Experimental Analysis of Phase Error in Centralized and Distributed SDR Systems

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2023 32nd International Conference on Computer Communications and Networks (ICCCN) | 979-8-3503-3618-4/23/$31.00 ©2023 IEEE | DOI: 10.1109/ICCCN58024.2023.10230196

***Abstract*—Understanding the behavior of phase errors between radio frequency (RF) chains in software-defined radios (SDRs) is crucial to the success of implementing many phase-sensitive applications, such as beamforming. Even if SDRs are provided the same clocking signal, initial local oscillator (LO) phase offsets across devices will inevitably be different. Despite its known effect on many wireless applications, there are only few works that experimentally discuss random phase errors in SDRs. To address this issue, we perform experiments and analyze the results of tens of experiments in an attempt to understand the nature of this phase offset. In particular, we target the USRP (Universal Standard Radio Peripheral) N310 platform as it provides up to 4 simultaneous transmit/receive chains that could be attractive for beamforming applications. We first model the system used in this study and demonstrate how phase errors can affect distributed beamforming gains. Then, we introduce our experimental setup, procedures and analysis of the results of the measured phase error. We do so first between two chains of the same/different transceiver boards within the same USRP, and then between chains of distributed USRPs that are geographically separated. We calculate the mean and standard deviation of this phase error, investigate its behavior over time, and demonstrate how the distribution of this error can vary based on whether it is measured in a centralized or a distributed fashion.**

I. Introduction

The use of software-defined radios (SDRs) to flexibly investigate the fundamental and next-generation technologies in the wireless domain has become extremely ubiquitous. As wireless applications are becoming more diverse and demanding, the need for robust and higher capacity wireless systems is becoming a crucial driver of the design of these systems. To increase the capacity of a wireless system using multiple antennas, one technology stands out: beamforming. In transmit beamforming, the phases of transmitted signals are adjusted so that they add up constructively at the receiver. For instance, a distributed UAV (unmanned aerial vehicle) swarm that seek to increase the signal-to-noise-ratio (SNR) at a target receiver.

In an ideal world, the phases of the incoming signals from the swarm UAVs should coherently combine resulting in N^2 times the received power compared to a single UAV transmitter. However, this is seldom the case. In many practical (and especially) distributed wireless systems, the different radios will ultimately be equipped with different local oscillators (LOs). Each LO will generate the desired carrier signal with an arbitrary phase offset from the other radios. Even with a clocking solution that provides the reference frequency to the phase-locked loops (PLLs) inside the SDRs, there will still exist a phase offset between these radios. This phase offset needs to be calibrated or compensated for so that the signals can constructively add at the target.

In this work, we carryout several experiments and characterize the phase offset behavior over different time durations within the same USRP (same and different daughter-boards) and across different USRPs that are distributed via equal-length cables in an indoor facility. The analysis and statistical characterization of phase error presented in this work can help researchers come up with efficient phase error compensation methods when implementing SDR-based applications that require phase coherency, such as beamforming. We show that: (*i.*) if the RF chains used belong to the same LO within the same USRP N310, the mean phase error is consistent across experiments and only changes by 2π with a small standard deviation not exceeding $2.2°$, (*ii.*) if the RF chains used belong to the same SDR device but use different LOs, the mean phase error is random per new experiment but stable over the course of an experiment with a standard deviation not exceeding $3°$, and (*iii.*) in a distributed setup where RF chains belong to different USRPs, this phase error is random per new experiment and not stable over the course of an experiment due to phase drifts and the resultant phase error jumps.

These findings can influence how phase offset calibration is done in SDR-based beamforming systems. For example, in the first two scenarios, a one time calibration per experiment is enough where, after obtaining the mean and standard deviation of the phase difference between two chains for any time duration, we can use that mean offset and expect a phase alignment of the transmitted signals within $3°$ of error. However, in the third distributed/different USRPs setup, a continuous calibration is needed as the mean and standard deviation of this error will keep changing throughout an experiment due to the observed phase drifts.

II. Related Work

The issue of the random phase offset[1] between RF chains is not new. It has been analytically and empirically studied in various research articles where angle-of-arrival (AoA) and beamforming applications were under investigation.

