Magnetorheological polydimethylsiloxane micro-optical resonator

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We investigate the possibility of using magnetorheological polydimethylsiloxane (MR-PDMS) spheres as micro-optical resonators. In particular, the effect of a magnetic field on the whispering gallery modes (WGM) of these resonators is studied. The applied field induces mechanical deformation, causing shifts in the WGM. The microspheres are made of PDMS with embedded magnetically polarizable particles. An analysis is carried out to estimate the WGM shifts induced by an external magnetic field. An experiment is also carried out to demonstrate the magnetic field-induced WGM shifts in an MR-PDMS microsphere. The results indicate that MR-PDMS microspheres can be used as high-Q-factor tunable optical cavities with potential applications in sensing. © 2010 Optical Society of America

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Magnetorheological (MR) elastomers are polymeric matrices filled with polarizable microparticles, which have attracted attention in recent years owing to the wide range of potential applications [1–6]. In the presence of a magnetic field, the embedded particles become polarized, exerting force upon one another because of dipolar interactions. These forces lead to changes in the mechanical and magnetic properties of the MR elastomer that are reversible. In this Letter, we demonstrate an MR polydimethylsiloxane (MR-PDMS) microsphere that can be used as a tunable high-quality (Q) factor optical resonator. The polarizable particles are mixed in with the PDMS when it is in liquid form and cured to solidify into a nearly spherical shape of several hundreds of micrometers in diameter. The sphere is then coated with a thin layer of pure PDMS with a thickness of several micrometers. Light from a tunable laser is tangentially coupled into this outer layer using a tapered single-mode optical fiber in order to excite the optical modes of the sphere. As in previous studies of microsphere whispering gallery mode (WGM), the optical modes are observed as sharp dips in the transmission spectrum [7–10].

When the MR-PDMS microsphere of radius \( R \) and index of refraction \( m \) is exposed to a magnetic field, its shape and refractive index are perturbed, leading to a shift in its optical resonances (WGM). As shown in [9,10] the change in the resonance wavelength due to the birefringence effect (\( \Delta m/m \)) is negligible compared to the strain effect (\( \Delta R/R \)). Taking this into account, the relative change in the WGM can be written as

\[ \Delta \lambda / \lambda = \Delta R / R, \]  

where \( \lambda \) is the corresponding resonant vacuum wavelength.

In the analysis, we consider a MR-PDMS microsphere under a static, uniform magnetic field of \( H_0 \) as shown in Fig. 1. Laser light coupled into the sphere from the optical fiber travels on a plane normal to the \( z \) direction, as shown. The surrounding fluid is air with magnetic permeability close to that of a vacuum (\( \sim \mu_0 \)). The relative magnetic permeability of the MR-PDMS material is \( \mu_r \). The external magnetic field sets up a body force within the sphere and a magnetic pressure acting on its surface. An expression for the WGM shift induced by the applied magnetic field, \( H_0 \), can be obtained by solving the Navier equation to obtain the deformation of the sphere:

\[ \nabla^2 u + \frac{1}{1 - 2\nu} \nabla (\nabla \cdot u) + \frac{f}{G} = 0 \]  

with boundary conditions

\[ \sigma_{rr} = p \quad \text{and} \quad \sigma_{r\theta} = 0 \quad \text{at} \quad r = R, \]  

where \( u \) is the displacement, \( \nu \) is the Poisson ratio, \( G \) is the shear modulus, \( \sigma_{rr} \) and \( \sigma_{r\theta} \) are the normal and shearing components of stress, and \( p \) is the magnetic pressure acting on the sphere surface. The magnetic-field-induced body force \( f \), in a homogeneous, isotropic material is given by [11]

\[ f = -\frac{1}{4} (b_1 + b_2) \nabla H^2. \]  

Here \( H \) is the magnitude of the field within the sphere. Coefficients \( b_1 \) and \( b_2 \) are defined as \( b_1 = \partial \mu / \partial e_{ii} \) and \( b_2 = \partial \mu / \partial e_{kk} \), where \( e_{ii} \) and \( e_{kk} \) are the normal components of strain [11]. Under a uniform external magnetic field, the field distribution within the solid sphere is also uniform. Hence \( f = 0 \). The solution to this reduced form of Eq. (1) for an azimuthally symmetric (\( \phi \)-direction) case is

\[ u_r = \sum [A_n(n + 1)(n - 2 + 4\nu)r^{n+1} \] 

\[ + B_n n r^{n-1} P_n (\cos \theta) ] , \]  

where, \( u_r \) is the local displacement in the radial direction and \( P_n \) is the Legendre polynomial. Constants \( A_n \) and \( B_n \) can be determined using the boundary conditions and the stress-strain relationships for an elastic solid [9]. The magnetic pressure acting on the sphere surface due to a uniform magnetic field \( H_0 \), as shown in Fig. 1, is given by [11].

