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Direct measurement of wall shear stress in a reattaching flow with a photonic sensor

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Abstract
Wall shear stress measurements are carried out in a planar backward-facing step flow using a micro-optical sensor. The sensor is essentially a floating element system and measures the shear stress directly. The transduction method to measure the floating element deflection is based on the whispering gallery optical mode (WGM) shifts of a dielectric microsphere. This method is capable of measuring floating element displacements of the order of a nanometer. The floating element surface is circular with a diameter of \( \sim 960 \mu m \), which is part of a beam that is in contact with the dielectric microsphere. The sensor is calibrated for shear stress as well as pressure sensitivity yielding 7.3 pm Pa\(^{-1}\) and 0.0236 pm Pa\(^{-1}\) for shear stress and pressure sensitivity, respectively. Hence, the contribution by the wall pressure is less than two orders of magnitude smaller than that of shear stress. Measurements are made for a Reynolds number range of 2000–5000 extending to 18 step heights from the step face. The results are in good agreement with those of earlier reports. An analysis is also carried out to evaluate the performance of the WGM sensor including measurement sensitivity and bandwidth.

Keywords: whispering gallery mode, morphology-dependent resonances, wall shear stress, microsensor, separating flow, reattaching flow

(Some figures may appear in colour only in the online journal)

1. Introduction
Accurate measurement of wall shear stress in wall bounded flows still remains as a challenge in fluid dynamics. The capability to measure temporally- and spatially-resolved wall shear stress plays a crucial role for a broad range of fluid mechanics applications ranging from flow control to smart designs for drag reduction. Currently, there still is a need for further developments in low-noise, high resolution sensors applicable to a wide range of flows [1–3]. Most of the currently used sensors are based on indirect measurements where the wall shear stress is determined from the measurement of another flow property. Some examples of these are hot-wire/film-based anemometry [4] or heat flux gages [5], surface acoustic wave sensors [6] and laser-based velocity sensors [7–11]. Another frequently used method is surface oil film interferometry that provides time averaged measurement [12, 13]. MEMS-based sensors have also been proposed for indirect measurements (thermal [14, 15], micropillar [16–21] and optical wall shear sensors [22]). Of these, thermal sensors offer simplicity in fabrication but are based on heat transfer analogy, and their calibration can be difficult. Micropillar/artificial hair type sensors are arrays of vertical cantilevers that are attached to a region of the measurement surface, and the shear stress is calculated from their deflection [16–21]. A promising sensor type is the laser Doppler anemometry-based optical MEMS (MOEMS). However, its application can be challenging in high Reynolds number flows due to the difficulty of generating sufficiently small measurement volumes.

The most commonly used direct shear stress sensors employ a floating element that is flush with the wall, and the shear stress is determined from the deflection of the floating element. Different transduction methods are used to measure the floating element deflection, such as capacitive element [23–27], piezoresistive [28, 29], differential optical shutter and fringe moiré [30] techniques. A number of these techniques are quite promising but they are still work in progress. Some of the MEMS approaches described above may suffer from electromagnetic noise interference, tunnel
vibration and undesirable flow through sensor gaps. We recently developed a whispering gallery mode (WGM)-based wall shear stress sensor [31] which employs a variation of the floating element-based measurement technique. This micro-optical sensor is capable of measurements resolved in time and space, can have a large dynamic range, and is capable of mitigating some of the drawbacks of the current skin friction measurement techniques. The signal output is optical rather than electronic, which makes the sensor immune to electromagnetic interference. The mechanical principle is similar to the floating element technique; force exerted by the flow on a small surface element flush with the wall is measured. However, whereas the typical floating element sensor requires considerable movement/deflection to measure the force exerted by the flow, the present sensor requires movements only in the order of nanometers for measurement. The sensor can be applied to a wide range of flows; the measurement resolution, dynamic range and bandwidth can be optimized for a given flow by varying the sensing element size and material. Wall shear stress is not the first application of the WGM microspheres; in recent years, several other WGM-based optical sensors have been reported. These sensor applications include protein adsorption [32, 33], trace gas detection [34], impurity detection in liquids [35], structural health monitoring of composite materials [36], detection of electric field [37], magnetic field [38, 39] and temperature [40, 41] as well as mechanical sensing, such as pressure [42, 43] and force [44, 45].