For example, in [1], an approach for implementing an antenna array using two SDRs was proposed. The phase difference of arrival was used to determine the AoA of the incoming signal. A known reference signal transmitter was needed to create the knowledge of the phase difference

[1] We use phase offset and phase error interchangeably throughout this work.

between the nodes. This phase difference was found to be stable for one minute. We will show here that phase drifts can occur even within a one minute duration. In [2], AoA estimation was implemented using the B210 SDR platform with a brief analysis of the nature of phase error between two chains of the same SDR. In [3], an approximation was proposed to account for all phase offsets between distributed nodes and the performance of beamforming under various fixed phase offset values and different spatial distributions of nodes was investigated. In [5], a distributed MIMO system was implemented. The system considered phase offset between different nodes using a master-slave architecture and with the aid of preamble frames. A cross-correlation method was used in [6] to calculate the phase difference between two nodes for angle-of-arrival estimation. In [7], a channel calibration procedure was implemented on WARP boards to create a multi-antenna base station. A dedicated circuit was proposed by [8] to extract the frequency and phase of a reference signal. In [13], a software flow graph that is based on GNU Radio blocks was used to perform time and frequency correction in a distributed system. In [16], we leveraged the body of a UAV to increase beamforming gains through rotation when phase offsets exist at distributed transmitters.

Although these works provide good insight into issues related to system design and implementation of centralized and distributed beamforming using SDRs with phase offset considerations, there is little information about the experimental and actual nature of this phase offset. That is, how it behaves over time and across trials of the same and/or different nodes that might be spatially distributed. For this reason, some studies, such as [11], [12] focused their efforts on investigating this issue. However, in [11], the presented results only investigated one minute duration of recorded instantaneous phase values of two receivers that were adjacent to each other. Also, a very limited number of trials was performed. In [12], a longer duration experiment was performed and the phenomena of phase drifts was thoroughly discussed. However, LO sharing was used as a method of distributing the LO signal across the 2 USRPs used. Specifically, the X310 platform was used which enables LO sharing and leverages integer frequency dividers in its PLL. Integer frequency dividers allow for a well established and stable relationship between the input reference frequency and the output frequency of the PLL circuit [12]. This is not the case for many other SDR platforms on the market, such as USRP E312, B200, B200mini, B210, and N310. These devices do not have LO sharing and use fractional frequency dividers, allowing for greater flexibility in frequency generation but lack a well defined input/output phase relationship.

III. System Model

We first briefly describe the system investigated in this study. Then, we model a distributed beamforming system and show, analytically and experimentally, the impact of phase error on beamforming gains.

A. Phase Error in Distributed Nodes

We create a message signal $m(t)$ and use it to modulate a carrier signal $c(t) = cos(2\pi f_c t + \phi_c(t))$. Here, $\phi_c(t) = \theta_c + \theta_{pn,c}(t)$, where θ_c is an arbitrary phase offset value from a reference, say $0°$ and $\theta_{pn,c}(t)$ is the phase noise. The modulated signal will be $y(t) = m(t)c(t)$. When we demodulate the signal with a carrier $c_r(t) = cos(2\pi f_r t + \phi_r(t))$, where $\phi_r(t) = \theta_r + \theta_{pn,r}(t)$ then, the resultant signal is:

$$r(t) = m(t)cos(2\pi f_c t + \phi_c(t))cos(2\pi f_r t + \phi_r(t)) \tag{1}$$

Which, using trigonometric identities, can be expressed as:

$$\begin{aligned} r(t) = \frac{1}{2}m(t)[&cos(2\pi f_c t + \phi_c(t) - 2\pi f_r t - \phi_r(t)) + \\ &\underbrace{cos(2\pi f_c t + 2\pi f_r t + \phi_c(t) + \phi_r(t))}] \end{aligned} \tag{2}$$

The signal $r(t)$ after demodulation will pass through a Low-Pass Filter (LPF) where high frequency components (underlined above) will be removed and we are left with:

$$r_{LPF}(t) = \frac{1}{2}m(t)[cos(2\pi f_c t + \phi_c(t) - 2\pi f_r t - \phi_r(t)) \tag{3}$$