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\[ p = \frac{\mu_0(\mu_r - 1)}{2} [H^2 + (\mu_r - 1)H^2 \cos^2 \theta] + \frac{b_1 - b_2}{2} H^2 \cos^2 \theta + \frac{b_2}{2} H^2. \]  

(6)

To satisfy the boundary condition of Eq. (2), we rewrite this equation using a Legendre series as follows:

\[ p = \left[ \frac{\mu_0(\mu_r - 1)}{2} + \frac{b_2}{2} \right] H^2 + \left[ \mu_0(\mu_r - 1)^2 + \frac{(b_1 - b_2)\mu_0}{\mu_0} \right] \frac{1}{3} P_2 H^2 + \left[ \mu_0(\mu_r - 1)^2 + b_1 - b_2 \right] \frac{1}{4} H^2. \]  

(7)

where \( P_2 = (3\cos^2 \theta - 1)/2 \). Because we consider polymers that are nearly incompressible, the contribution of the first and the last terms of the above expression to strain \((\Delta R/R)\) is negligible, and hence can be dropped. Further, expressing the magnetic field within the sphere in terms of the external inductive magnetic field \( B_0 = \mu_0 H_0 \), the magnetic pressure term can be simplified to

\[ p = \frac{b_1 - b_2}{2} \mu_0(\mu_r - 1)^2 \frac{3B_0^2}{3\mu_0(\mu_r + 2)^2} P_2. \]  

(8)

Using the above expression for the boundary condition of Eq. (3) and following the same procedure as in [9] to obtain the relative change of sphere radius at \( \theta = \pi/2 \), Eq. (1) can be rewritten as

\[ \frac{\Delta \lambda}{\lambda} = \frac{\Delta R}{R} = -\frac{7 - 4\nu}{4G(7 + 5\nu)} \left[ (\mu_r - 1)^2 + \frac{b_1 - b_2}{\mu_0} \right] \frac{3B_0^2}{\mu_0(\mu_r + 2)^2}. \]  

(9)

This expression gives a simple relationship between the WGM shift and the applied inductive magnetic field. Note that the above solution is for the case when the magnetic field is normal to the plane of light circulation in the sphere. To evaluate the above expression, the mechanical properties \((G \text{ and } \nu)\) and magnetic permeability \((\mu_r)\) of MR materials must be known under the effect of a magnetic field. To the best of our knowledge, there is no information on these parameters for MR-PDMS. The shear modulus of a polymer filled with randomly distributed spherical microparticles can be described with good approximation by \( G = G_0(1 + 2.5\varphi + 14.1\varphi^2) \) where \( G_0 \) is the shear modulus of the pure polymer and \( \varphi \) is the volume fraction of particles [12]. However, when such MR polymers are subjected to a magnetic field, the shear modulus is modified. In an earlier experimental study, a doubling in the shear modulus of iron particle-filled elastomer (with 20% volume fraction) was observed when the material was subjected to an inductive magnetic field of 670 mT [5,6]. This hardening of the material is caused by the magnetic interactions among dipoles induced by the magnetic field. The change of magnetic permeability induced by deformation (expressed through coefficients \( b_1 \) and \( b_2 \)) is due to changes in the distance among particles, which, in turn, change the magnetic reluctance and hence, the magnetic permeability.

In the present experiment we use a microsphere made of PDMS with base-to-curing agent ratio of 60:1 that is filled with polarizable particles (MQP-S-11-9). The process of manufacturing the PDMS sphere is the same as that described in [9]. The volume fraction of the particles is approximately 30%. After the PDMS curing.
process, the 400 μm radius MR-PDMS microsphere is coated with a thin layer (~5 μm) of PDMS to assure that the coupled light travels within this outer shell. Figure 2 shows a photograph of the microsphere attached to a silica stem. The microsphere is placed adjacent to a solenoid in a nearly uniform magnetic field, as shown in Fig. 3. By changing the voltage applied to the solenoid, the magnitude of the inductive magnetic field is changed and recorded by a Gauss meter. The WGM shifts were monitored through the fiber transmission spectrum, as in our earlier studies [7–10]. Figure 4 shows the transmission spectrum of the observed WGM shift with no field and with field strength of 4 mT. The observed MR-PDMS microsphere sensor quality factor is $2 \times 10^6$.

Figure 5 shows the experimental results along with predictions from Eq. (9). For the latter, the shear modulus is determined from $G = G_0 (1 + 2.5\phi + 14.1\phi^2)$ with $G_0 = 1000$ Pa, $\phi = 0.3$, $\nu = 0.49$; $b_1 - b_2$ is taken to be zero, and the WGM shifts are calculated for two different relative permeability values defining the range provided by the manufacturer; $\mu_r = 1.15–1.22$. The estimated WGM shifts from Eq. (9) agree reasonably well with the experimental results. The study shows that the optical modes of MR-PDMS microspheres can easily be tuned by using an external magnetic field with several possibilities, such as magnetic field detection. The data of Fig. 5 indicate a field resolution of ~1 mT with $Q = 10^7$. This resolution can be improved by optimizing sphere material and using particles with larger $\mu_r$. For example, the use of 30% volume fraction NiZn ferrite powder in polymeric base material would give $\mu_r = 3$ [13], resulting in micro-Tesla field resolution.

References