Even though there are a number of direct shear stress sensors under development, to the best of our knowledge, there are no reports on their deployment in separating and reattaching flow studies. In this paper, we demonstrate the use of the WGM-based wall shear stress sensor in a planar backward-facing step flow. This type of flow contains most complex features of separating–reattaching flows while having a relatively simple geometry. Moreover, the backward-facing step flow has been studied extensively by other measurement methods, thus allowing comparisons to results available in the literature. In addition to the measurements in the reattaching backward-facing step flow, an analysis is also carried out to evaluate the performance of the WGM sensor including shear stress sensitivity, pressure sensitivity and dynamic response.

2. WGM sensor concept

The sensing principle is based on tracking the optical mode (WGM) shifts of a dielectric sphere. These optical mode shifts are due to perturbations of the sphere morphology caused by an applied force (in this case, wall shear stress) on the sphere. By tracking the WGM shifts, the shear stress magnitude can be measured. The optical setup for the current WGM sensor is presented in figure 1. The setup is similar to that used in our previous prototypes ([31, 43–45]). The optical modes are excited by a tunable diode laser with \( \sim 1.3 \mu m \) central wavelength, which is coupled to a single mode optical fiber. A section of the fiber is tapered down to \( \sim 10 \mu m \) diameter, where a dielectric microsphere is brought in contact with the fiber. The tapered section of the fiber facilitates light coupling between the fiber and the dielectric sphere. The end of the optical fiber is terminated with a fast photodiode (PDA10CS by Thorlabs). The wavelength of the laser is tuned over a narrow range (\( \sim 0.1 \) nm) and the optical modes are observed as sharp dips in the intensity of the transmitted light (see figure 1). A minute change in the morphology of the microsphere (such as perturbations in the size or index of refraction due to wall shear stress) results in a shift in the optical modes. By monitoring these shifts, the magnitude of the wall shear stress can be determined.

A simple description of microsphere WGM is afforded by ray optics: laser light coupled into the microsphere nearly tangentially experiences multiples of total internal reflection inside the microsphere (provided that the index of refraction of the sphere is greater than that of the surrounding medium) and circumnavigate the interior surface of the sphere. An optical resonance (WGM) is observed when the round trip distance traveled is an integer multiple of the wavelength of the light. For the case \( a \gg \lambda \) (where \( a \) is the sphere radius and \( \lambda \) is the wavelength of the light), the approximate condition for optical resonance (WGM) can be expressed as

\[
2\pi n_0 a \approx l\lambda,
\]

where \( n_0 \) is the sphere’s refractive index and \( l \) is an integer representing the circumferential mode number. Any perturbation to the size or the refractive index of the microsphere will lead to a shift in the resonance wavelength as

\[
\frac{d\lambda}{\lambda} = \frac{dn_0}{n_0} + \frac{da}{a}.
\]

Thus, any external effect (such as wall shear stress) that perturbs the sphere morphology can be measured by monitoring the optical modes.

An attractive feature of the WGM-based sensors is their large optical quality factors \( Q = \lambda/\delta\lambda \). The \( Q \)-factor is determined by the energy loss in the optical cavity (dielectric sphere). Smaller energy losses lead to larger \( Q \)-factors and hence narrower optical mode linewidths. Narrower linewidths in return afford better sensor resolution. WGM \( Q \)-factors approaching material loss limit of \( 10^{10} \) have been reported with fused silica microspheres [46]. In the present work, \( Q \)-factors as high as \( 10^6 \) have been observed with PDMS microspheres.
3. Sensor design