If the difference between the transmit carrier and the receiver carrier $f_c - f_r$ is denoted as δ_{f_c}, then the baseband signal, which we will record through its in-phase (I) and quadrature (Q) samples, can be rewritten as:

$$r_{LPF}(t) = \frac{1}{2}m(t)[cos(2\pi\delta_{f_c} t + \phi_c(t) - \phi_r(t))] \tag{4}$$

In a frequency-synchronized network with N receivers, the generated carrier frequencies of these receivers will be the same, which would result in the same CFO between the transmitter and all receivers, (*i.e.,* $\delta_{f_{c1}} = \delta_{f_{c2}} = \delta_{f_{cN}} = \delta_{f_c}$). However, there will be different arbitrary phase offset values along with phase noise. The above baseband signal for the R^{th} receiver ($R \in 1, 2, 3, .., N$) can then be rewritten as:

$$r_{LPF,R}(t) = \frac{1}{2}m(t)[cos(2\pi\delta_{f_c} t + \phi_c(t) - \theta_{rR} - \theta_{pn,rR}(t))] \tag{5}$$

In our experiments, we measure the phase difference between the received signals of many receiver chains and characterize its statistical behavior. The phase difference between two received signals, $r_1(t)$ and $r_2(t)$, is computed as follows:

$$\Delta_{\phi 1,2}(t) = \text{unwrap}[\angle(HT(r_1(t)))] - \text{unwrap}[\angle(HT(r_2(t)))] \tag{6}$$

Where unwrap$\angle(HT(x))$ indicates the unwrapped phase of the Hilbert transform of the signal x.

B. Impact of Phase Error on Distributed Beamforming

To illustrate the impact of phase-offset on distributed transmit beamforming, let us consider a system of two transmitters and one receiver, each with its own independent LO. All nodes operate at the same carrier frequency f_c but each has a certain phase offset from a certain reference (*e.g.,* $0°$). Let the phase offset at the first transmit node, N1, be denoted as ϕ_1 . Let the second transmitter and its phase offset be denoted as N2 and ϕ_2 respectively. The receiver node, R, has a phase offset denoted by ϕ_R. The goal is for N1 and N2 to send a baseband signal $m(t)$ so that it adds up coherently at the receiver. Assume that the channels h_{N1-R} and h_{N2-R} are frequency flat and can be described by $h1$ and $h2$ respectively.

When Node i sends $m(t)$, R receives $r_i(t) = h_i m(t) exp^{j(\phi_i - \phi_R)}$. Receiver R then estimates the channel and sends back its estimates $\hat{h_1}$ and $\hat{h_2}$. Where $\hat{h_1} = h_1 exp^{j(\phi_R - \phi_1)}$ and $\hat{h_2} = h_2 exp^{j(\phi_R - \phi_2)}$. To achieve beamforming at the receiver and have $m(t)$ add up at the receiver node, N1, after receiving its downlink channel estimate, will transmit $\hat{h_1^*} m(t)$ and N2 will transmit $\hat{h_2^*} m(t)$. The received signal at R will be [10]:

$$r(t) = (|h_1|^2 exp^{j(2\phi_1 - 2\phi_R)} + |h_2|^2 exp^{j(2\phi_2 - 2\phi_R)}) m(t) \quad (7)$$

Taking the squared magnitude of $r(t)$ yields:

$$|r(t)|^2 = |m(t)|^2 (|h_1|^4 + |h_2|^4 + 2\cos(2\phi_1 - 2\phi_2)|h_1|^2 |h_2|^2) \quad (8)$$

The received signal power after beamforming with perfect and erroneous Channel State Information (CSI) at various phase offset values between the transmitters, using (8), is shown in Fig. 1(a). We can see that up to 30° of phase offset results in a small reduction (approx. 1 dB) in the beamforming gain. As the phase offset increases, however, we start to see significant reductions. After 50° of phase error, the beamformed signal is reduced by more than 3 dB which is the ideal expected gain in a 2×1 system, rendering the beamforming system useless. The same results and intuition applies to imperfect (*i.e.*, $\hat{h_n} \neq h_n$) CSI systems as it can be seen by the red lines in the same figure. We emphasize that regardless of the CSI quality, the inherit LO phase offset at each node will result in the above mentioned effects to some degree.

