Whispering gallery modes of microspheres in the presence of a changing surrounding medium: A new ray-tracing analysis and sensor experiment

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A simple plane wave, ray-tracing approach was used to derive approximate equations for the dielectric microsphere whispering gallery mode (WGM) resonant wavenumber and quality factor, as dependent on the surrounding medium’s refractive index. These equations are then used to determine the feasibility of a micro-optical sensor for species concentration. Results indicate that the WGMs are not sensitive enough to refractive index changes in the case of gas media. However, they can be sufficiently sensitive for measurements in liquids. Experiments were carried out to validate the analysis and to provide an assessment of this sensor concept. © 2010 American Institute of Physics. [doi:10.1063/1.3425790]

I. INTRODUCTION

Micro-optical sensor concepts based on the so-called whispering gallery mode (WGM) shifts in spherical dielectric resonators were recently demonstrated for temperature,1 force,2,3 pressure,4 and electric field.5 In these studies, a single mode optical fiber is used to couple tunable laser light into the sphere. The WGMs are observed as sharp dips in the transmission spectrum through the fiber. The observed linewidth, Δλ, of these dips is related to the quality factor, Q = λ/Δλ. A small perturbation in the shape, size, or the index of refraction of the microsphere causes a shift in the resonance positions allowing for the precise measurement of the external condition causing the change in the morphology of the sphere. In addition to the morphology of the sphere, the WGMs are affected also by changes in the refractive index of the surrounding medium. Thus, a change in the composition of the surrounding medium, if sufficiently large, will induce a detectable shift in the WGMs. This effect can be used, in principle, to monitor species concentration changes in the medium surrounding a dielectric microcavity. Krioukov et al.,6 recently proposed a species concentration sensor using this approach. They monitored refractive index changes around an integrated microdisk-waveguide system by exciting the microdisk WGM and observing them through the scattered light from the disk. In the present approach, we use a dielectric sphere along with a single mode optical fiber as a light input-output device.

Figure 1(a) depicts the geometric optics view of the WGM resonance condition. In this view, the laser light circles the interior of the sphere through multiples of total internal reflections and returns in phase. The approximate condition for optical resonance (or WGM) is

\[ 2\pi R n_1 = l \lambda, \]

where, \( R \) and \( n_1 \) are the sphere radius and refractive index, \( \lambda \) is the vacuum wavelength of light, and \( l \) is an integer. An incremental change in the refractive index of the sphere will result in a shift in the resonant wavelength; \( \Delta n_1 / n_1 \sim \Delta \lambda / \lambda \).

Similarly, a change in the radius of sphere will also result in a change in the resonant wavelength; \( \Delta R / R \sim \Delta \lambda / \lambda \). Therefore, any change in the physical condition of the surrounding medium that induces a \( \Delta n_1 \) or \( \Delta R \) of the microsphere can be sensed by monitoring the WGM shifts.

Equation (1) is an approximation that helps us evaluate the first order effects on WGMs of the sphere (those due to a

\[ \Delta \lambda \sim \frac{2\pi R n_1}{l} \]
Therefore, for a given laser wavelength and coupling angle, produced shifts are in the order of refractive index of the surrounding medium, and basic electromagnetic equations. 

II. ANALYSIS

A. WGM shift

Starting with a quantum analogy-based theory developed by Johnson" describing the optical resonances of a sphere, Teraoka et al.,8 carried out a perturbation approach to study WGM shifts due to changes in the refractive index of the medium surrounding the sphere. More recently, Schweiger and Horn9 used a wave theory approach to obtain similar results. We present here a simple approach using geometric optics, along with basic electromagnetic wave reflection expressions, to obtain equations that accurately describe the resonant wavenumber and quality factor \( Q \) of the WGM. That is, using simple physical modeling of reflections at an interface, we derive a modified version of Eq. (1), together with an expression for the associated \( Q \), in which the effect of \( n_2 \) is included. The analysis is valid for \( R \gg \lambda \) and the sphere is treated as a cylinder for conceptual simplicity with the light traveling on a plane parallel to the end surfaces of the cylinder. When the order number \( l \) of resonance is large, the cylindrical resonance condition would accurately model a spherical resonance as well. The analysis yields two distinct equations for the transverse electric (TE) and transverse magnetic (TM) modes.

