.14×10^{8}
Group refractive index(n_g)	4	Threshold current	15.8 mA
Facet loss	45 cm^{-1}	Carrier lifetime at threshold (τ)	2.2 ns
Internal loss	40 cm^{-1}	Photon lifetime(τ_P)	1.6 ps

Numerical Example

- If we want to calculate the relaxation frequency with the parameters provided from the previous slide at 25mA:

We rewrite the relaxation frequency in terms of carrier lifetime and photon lifetime:

$$\boxed{f_R = \frac{1}{2\pi}\left[\frac{1}{\tau\tau_p}\left(\frac{I}{I_{th}}-1\right)\right]^{1/2}}$$

Plug in the values:

$$f_R = \frac{1}{2\pi}\left[\frac{1}{2.2\times10^{-9}\times1.6\times10^{-12}}\left(\frac{25}{15.8}-1\right)\right]^{1/2}$$

$$f_R = 2.0\times10^{9}\,Hz$$

About Relaxation Frequency

- Direct modulation:
  - The transient and modulation responses rely on relaxation frequency.
  - The modulation efficiency drops for modulation frequencies > f_R
  - The resonance manifests itself as a transient oscillation during laser switching and also as an enhancement of the modulation response to a small sinusoidal current in the relevant frequency range.
- Suppression of f_R peak:
  - Spontaneous emission, carrier diffusion and spectral-hole burning
- Other things related to modulation response:
  - Line-width enhancement factor (for interdependence between AM and FM ?)
  - Gain suppression
  - Frequency chirping
  - Ultrashort pulse generation
  - Transport effects

Gain Suppression

- Nonlinear-gain effects:
  - Optical gain in the cavity by an optical wave can lead to lower the optical intensity. This could be caused by gain saturation and suppression.
  - The total electron density is constant and the reduction of the density of resonant carriers (including electrons and holes).
  - A consequence of a small but finite intraband scattering time; Inhomogeneous broadening
  - Limit the modulation bandwidth
  - Instead of $G=a(N-N_{tr})$, we have

$$\boxed{G(N)=\frac{G_0+G^{'}(N-N_0)}{1+\varepsilon P}}$$

$G_0=G(N_0)$, $G^{'}=(\partial G/\partial N)_{N=N0}$,

ε: gain suppression coefficient usually evaluated experimentally

P:photon density

  - $1+\varepsilon P$ accounts for nonlinear gain saturation when photon density is high
- Use this nonlinear gain with the rate equation, we can get

$$\boxed{(\omega_R)_{\max}=\frac{a}{\sqrt{2\varepsilon}}}$$

Frequency Chirping

- Frequency chirping:
  - During modulation the current affects the optical frequency and cause the frequency shift. Periodic oscillations of the N and P response to modulation of the current.
  - Consider the phase change, we get the frequency chirp($\delta\nu$):

$$\boxed{\delta\nu(t)=\delta\nu_0\sin(\omega_m+\theta_c)}$$

$\delta\nu_0$: the maximum shift and peak at f_R

ω_m: modulation frequency

  - If we look at the maximum frequency chirp

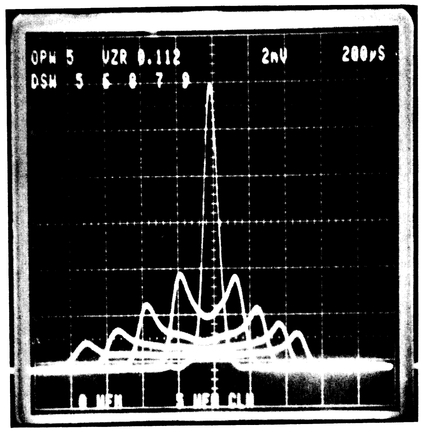
$$\boxed{\delta\nu_0\propto\frac{\beta_c I_P}{4\pi qGP}f(\omega_m,P,R_{sp},G,\omega_R,G^{'})}$$

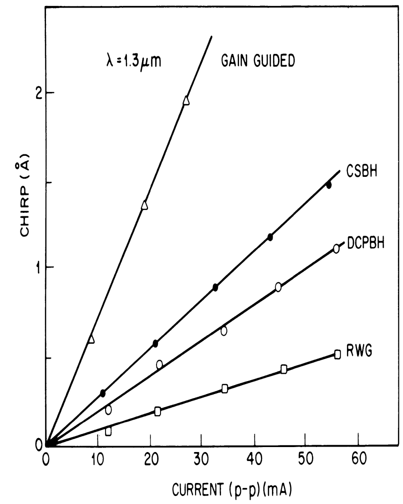
β_c: line enhancement factor

I_P:peak value of the modulation current

- Several factors involve with frequency chirp
  - The increase of amplitude I_P broaden the optical spectrum as figure. Asymmetric and double-peak profile.
  - For gain-guided R_{sp} is larger than index-guided.
  - β_c and τ_P, which is related to G, can be accounted for index-guided.