We also conducted a 2×1 distributed transmit beamforming experiment and investigated the impact of phase error between two chains on beamforming gain. We induced a controlled phase offset between two chains that were connected to the same LO in an SDR platform that was connected to two Tx antennas. The induced phase offset was introduced at baseband through a GNU Radio block. The Rx antenna was about 3 meters away. We recorded the received power as the beamforming system was experiencing a varying phase error from $-\pi$ to π in $\frac{\pi}{9}$ increments. The results of one experiment at one Rx antenna position is shown in Fig. 1(b). We can see that the beamformed received power decreased by 20 dB as the phase error changed from 0° to around 150°.

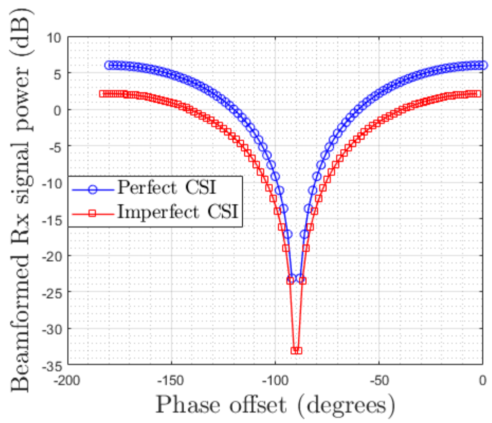
The above examples show the significant impact of phase error on distributed beamforming systems. Other works [3], [9], including ours [16], have shown such effects too. However, the nature of this error over time and across SDR devices have yet to be understood. This is the goal of this work.

IV. Experimental Results

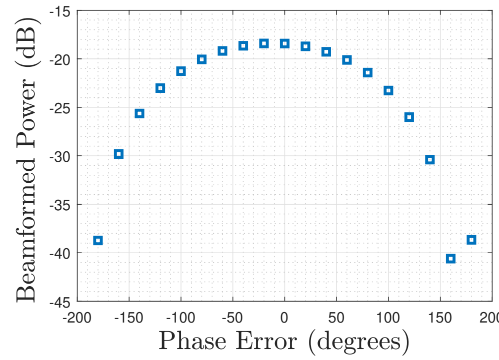
The measured behavior of phase offset within the same SDR and across different SDRs over multiple experiments and time durations is presented here. We start with one USRP and then consider distributed USRPs.

A. Phase Error in a Single Node/USRP

We first measure the phase offset between two RF chains of the same daughterboard, and two RF chains of two different daughterboards (different LOs) of the USRP N310. A daughterboard here means an AD (Analog Devices) transceiver, which has its own LO. Fig. 2 illustrates the experimental setup for this part.



(a)



(b)

Fig. 1. Beamforming gain versus phase error in 2-by-1 distributed transmit beamforming system. (a) calculated using (8), (b) experimental results.

Procedure: We transmit a sinusoidal signal which goes through a power splitter and reaches the two receiving RF chains. Going through an RF chain, the received signal will be multiplied by the carrier frequency plus/minus any CFO, and then low-pass filtered. The baseband signal is recorded in the form of I/Q samples. We work with the analytic signal of the I or Q component by using the Hilbert transform. We get the instantaneous phase of the received signal and determine the phase offset between the two received signals by subtracting one from the other after unwrapping, as denoted in (6). All experiments are at a sampling rate of $f_s = 240$ kS/s. The message frequency is $f_m = 10$ kHz and the carrier frequency is $f_c = 2.4$ GHz. The experiment duration ranged from 1 minute (in normal trials) to 5 minutes (in long trials). A new experiment is created by turning off the USRP for about 10 seconds and turning it back on. The room temperature was monitored throughout all experiments via a thermometer that was placed on the wall near the USRPs, and all experiments were conducted under the same room temperature.