Consider the reflection of a plane wave from a dielectric interface, as shown in Fig. 1(b), when the angle of incidence \( \theta_i \) is larger than the critical angle. That is

\[ \theta_i > \theta_c = \sin^{-1} \sqrt{\frac{n_2}{n_1}} \sin^{-1} \frac{n_2}{n_1}, \]  

where \( n \) and \( \varepsilon \) are the refractive indices and the dielectric constants of the resonator (1) and surrounding media (2). The reflection from such a planar interface should accurately model each individual reflection in the ray-optics description shown in Fig. 1(a). The reflection coefficients \( \Gamma_{TE} \) and \( \Gamma_{TM} \) for the TE and TM polarizations are expressed using equivalent impedances of the two media as10

\[ \Gamma_{TE} = \frac{Z_{TE2} - Z_{TE1}}{Z_{TE2} + Z_{TE1}}, \quad \Gamma_{TM} = \frac{Z_{TM2} - Z_{TM1}}{Z_{TM2} + Z_{TM1}}, \]

where,

\[ Z_{TE1} = \frac{Z_0}{n_1 \cos \theta_i}, \quad Z_{TE2} = -\frac{Z_0}{j \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}, \]

and

\[ Z_{TM1} = \frac{Z_0 \cos \theta_i}{n_1}, \quad Z_{TM2} = -\frac{jZ_0 \sqrt{n_2^2 \sin^2 \theta_i - n_1^2}}{n_2^2}, \]

with \( Z_0=377 \ \Omega \) as the wave impedance of free space. Our interest is the phase associated with each reflection coefficient for large \( N \).

Combining Eqs. (3) and (4) for the TE polarization,

\[ \Gamma_{TE} = \frac{n_1 \cos \theta_i + j \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}{n_1 \cos \theta_i - j \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}, \]

\[ \arg(\Gamma_{TE}) = \pi - \frac{2 \tan^{-1} \cos \theta_i}{\sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}. \]

Referring to Fig. 1(a), we note that, for \( N \gg 1 \), the incidence angle \( \theta_i \) is

\[ \theta_i = \frac{\pi}{2} - \frac{\pi}{N} \approx \frac{\pi}{2}. \]

Therefore, \( \arg(\Gamma_{TE}) \) can be approximated as follows:

\[ \arg(\Gamma_{TE}) \approx \pi - \frac{2 \pi/N}{\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}}. \]

Similarly, combining Eqs. (3) and (5) for the TM polarization,

\[ \Gamma_{TM} = \frac{-n_2 \cos \theta_i - jn_1 \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}{n_2 \cos \theta_i - jn_1 \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}, \]

yielding the argument

\[ \arg(\Gamma_{TM}) = -2 \tan^{-1} \frac{\cos \theta_i}{\sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}. \]

Again, Eq. (11) can be approximated using Eq. (8) as,

\[ \arg(\Gamma_{TM}) \approx -\frac{2 \pi/N}{\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}}. \]

Equation (9) shows that the reflection phase is close to \( \pi \) for the TE mode. We also know that the magnitude of reflection is unity (total internal reflection). Therefore, we may consider the case as a perturbation of an ideal conducting wall for which \( |\Gamma| = 1 \) and \( \arg(\Gamma) = \pi \). Accordingly, the TE resonant condition for the dielectric sphere of Fig. 1(a) may also be considered as a perturbation of that for a sphere with an ideal conducting wall. Using multiple ray-reflection model shown in Fig. 1(a), we consider the resonant condition for the conducting sphere first. The resonance radius is analytically known from the first zero of the \( l \)th order Bessel function10,11 that is \( k_i R = l + 1.856^{l/2} \). Here, \( k_i = 2\pi/\lambda_1 \) is the
wavenumber and $\lambda_i$ is the wavelength in the sphere. Thus, from Fig. 1(a) and using Eq. (8), the resonant condition can be expressed as

$$N2k_iR \cos \theta_i \equiv N2k_iR \frac{\pi}{N} = 2\pi k_i R = 2\pi (l + 1.856l^{1/3}).$$  

(13)