The sensor, shown in figure 2, is a variation of the wall shear stress sensor prototype reported in [31]. The sensor cavity is fitted into a hole on the test section wall with its outside surface flush with the wall. The bottom surface of the cylindrical cavity is made of 1 mm thick aluminum. The active components of the sensor cavity are the polydimethylsiloxane (PDMS) microsphere, a 5 mm polymethylmethacrylate (PMMA) beam with 960 μm diameter that acts as a lever and sensing surface, and a steel axle that serves as the pivot. The PMMA beam is attached to the axle using loctite epoxy. A brass back plate (0.4 by 0.4 mm²) is attached to the PMMA beam at its midpoint again using loctite epoxy to provide a flat contact surface pressing against the microsphere. The back plate compresses the microsphere against the backstop with the force transmitted through the beam to measure the one-dimensional wall shear stress. The microsphere is preloaded (compressed) as the sensor is built so that both positive and negative shear stress can be measured. The ends of the 500 μm diameter axle pass through the 600 μm diameter holes punched out on the aluminum housing. The clearance between the holes and the axle is filled with PDMS with 40:1 base-to-curing agent mixing ratio (by volume) to hold the axle in place. The sensing surface is aligned flush with the test section wall. As the PMMA beam pivots about the axle, it transmits the shear force experienced by its flat tip (sensing surface) to the microsphere slightly deforming it (da/α) and causing a shift in the sphere WGM. A latex membrane with 60 μm thickness covers the circular gap of about 200 μm between the sensing element and the wall to prevent flow through the cavity.

4. Analysis

An analysis is carried out to study the performance of the optical wall shear stress sensor and the influence of local pressure on measurement accuracy. In turbulent flows, the magnitude of the local pressure (both mean and fluctuating) can be significantly larger than that of the wall shear stress. Hence, the effect of pressure force acting on the surface of a shear stress sensor can be considerable.

4.1. Sensitivity

The sensor is modeled as a mechanical system that is comprised of the PDMS sphere and the PMMA beam. The free body diagram of the mechanical system is shown in figure 3 where θ represents the built-in misalignment angle at the beam tip relative to its axis. This misalignment is the main source of pressure effect on measured shear stress. The moments \( M_p \) and \( M_τ \), about the pivot due to gage pressure, \( P \), and wall shear stress, \( τ \), acting on the sensing surface of the beam are

\[
\vec{M}_p = P\pi \frac{D^2}{4} l \sin \theta
\]

\[
\vec{M}_τ = τ\pi \frac{D^2}{4} l \cos ϕ_0
\]

where \( D \) and \( l \) are the diameter and length of the beam, respectively. The resultant moment is counteracted by the sum of the moments due to the sphere reaction \( M_s \), the torsion spring at pivot \( M_r \) and the weight of the beam \( M_b \):

\[
\vec{M}_b + \vec{M}_s + \vec{M}_τ = -mg \frac{l}{2} \sin ϕ_0 - F_s \frac{l}{2} \cos ϕ_0 - k_r \sin ϕ_0
\]

where \( F_s \) is the reaction force of the sphere, \( m \) is the mass of the beam, \( g \) is the gravitational acceleration and \( k_r \) is the torsional spring constant of the pivot. The initial equilibrium of the moments yields

\[
\vec{M}_r + \vec{M}_p + \vec{M}_b + \vec{M}_s + \vec{M}_τ = 0
\]