Frequency Chirping 2



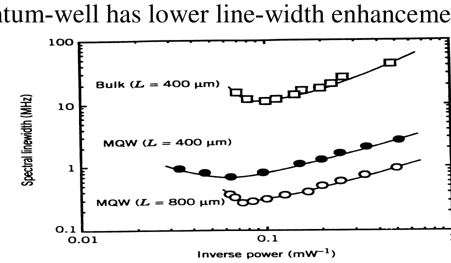


Quantum Well

- How does quantum well help the relaxation frequency?
  - Quantum well lasers have a high differential gain which gives high modulation bandwidth
  - Strained quantum well laser gives more modulation bandwidth

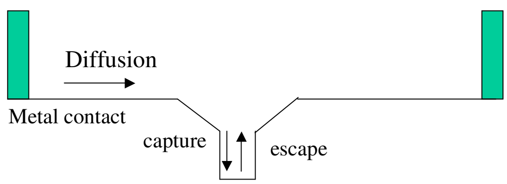
$$\boxed{G' = \frac{\xi}{\rho_c + \rho_v}\left(\frac{\rho_c}{(e^{E_{fc}/kT}+1)^{1/D}} + \frac{\rho_v}{e^{E_{fv}/kT}+1}\right)}$$

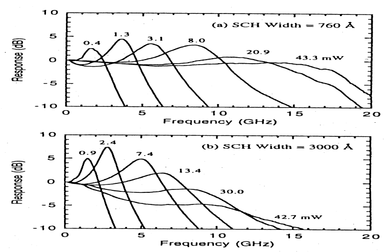
  - The differential gain can be expressed as above. For regular QW, D is about 10. D is about 2 for strained QW. D is the ratio of the hole to the electron densities of states.
  - Multiquantum-well has lower line-width enhancement factor



Transport Effects

- For QW lasers, the transport effects need to be considered.
  - A finite capture and escape time of the carriers between the SCH region and quantum wells.
  - Carrier diffusion across SCH ⇒ capture, thermionic emission ⇒ escape
  - Use three rate equations. The carrier equation is replaced by tow equations. One is for barrier regions and the other is in the active region.
  - These time are small compared to the consequence of direct modulation.
  - But, they can contribute to differential gain. The transport across SCH reduce the differential gain and decrease the relaxation frequency. So, wider SCH region has lower relaxation frequency.





Linewidth Enhancement

- Linewidth
  - From intensity variation(complex dielectric constant)
  - Spontaneous emission cause intensity and phase fluctuation
  - Physical model

$$\boxed{\Delta f = \frac{\hbar w v_g \alpha_m R_{sp}}{8\pi P_o}(1+\alpha_H{}^2)}$$

P_0 : optical output power
α_H: line width enhancement factor

  - Linewidth enhancement factor relate to change of the refractive index per injected carriers vs. the differential gain

$$\boxed{\alpha_H = -\frac{4\pi}{\lambda}\frac{dn/dN}{dg/dN}}$$

  - α_H: tensile < compressive < strained < bulk
  - α_H affects SMSR, RIN and FM chirping
- Improvement to minimize the enhancement factor:
  - Strained quantum well active region
  - Minimize the carrier induced index change.

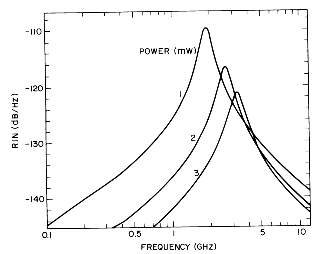
Noise

- All of the fluctuations base on the quantum nature of the lasing process itself.
- Intensity noise:
  - Reaches its peak around threshold
  - A peak near the relaxation-oscillation frequency
  - Characterized by relative intensity noise (RIN), which decrease with an increase in laser power.
- Phase noise:
  - Spectral boradening for each longitudinal mode
  - Line width

$$S_{v_{FW}} = \frac{1}{2\pi}\frac{\Gamma R_{sp}}{4\pi P}\left(1+\alpha_H^{2}\left|H(w)\right|^{2}\right)$$

From spontaneous emission phase fluctuation.

From carrier noise.



References

- L.A. COLDREN, S.Q. CORZINE, Diode Lasers and Photonic Integrated Circuits, Chs. 2 and 5, John Wiley & Sons, Inc., 1995
- Peter S.Zory, Quantum Well Lasers, Chs. 5, Academic Press, Inc., 1993
- G.P.Agrawal, N.K.Dutta, Long-wavelength Semiconductor Lasers, Chs.6, Van Nostrand Reinhold Company, New York, 1986
- Shun Lien Chuang, Physics of Optoelectronic Devices, Chs.11, John Wiley & Sons, Inc., 1995
- Tso-Min Chou, Quantum Well Lasers lecture notes, Dallas, Spring 2002