The factor $1.856l^{1/3}$ in Eq. (13) can be interpreted as a first order correction that accounts for the spherical nature of the wave front and the curvature of the resonator boundary, which is different from the simple plane wave treatment of Fig. 1(b). For large mode numbers ($l \gg 1$) this term would be small compared to $l$, resulting in the approximate resonance condition of Eq. (1). The resonant condition for the TM polarization is close to zero for large differences. From Eq. (15) the perturbation of a sphere with an ideal magnetic wall for the TM resonant radius for the magnetic-walled resonator would also correspond to the first zero of the $l$th order Bessel function, with $k_i R \equiv l + 1.856l^{1/3}$. So we have,

$$N[2k_iR \cos \theta_i - \arg(\Gamma_{TM})] = 2\pi l + 1.856l^{1/3},$$  

(18)

and

$$k_i R \equiv \frac{2\pi (l + 1.856l^{1/3}) + \arg(\Gamma_{TM})N}{2N(\frac{\pi}{N})}$$

$$\equiv l + 1.856l^{1/3} - \frac{(\frac{n_2}{n_1})^2}{\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}}.$$  

(19)

The above equation is the TM counterpart of Eq. (15). Again, the resonance shifts due to an incremental change in the refractive index of the surrounding medium can be obtained by differentiating Eq. (19)

$$\Delta(k_i R) \equiv \frac{\Delta(k_i R)}{k_i R} = -\frac{n_1 n_2 \Delta n_2}{\left[\frac{n_2}{n_1}\right]^2 \left[\left(n_1^2 - n_2^2\right)^{3/2} k_0 R\right]}.$$  

(20)

or

$$\frac{\Delta(k_i R)}{k_i R} = \frac{\Delta(k_i R)}{k_i R} = -\frac{n_1 n_2 \Delta n_2}{\left[\frac{n_2}{n_1}\right]^2 \left[\left(n_1^2 - n_2^2\right)^{3/2} k_0 R\right]}.$$  

(21)

Equations (17) and (21) describe the dependence of the resonant frequency (WGM) shift on the refractive index of the medium surrounding the sphere for TE and TM polarizations, respectively. Note that $l$ is kept constant in obtaining the above derivatives. In an earlier study, Teraoka et al. also carried out a perturbation analysis based on an equivalent quantum theory model of the optical modes of spheres to obtain expressions describing WGM dependence on $\Delta n_2$. For $k_0 R \gg 1$, Eq. (16) and the implicit formula obtained by Ref. 8 differs by the ratio of $\sqrt{[(l-0.5)/(k_0 R)]^2 - n_2^2}/(n_1^2 - n_2^2)$. However, when the order number is approximated by $l = n_1 k_0 R$ their expression for WGM shift becomes identical to Eq. (16).

### B. Quality factor

The resonance analysis presented above models the interface between the microsphere and the external medium [Fig. 1(a)] as a planar surface [Fig. 1(b)]. This results in total internal reflection, with magnitudes of the reflection coefficients equal to unity. However, each reflection generates some radiation, attributed to the curvature of the interface. We model the radiation loss with an equivalent radiation resistance $R_{rr}$ and $R_{rm}$, for TE and TM polarizations, respectively. The reflection coefficients for the TE and TM modes, taking into account the equivalent radiation resistance are given as

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The impedances $Z_{TE1}$, $Z_{TM1}$, and $Z_{TM2}$ are given by Eqs. (4) and (5). For $N \gg 1$ and $\theta = \pi/2 - \pi/N$, the magnitudes of the reflection coefficients can be approximated as

$$|\Gamma_{TE}| \approx 1 - \frac{2\pi R_{re}}{Z_1 N}, \quad \text{and} \quad |\Gamma_{TM}| \approx 1 - \frac{2R_{rm} Z_1 \pi}{N |Z_{TM2}|},$$

which $Z_1 = \sqrt{\mu_0/\varepsilon_1}$ is the intrinsic impedance of the microsphere material. Equation (23) indicates reflection coefficients smaller than unity, which is physically more realistic. The complex resonance condition for the TE mode may now be accurately established, using the reflection coefficient magnitude of Eq. (23)

$$\exp[-jN2k_1R \cos \theta + jN \arg(-\Gamma_{TM})]/|\Gamma_{TE}|^N = f(\omega_r + j\omega_i) = \exp(-j2\pi) = 1,$$