The force acting on the sphere, \( F_s \), results in a radial deformation, causing a shift in the WGM of the sphere as given by equation (2). In a previous study, we used Hertz contact theory to estimate the deformation of a sphere under compressive force between two plates [45]. The same analysis can be applied to obtain an analytical expression for the sensitivity of the wall shear stress sensor. The deformation
of the sphere can be obtained by solving the steady Navier equation:
\[
\nabla^2 u + \frac{1}{1 - 2\nu} \nabla (\nabla \cdot u) = 0, \tag{7}
\]
where \( u \) is the displacement of a given point within the sphere and \( \nu \) is the Poisson ratio of the material. For the azimuthally symmetric loading, the general solution of equation (7) is [47, 48]
\[
\sigma_r = 2G \sum_n [A_n(n+1)(n^2 - n - 2 + 2\nu)r^n + B_n(n-1)r^{n-2}]P_n(\cos \vartheta), \tag{8}
\]
where \( r \) and \( \vartheta \) are the radial and polar coordinates, respectively (figure 4), \( u_r \) is the radial component of displacement, \( P_n \) is the Legendre polynomial and \( A_n \) and \( B_n \) are constants determined by the boundary condition at the sphere surface. Stress distributions within the sphere can be obtained from equation (8) as
\[
\sigma_{rr} = 2G \sum_n [A_n(n+1)(n^2 - n - 2 + 2\nu)r^n + B_n(n-1)r^{n-2}]P_n(\cos \vartheta), \tag{9a}
\]
\[
\sigma_{\vartheta \vartheta} = 2G \sum_n [A_n(n^2 + 2n - 1 + 2\nu)r^n + B_n(n-1)r^{n-2}] \frac{dP_n(\cos \vartheta)}{d\vartheta}, \tag{9b}
\]
where \( G \) is the shear modulus of the sphere material. The boundary conditions for the stress are as follows: the normal stress at sphere surface within the contact area is \( \sigma_r(a) = \frac{4}{3\pi a_0^3} \sqrt{a^2 - a_0^2 \sin^2 \vartheta} \) and is zero elsewhere. The stress distribution can be obtained by expanding the stress at the contact area as a Legendre series in the spherical coordinates with coefficients [45]
\[
H_n = \frac{2n+1}{2n+3} \int_0^{\pi/2} P_n(\cos \vartheta) \sin \vartheta \, d\vartheta \tag{10}
\]
where \( a \) and \( a_0 \) are the sphere radius and the radius of the contact surface, respectively (figure 4). The radial deformation of the sphere at \( \vartheta = \pi/2 \) can be obtained by satisfying the boundary conditions. Then, using equation (2), the sensitivity to force applied to the sphere can be expressed as
\[
\frac{\partial \lambda}{\partial F_s} = \lambda \sum_n \frac{H_n}{4G(n^2 + 2n\nu + n + \nu + 1)} \times \left[ \begin{array}{c} 2 - 4n^2 + \nu(4n^2 - 2n - 4) \\ n - 1 \end{array} \right]P_n(0), \tag{11}
\]
where \( \partial F_s \) is the change in the force acting on the sphere. Thus, equation (11) gives the shift in the WGM due to a change in the force, \( \partial F_s \), at the point contact of the sphere. The refractive index-induced WGM shifts (due to birefringent effect) are negligible compared to those due to radial deformation of the sphere [45]. Therefore, the solution given by equation (11) only considers WGM shift due to radial deformation, \( \partial r \), in determining \( \partial \lambda \).

The shear stress sensitivity can be obtained by setting \( m_\phi \) to zero in equation (6) and solving for the force acting on the sphere:
\[
F_i = \tau \pi \frac{D^2}{2} - mg\phi_0 - \frac{2k_r}{l} \phi_0. \tag{12}
\]

For small deformations, the dependence of sphere WGM on the applied force is essentially linear [44, 45]. Thus, we model the sphere as a linear spring as follows:
\[
F_s = l\frac{1}{2}k_0\phi_0, \tag{13}
\]
where \( k_0 \) is the equivalent spring constant of the sphere. Inserting equation (13) into equation (12) and differentiating
\[
\frac{1}{2}k_0 \frac{d\phi}{d\varphi} = \frac{\pi D^2}{2} \frac{d\tau}{d\varphi} - mg \frac{d\phi}{d\varphi} - \frac{2k_0}{l} \frac{d\phi}{d\varphi}. \tag{14}
\]

Using Hertz contact theory, the spring constant of a 40:1 PDMS sphere with diameter of \( \sim 900 \) \( \mu \)m is determined to be \( k_r \approx 3.5 \) N m\(^{-1}\). In the present sensor, the PMMA beam has a diameter of 0.96 mm and a length of 5 mm yielding a mass of \( m \approx 4.268 \times 10^{-6} \) kg. With these, a term-by-term order of magnitude estimate of equation (14) yields
\[
O(10^{-5}) \frac{d\varphi}{d\varphi} = O(10^{-6}) \frac{d\tau}{d\varphi} O(10^{-5}) \frac{d\varphi}{d\varphi} O(10^5) k_0 \frac{d\varphi}{d\varphi}. \tag{15}
\]

The first term on the right-hand side of the above equation is the driving force. In order for the sensor to function, the last term on the right-hand side (representing the pivot reaction force) has to be of the same order or smaller than the left-hand side term (representing the force applied on the sphere). Thus, comparing the left-hand side term to the second term on the right, we determine that the latter is negligible. Then, the force on the sphere due to shear stress becomes
\[
\left(\frac{dF_i}{r}\right) = \pi \frac{D^2}{2} \frac{d\tau}{d\varphi} = \frac{2k_r}{l} \frac{d\phi}{d\varphi}. \tag{16}
\]

Although the torsional spring constant, \( k_r \), is not known \( \text{a priori} \), it can be estimated from the sensor calibration as discussed in section 5.3. The shear stress sensitivity is determined by inserting equation (16) into equation (11).