with

$$f(\omega_r + j\omega_i) = |\Gamma_{TE}|^N \left(1 - \frac{2\pi R_{re}}{Z_1 N} \right) \approx \left(1 - \frac{2\pi R_{re}}{Z_1} \right),$$

$$f(\omega_r + j\omega_i) = 1 = f(\omega_r) + j\omega_i \frac{\partial f(\omega_r)}{\partial \omega} = \left(1 + \frac{2\pi R_{k1} \omega_i}{\omega_r} \right),$$

$$\times \left(1 - \frac{2\pi R_{re}}{Z_1} \right) \approx 1 + \frac{2\pi R_{k1} \omega_i}{\omega_r} - \frac{2\pi R_{re}}{Z_1},$$

where $\omega_r$ and $\omega_i$ are real and imaginary angular frequencies. Then, the quality factor, $Q$ can be expressed as

$$Q = \frac{\omega_r}{\omega_i} = \frac{Z_{TM2} R_{k1}}{R_{re}}. \quad \text{(27)}$$

The above equation shows that in order to evaluate the quality factor, the equivalent radiation resistance $R_{re}$ has to be calculated. With this objective, we consider a TE spherical mode for $r>R$, with electric field distribution for $\theta = \pi/2$ expresses as

$$E_\varphi = A h^{(2)}_l(k_2 r)e^{ij\phi}, \quad \text{(28)}$$

where $A$ is a constant and $h^{(2)}_l$ is the outgoing spherical Hankel function of order $l$. When $r \rightarrow \infty$ the above equation may be simplified to

$$E_\varphi(r \rightarrow \infty) \approx Aj^{l+1} e^{-ik_2 r}/k_2 r. \quad \text{(29)}$$

The power density associated with the field is given by

$$P_d = \frac{|E_\varphi|^2}{Z_2} = \frac{A^2}{Z_2^2 (k_2 r)^2},$$

where $Z_2 = \sqrt{\mu_0/\varepsilon_2}$ is the intrinsic wave impedance of the medium surrounding the sphere. The power radiated from the sphere, per unit polar angle $\Delta \theta = 1$, can now be expressed in terms of the power density.

$$P_{rad} = P_d 2\pi r^2 = \frac{|A|^2 2\pi}{Z_2}.$$ \quad \text{(31)}$$

The above $P_{rad}$ can also be written in terms of the electric or magnetic field components at $r=R$, and the equivalent radiation resistance $R_{re}$.

$$P_{rad} = |E_\varphi(R)|^2 R_{re} 2\pi r^2 = \frac{|E_\varphi(R)|^2}{|Z_{TM2}|^2} R_{re} 2\pi r^2 = \frac{|A|^2 |h^{(2)}_l(k_2 R)|^2}{|Z_{TM2}|^2} R_{re} 2\pi r^2.$$ \quad \text{(32)}$$

Combining Eqs. (31) and (32), the equivalent radiation resistance can be expressed as

$$R_{re} = \frac{|Z_{TM2}|^2}{|h^{(2)}_l(k_2 R)|^2 Z_2 (k_2 r)^2}.$$ \quad \text{(33)}$$

Using the above expression for $R_{re}$ in Eq. (27) the quality factor $Q$ can be derived as follows:

$$Q = \frac{\omega_r}{\omega_i} = \frac{Z_{TM2} R_{k1}}{R_{re}} = \left(\frac{|h^{(2)}_l(k_2 R)|^2 Z_2 Z_1 (k_2 R)^2 (R_{k1})}{|Z_{TM2}|^2} \right),$$

$$\approx \gamma^{(2)}_l(k_2 R)(k_2 R)^3 \left[\frac{\varepsilon_1}{\varepsilon_2} - 1 \right],$$

where $\gamma_l(.)$ is the spherical Bessel function of the second kind. Note that we approximated $|Z_{TM2}|$ in the above derivation for $\theta = \pi/2$ to obtain

$$Z_{TM2} = \frac{\omega_r}{\sqrt{\varepsilon_1^2 \theta_1 - k_2^2}} \approx \frac{Z_2}{\sqrt{\left(\frac{k_1}{k_2}\right)^2 - 1}} = \frac{Z_2}{\sqrt{\varepsilon_1/\varepsilon_2}} - 1.$$ \quad \text{(35)}$$

Using a similar procedure as for the TE mode, an expression for the quality factor $Q$ for TM mode can also be obtained.

$$\exp[-jN2k_1R \cos \theta + jN \arg(-\Gamma_{TM})]/|\Gamma_{TE}|^N = f(\omega_r + j\omega_i) = \exp(-j2\pi) = 1,$$

$$f(\omega_r + j\omega_i) = |\Gamma_{TM}|^N \left(1 - \frac{2\pi Z_1 R_{re}}{|Z_{TM2}|^2 N} \right) \approx \left(1 - \frac{2\pi Z_1 R_{re}}{|Z_{TM2}|^2} \right),$$

$$f(\omega_r + j\omega_i) = 1 = f(\omega_r) + j\omega_i \frac{\partial f(\omega_r)}{\partial \omega} = \left(1 + \frac{2\pi R_{k1} \omega_i}{\omega_r} \right),$$

$$\times \left(1 - \frac{2\pi R_{re}}{Z_1} \right) \approx 1 + \frac{2\pi R_{k1} \omega_i}{\omega_r} - \frac{2\pi R_{re}}{Z_1},$$

$$Q = \frac{\omega_r}{\omega_i} = \frac{|Z_{TM2}|^2 R_{k1}}{R_{re} Z_1}. \quad \text{(39)}$$

As in the TE mode case, we consider here a TM spherical mode for $r>R$, with its magnetic field expressed for $\theta = \pi/2$. 

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\( H_\theta (r > R, \theta = \pi/2) = \mathcal{A} h^{(2)}_l(k_r r) e^{i \phi}. \)

The above equation may be simplified for \( r \gg R \), and the corresponding power density \( P_d \) and radiated power \( P_{rad} \) per unit elevation angle can be expressed as

\[
P_d = \frac{|A|^2 Z_2}{(k_2 r)^2}, \quad P_{rad} = \frac{P_d 2 \pi r^2}. \tag{41}
\]

\( P_{rad} \) may also be expressed using the fields at \( r=R \) and the equivalent resistance \( R_{rm} \).

\[
P_{rad} = |H_\theta(R)|^2 R_{rm} 2 \pi R^2 = |A|^2 |h^{(2)}_l(k_2 R)|^2 R_{rm}^2 2 \pi R^2. \tag{42}
\]

Combining Eqs. (41) and (42), yields the equivalent resistance, \( R_{rm} \), which can then be used in Eq. (38) to find the expression for the quality factor \( Q \) for the TM mode.

\[
R_{rm} = \frac{Z_2}{|h^{(2)}_l(k_2 R)|^2(k_2 R)^2}. \tag{43}
\]

\[
Q = \frac{R k l |Z_{TM2}|^2}{R_{rm} Z_1} = \frac{|h^{(2)}_l(k_2 R)|^2 |Z_{TM2}|^2(k_2 R)^2(Rk_l)}{Z_l Z_2}
\]

\[
= n_l^2(k_2 R)(k_2 R)^2 \left( \frac{\varepsilon_{r1}}{\varepsilon_{r2}} - 1 \right) \left( \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \right). \tag{44}
\]

Again, we approximated \( |Z_{TM2}| \) in the above derivation for \( \theta = \pi/2 \) to obtain:

\[
|Z_{TM2}| = \frac{\sqrt{k_1^2 \sin^2 \theta - k_2^2}}{2 \omega_2} = Z_2 \sqrt{k_1^2 \frac{k_1}{k_2} - 1}.
\]

\[
= Z_2 \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} - 1. \tag{45}
\]