For pressure sensitivity, we set the shear stress term in equation (6) to zero and again solve for the force acting on the sphere,
\[
F_s = P\pi \frac{D^2}{2} - mg\phi_0 - \frac{2k_r}{l} \phi_0. \tag{17}
\]
Note that the pressure will always be acting perpendicular to the tip of the beam surface. That is, the pressure effect will be independent of $\phi_0$. Following the same linear spring model as before we have

$$\frac{1}{2}k_1 \phi = \frac{dP \pi D^2}{2} \theta - mg \phi - \frac{2k_r}{l} \phi.$$  \hspace{1cm} (18)

An order of magnitude analysis similar to that for shear stress yields

$$O(10^{-2}) \phi = O(10^{-6}) \frac{dP \pi D^2}{2} \theta - O(10^{-5}) k_r \phi.$$  \hspace{1cm} (19)

The first term on the right-hand side of the above equation is the driving term. The second term on the right-hand side is negligible compared to the left-hand term and can be neglected. Thus, force on the sphere due to pressure is

$$(dF_p)_p = \frac{1}{2} k_1 \phi = \frac{dP \pi D^2}{2} \theta - \frac{2k_r}{l} \phi.$$  \hspace{1cm} (20)

The pressure sensitivity can be determined by inserting equation (20) into equation (11).

Equations (16) and (20) do not have any terms dependent on initial beam misalignment angle, $\phi_0$. Therefore, both equations hold also for negative values of the beam misalignment.

### 4.2. Dynamic response

In order to determine the dynamic response (bandwidth) of the sensor, we consider the arrangement in figure 3 as a linear system. The equation of motion for the system in figure 3 is

$$I_b \ddot{\phi} + k_1 \phi + k_2 \phi + b \dot{\phi} + l = 0,$$  \hspace{1cm} (21)

where $b$ and $I_b$ are the damping constant and the moment of inertia of the beam, respectively and $l$ is the displacement of the beam at the sphere contact point. With

$$\phi = \frac{2x}{l}; \quad I_b = \frac{m l^2}{3}; \quad \dot{\phi} = \frac{2 \dot{x}}{l},$$  \hspace{1cm} (22)

the equation of motion becomes

$$m \ddot{x} + \frac{4}{3} k_{eff} \dot{x} + \frac{2}{3} b \dot{x} = 0,$$  \hspace{1cm} (23)

where $k_{eff}$ is the effective spring constant of the system, $k_{eff} = k_1 + \frac{4k_r}{l}$. The natural frequency of the equivalent system is $\omega = \sqrt{\frac{3}{2l}(\frac{km}{m} - \frac{36l}{}\text{tan}^2)}$. In this system, the magnitude of the damping coefficient, $b$, would be dominated by the viscosity of the PDMS material. The authors have not found any information on the damping coefficient of 40:1 PDMS. The bandwidth of the equivalent undamped system, $\omega_0 = \sqrt{\frac{3}{2l}(\frac{km}{m})}$, can be obtained by using $k_1/l^2 \approx 2.7 \text{ N m}^{-1}$ (determined from the experimental results presented below), which yields $\omega_0 \approx 1.5 \text{ kHz}$. This is the upper bound of the 40:1 PDMS sensor’s dynamic response.