Fig. 2 shows the TE resonance quality factor as a function of microsphere radius for a silica sphere surrounded by air \((n_2/n_1=0.69)\). In the \( Q \) calculation, the order number, \( l \), is determined using (a) the zeroth order approximation that includes only the reflection at the interface [Eq. (1)]; (b) first order approximation that includes reflection and surface curvature at the interface [Eq. (13)]; and (c) second order approximation that includes reflection, surface curvature as well as phase [Eq. (15)]. The resonance condition of Eq. (1) significantly overestimates the quality factor. The conditions of Eqs. (13) and (15) provide comparable values of the quality factor. However, the phase contribution in Eq. (15) includes the additional effect of the external medium, which is valuable in the study of resonance perturbation due to variation in the external medium. The results using Eq. (15) are comparable to those from a much involved theory using a volume-current analysis of the spherical mode. This illustrates the accuracy of the present method, obtained with analytical and conceptual simplicity. Figure 3 shows the effect of refractive index ratio \((n_2/n_1)\) and order number \((l)\) on TE resonance quality factor. The quality factor, \( Q \), increases with increasing \( l \) and decreasing \( n_2/n_1 \). For large \( l \) and small \( n_2/n_1 \) the \( Q \) value due to radiation is much larger than that due to material loss. Accordingly, in this range the overall \( Q \) is primarily determined by the material loss, and is essentially insensitive to the variations in the surrounding medium. On the other hand, for \( n_2/n_1 \) ratios close to unity, the contribution of radiation to \( Q \) value would be stronger making the overall quality factors smaller and more sensitive to \( n_2 \). This may adversely affect the accuracy of \( n_2 \) measurements using WGM shifts. Yet, \( Q \) changes may be significant enough to use them as a sensing parameter.

III. EXPERIMENTAL INVESTIGATION

A. Experimental setup

The output of the tunable distributed feedback laser diode was coupled into a single mode optical fiber with core diameter of \(~9 \mu m\) and cladding of \(125 \mu m\). The laser beam is split in two by a beam splitter (90%/10%) so that the 10% of the beam intensity goes to a photodiode to monitor the laser beam intensity and use it as a reference signal. The fiber with the 90% intensity was sent to the “sensor” side of the fiber with the microsphere in contact with it. At its output end, this fiber is connected to a fast photodiode to monitor the transmission spectrum. This signal through the sensor fiber was normalized using the reference signal. The microsphere was brought very close to the tapered fiber at a distance of the order of several nanometers to facilitate good light coupling between fiber and resonator. The laser is tuned by ramping the current into it using a laser controller. The controller, in turn, was driven by a function generator that provided a saw-tooth input to the controller. The output of both photodiode was monitored by 16 bit digitizer (data acquisition card) that was part of a host PC. The microspheres
were manufactured by melting a small section of an optical fiber using a microtorch as was done in our previous studies.\textsuperscript{2–3} The sphere diameters ranged between 350 and 500 \textmu m in the present experiments.

B. Experimental results

The details of the test section are shown in Fig. 4. In these experiments, the effect of potassium carbonate concentration (in water) on the WGM shifts is studied. The microsphere is placed in deionized water in a container. A potassium carbonate solution was gradually added to the container and allowed to diffuse for uniform concentration before the WGM shifts are observed. The temperature of the solution was monitored by a thermocouple and kept a constant value of 22 °C. The transmission spectrum was digitized and stored on the host PC to determine the WGM shift. The measurements were repeated with increasing concentrations of the solution.

Using \( k = 2\pi / \lambda \), Eqs. (18) and (22) can be rewritten in terms of wavelength, \( \lambda \) as;

\[
\frac{\Delta \lambda}{\lambda} = \frac{n_2 \Delta n_2}{(n_1^2 - n_2^2)^{3/2}} \frac{\lambda}{2\pi R},
\]

for the TE and TM modes, respectively. Again, \( R \) and \( n_1 \) are the radius and index of refraction of the sphere, respectively. (Note that the relationship between the WGM shifts and index of refraction change is not a function of the coupling (launch) angle of the light into the sphere.)

Figures 5(a) and 5(b) show the experimentally determined refractive index of potassium carbonate solution. From the measured WGM shifts (\( \Delta \lambda \)), the changes in the refractive index (\( \Delta n_2 \)) are determined using both the TE mode using Eq. (46) and shown in Fig. 5(a) and the TM mode [using Eq. (47) and shown in Fig. 5(b)]. At this point, we cannot determine if a mode observed in the transmission spectrum is TE or TM. As one would expect, the trends given by the two modes are essentially the same. In Fig. 5, refractive index dependence on potassium carbonate concentration from the chemical literature\textsuperscript{13} is also presented for comparison. (Note that the data in Ref. 13 are for \( \lambda \approx 590 \) nm.) The agreement between the experiments and the literature data is stronger in Fig. 5(a) than that in Fig. 5(b). This seems to indicate that in all the experiments, the observed WGMs happen to be TE modes. However, in the results summarized by this figure, we cannot independently identify the observed WGMs as either TE or TM modes. In order to clarify this point, an additional experiment was carried out where \( \Delta \lambda \) shifts were measured while simultaneously identifying the observed WGMs as TE or TM. To accomplish this, in addition to the transmission spectrum through the fiber, scattered light from the microsphere was also monitored. This made it possible to choose the appropriate equation [Eq. (46) or (47)] to determine \( \Delta n_2 \). The experimental arrangement is shown in Fig. 6. At resonance, the constructive build-up of the electromagnetic field in the sphere leads to increased scattering (radiation) through the sphere surface (corresponding to the WGM dips in the transmission spectrum). The scattered light from the sphere is filtered spectrally (using a laser line filter) and spatially (using a pinhole) and then imaged on a photomultiplier tube using a pair of lenses. A linear polarizer that is placed between the sphere and the front collecting lens allows for the identification of the observed mode as TE or TM. Note that the polarization of a mode in the sphere is predominantly in the direction normal to the plane in which the light circulates [as defined in Fig. 1(a)].
The container in Fig. 6 is initially filled with an aqueous solution of methanol (\(\text{CH}_3\text{OH}/\text{H}_2\text{O}\)). Pure water is then gradually added and from the observed WGM shifts (\(\Delta \lambda\)) the refractive index, \(n_2\), is determined using either Eq. (46) or (47), as appropriate. Figures 7(a) and 7(b) show typical scattering spectra for two mixture-concentrations. Figure 7(a) is for the case when the polarizer is adjusted such that only the TM modes are observed. In Fig. 7(b), the polarizer is adjusted such that the TE spectra is observed. In both figures, the simultaneously observed transmission spectra are also plotted. In these figures, C1 and C2 refer to two different concentrations of methanol solutions (C2 representing a higher concentration of water).

Figure 8 shows the experimentally determined refractive index of different concentrations of aqueous solution of methanol along with those values from the literature. Experimental data is obtained by the sensor using both the TE and TM modes. The figure shows that when the appropriate mode equation is used [i.e., Eq. (46) for the TE modes and Eq. (47) for the TM modes], the agreement between the published literature and the present results is strong. This result shows the potential of the proposed microsphere concentration sensor. However, additional work is needed to develop robust methods either to identify TE and TM modes in the transmission spectra or to selectively excite the sphere with TE or TM modes.

IV. CONCLUSIONS

The simple ray tracing, plane wave approach used in the present analysis results in equations for \(\Delta \lambda\) dependence on \(n_2\) that are identical to those approximate explicit expressions previously derived from a perturbation analysis based on a quantum-analogy model. However, we derive the explicit Eqs. (15) and (19) for the resonant wavelength using simple reflection principles, from which the \(\Delta \lambda\) dependence

![Figure 6](image)

**FIG. 6.** (Color online) Setup for index of refraction measurements with simultaneous TE and TM detection.

![Figure 7](image)

**FIG. 7.** (Color online) Typical scattering spectrum along with the corresponding transmission spectrum; (a) TM mode and (b) TE mode.
on $\Delta n_2$ can be obtained by differentiation. Equations (15) and (19) are essentially more accurate form of Eq. (1), that include additional terms. The second term is identical for both the TE and TM modes, and represents the effect of the curvature of the sphere surface. For large $l$, this term becomes negligible compared to the first term. The last term is the effect of refractive index of the surrounding medium on the resonance condition. For large $l$, this term also is dominated by the first term on the right hand side. The quality factor analysis shows that the energy losses due to surface radiation are very small (the $Q$-factors are generally large even for small order numbers). Equations (46) and (47) describe the dependence of WGM shifts on the refractive index change in the medium surrounding the sphere. The equations suggest that the proposed concentration sensor would not be sensitive enough for applications in a gas medium at or below atmospheric pressures. However, this also means that in other sensor applications (such as temperature, pressure, or force) gaseous contaminants in air would not have a significant effect on signal. For a typical liquid application Eqs. (46) and (47) are distinct enough that the nature (TE or TM) of the WGM tracked needs to be known so that the appropriate equation is used.

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**FIG. 8.** (Color online) Dependence of refractive index on molar concentration of aqueous methanol solution.