### 5. Experiments

#### 5.1. Experimental setup

Sensor calibration and experiments are performed in a two-dimensional channel whose schematic is shown in figure 5.
5.2. Pressure sensitivity

One of the challenges that direct wall shear stress sensors have to overcome is the sensitivity to pressure. The sensing surface is subjected to a normal pressure due to the pressure difference between the test section and the surrounding medium (atmospheric pressure). Note that the effect of the gradient of pressure is insignificant in the present sensor configuration since the gap between the sensor surface and the wall is covered by a membrane. To determine the pressure sensitivity, the sensor is mounted on the wall of a pressure chamber, which is a simple piston–cylinder device as shown in figure 7. The chamber pressure is varied by moving the piston. The results, along with the predicted pressure sensitivity (section 4.1) for misalignment angles of $\theta = 0.2^\circ$ and $0.5^\circ$ are shown in figure 8. The microsphere used is a $\sim 900 \mu$m diameter 40:1 PDMS. The comparison between the experimental result and the prediction indicates that the sensor surface misalignment is between $\theta = 0.2^\circ$ and $0.5^\circ$.

The sensor’s output has a linear dependence on pressure, and the pressure sensitivity is $d\lambda/dP = 0.0236 \text{ pm Pa}^{-1}$.

5.3. Sensor calibration

The sensor is calibrated in situ using the flow channel illustrated in figure 5. These measurements were carried out at the mid-span of the channel and at streamwise location where the flow is fully developed and two-dimensional (uniform streamwise velocity along the spanwise direction away from the side walls). The wall shear stress is calculated from

$$\tau = \frac{h dP(x)}{2 dx},$$

(24)

where $h$ is the channel height and $dP/dx$ is the streamwise gradient of pressure in the fully developed flow region. In figure 9, the flow rate through the channel is slowly increased (up to $Re_h \approx 20,000$) and the shear stress, $\tau$, determined from equation (24), is plotted against the WGM shift, $\Delta \lambda$. The figure shows that the sensor has a fairly linear response to wall shear stress with sensitivity $d\lambda/d\tau = 7.319 \text{ pm Pa}^{-1}$. Comparing this to figure 8, the shear stress sensitivity is more than two orders of magnitude larger than that of the pressure sensitivity. From the calibration plot the torsional spring constant at the pivot (figure 3) can be estimated using equation (16). From Hertz contact theory, $d\phi$ and $dF_s$ are calculated for a given $d\tau$ in the plot to obtain $k_{s}/l^2 \approx 2.7 \text{ N m}^{-1}$.

The sensor resolution is determined by the minimum resolvable shift, $\delta \lambda$. If, as a first approximation, we assume the minimum resolvable shift to be the linewidth of the optical mode, the sensor resolution is

$$\delta \tau = \frac{\lambda}{Q} \left( \frac{d\lambda}{d\tau} \right)^{-1}.$$  

(25)

However, the minimum resolvable shift in the WGM spectrum can be improved by utilizing robust signal processing methods such as that described in [43]. Our current setup is capable of detecting WGM shifts as small as $\sim 0.1 \text{ pm}$. Given the sensitivity, $d\lambda/d\tau$, in figure 9, the sensor has a wall shear stress resolution of $\sim 14 \text{ mPa}$.

5.4. Results

The mean pressure and shear stress are obtained for streamwise positions from $x/H = 1$ to 18. (As indicated in figure 6, $x$ is measured from the step face.) The measurements are repeated for Reynolds numbers of, $Re = 2500$, 3600 and 4600 (based on the step height and centerline velocity upstream of the step). The corresponding wall friction velocities and Reynolds numbers are, $u_\tau = 0.2, 0.27$ and 0.35 m s$^{-1}$ and $Re_\tau = 87, 120$ and 159 ($Re_\tau$ based on the friction velocity and upstream channel half-height). The measured coefficients for pressure and skin friction are presented in figures 10 and 11, respectively. Each data point represents the average of 1000 individual realizations, sampled at a rate of 100...
samples/second. In figure 10, the pressure data from Jovic and Driver [49] for Re = 5000 are also included for comparison. The pressure coefficient, $C_p$, is calculated as 

$$C_p = \frac{p - p_{\text{ref}}}{(1/2)\rho U_0^2},$$

(26)

where $\rho$ is the density of air, $U_0$ is the reference velocity, and $p_{\text{ref}}$ is the reference pressure. In the data presented by Jovic and Driver [49], the reference pressure was measured at $x/H = -5.1$. For comparison, we also use the reference pressure at the same location.

The corresponding skin friction coefficient, $C_f = \frac{\tau_{\text{rms}}}{(1/2)\rho U_0^2}$, distributions are shown in figure 11. As also found in previous studies, the reattachment length becomes larger with increasing Reynolds numbers. The present reattachment lengths are $x/H \approx 5.0, 5.8$ and 6.75 for the three Reynolds numbers. It is also observed that the absolute values of $C_f$ become smaller with larger Reynolds numbers. For comparison, measurements by Jovic and Driver [49] and Spazzini et al [50] are also presented in the figure. The present results agree well with these two earlier studies with the exception of the region close to the step; Spazzini et al observe a small secondary recirculation bubble near the step corner as evidenced by the second zero-crossing of $C_f$ around $x/H = 1$. It is possible that the spatial resolution of $\sim 0.16 H$ of the present sensor is not fine enough to resolve this small, secondary recirculation.

The forward flow probability (FFP) at various distances from the step for Re = 3600 is presented in figure 12. Each data point is obtained from a record of 1000 instantaneous measurements of the wall shear stress and FFP is the fraction of the forward flow in each record. The flow is fully reversed at $x/H \approx 3$. Downstream of this location, the FFP increases gradually reaching a value of 0.5 at $x/H \approx 5$ which coincides with the reattachment region. For $x/H \geq 10$ the flow is fully forward with no probability of reversal. The current measurements agree well with those of Spazzini et al [50] for the region $x/H \geq 3$. Closer to the step, however, their results indicate a larger fraction of forward flow than the present measurements. This trend is consistent with the $C_f$ results of figure 11 which imply smaller magnitudes of reversed flow velocities in Spazzini et al [50] study for $x/H \leq 3$.

Figure 13 shows the rms of the fluctuating component of wall shear stress, $\tau'$, normalized by the mean wall shear stress, $\bar{\tau}$, at $x/H = 18$ for Re = 3600. Time resolved wall shear stress measurements are carried out at the rate of 6000 samples/second and each data point in the figure represents a record length of $\sim 1$ s. Since the sensor bandwidth is estimated as $\omega_0 \approx 1.5$ kHz in section 4.2, this sampling rate should cover the full range of the sensor’s frequency response. Fluctuating shear stress reaches a maximum around $x/H \approx 3$ which coincides with the location of maximum negative mean shear stress and fully reversed flow (figures 11 and 12). Further downstream, the normalized fluctuation intensity asymptotes toward the value $\tau'/\bar{\tau} \sim 0.4$. For comparison, rms wall stress distributions obtained by Spazzini et al [50] (Re = 3500) and Tion et al [51] (Re = 3800) are also shown in the figure. Our results are in reasonably good agreement with those of previous studies except for $x/H \leq 3$ where the present results indicate larger fluctuating shear stress values.

The estimated relative contribution of fluctuating pressure to the measured dynamic wall shear stress is presented in figure 14 for several beam tip misalignment angles, $\theta$. The estimates are obtained by taking the ratio $(dF_r)/p_d(\delta F_r)$. The rms of shear stress and gage pressure are used for $d\tau$ and $dP$ (equations (16) and (20)) for each Reynolds number.
and streamwise position. Figure 8 indicates a misalignment angle in the range $0.2^\circ < \theta < 0.5^\circ$. Therefore, the relative contribution from the fluctuating component of pressure at the wall in the time resolved measurements is estimated to be less than 1%.

6. Conclusions

Measurements are made in the separating and reattaching region of a planar backward-facing step flow to investigate the efficacy of the WGM-based wall shear stress sensor. Shear stress and pressure calibrations show that the pressure sensitivity is more than two orders of magnitude smaller than that of the wall shear stress. The wall pressure effect on sensor performance is mainly due to beam tip misalignment and can be further reduced by employing advanced fabrication methods. The current skin friction measurements are in good agreement with those reported previously using indirect measurement methods. However, the sensing surface diameter of $\sim 960 \, \mu m$ in the current flow with the step height of 6.4 mm apparently does not provide sufficient resolution to detect the small secondary recirculation bubble near the step corner. Smaller sensor surfaces can be used to improve resolution although this would be a trade-off with sensitivity.

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