Phase Error between two RF chains of the same daughterboard: Due to chain imbalance, slight phase offsets exists between I/Q components of the same daughterboard. This phase offset, however, is stable over time. Stability here means that the mean value and its standard deviation are approximately the same over any time duration. The distribution of the phase offset over time between two chains of the

TABLE I
PHASE ERROR BETWEEN TWO RF CHAINS OF THE SAME USRP N310 DAUGHTERBOARD

Phase offset stat.	Exp.1	Exp.2	Exp.3	Exp.4	Exp.5	Exp.6	Exp.7	Exp.8	Exp.9	Exp.10
Mean phase offset	-2.749	-2.7481	-2.7471	-2.7483	-2.75	3.534	3.531	3.5301	-2.7480	3.5332
Standard dev.	0.0326	0.0296	0.0067	0.0395	0.0181	0.0222	0.0335	0.0135	0.0219	0.023

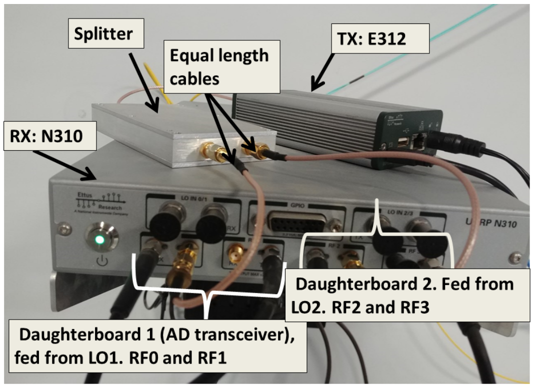


Fig. 2. Measurement setup of phase error for a single USRP.

same daughterboard follows a Normal distribution with a mean absolute error (MAE) between the measurements and the fit at the order of 10^{-5}. An example of this phase offset is shown in Fig. 3. In this experiment, the mean phase offset value was 0.1063 rad (approximately 6 degrees) and its standard deviation was 0.0046 rad. This was the minimum phase offset value experienced in all experiments.

Table I summarizes the results of ten experiments that investigated the phase offset between RF chains of the same N310 daughterboard. We make the following observations:

- In contrast to the phase offset obtained in the different boards setups, the phase offset between two chains of the same transceiver is consistent across different experiments. That is, even after restarting the USRP, it either stays the same or changes by 2π. This behavior can be seen in Table I where the phase offset is either approximately -2.74 or around 3.5 which is $-2.74 + 2\pi$. A similar observation was made in [11].
- Phase offset between two branches of the same transceiver exists. However, this phase offset is stable over time with its mean and standard deviation hardly changing over the course of up to 5 minutes. Moreover, the distribution of this offset over time follows a Normal distribution, as previously indicated. This observation is true for all experiments within the same USRP and for any tested time duration up to 5 minutes. Fig. 3(b) shows the histogram of the obtained phase offset value in one of the experiments which lasted for 1 minute.
- The standard deviation of the phase offset is small and does not exceed $1.8°$. Its average value over 10 experiments is 0.024 rad with a minimum value as low as 0.004 rad (0.2 degrees).

Remarks: If the above measured intra-board phase offset value is not compensated for in a distributed beamforming system, for instance, reductions in the expected beamforming gain will be inevitable. For example, at a phase offset value of $\Delta\phi = -2.749$ rad, an approximate 2 dB loss will be

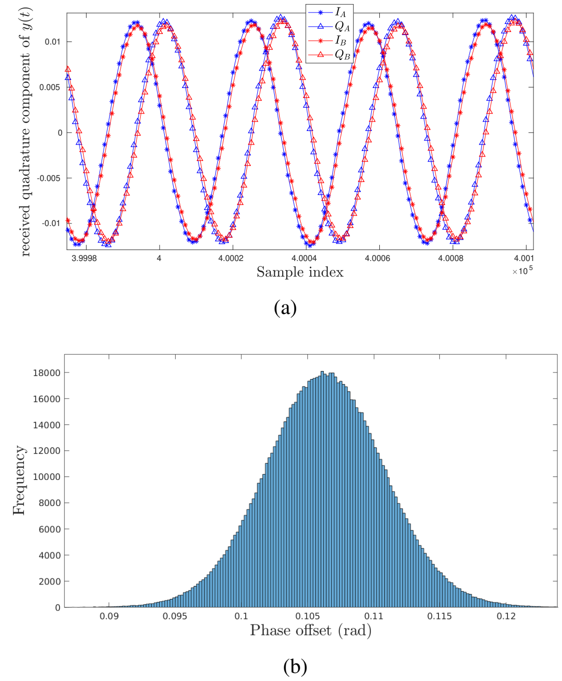
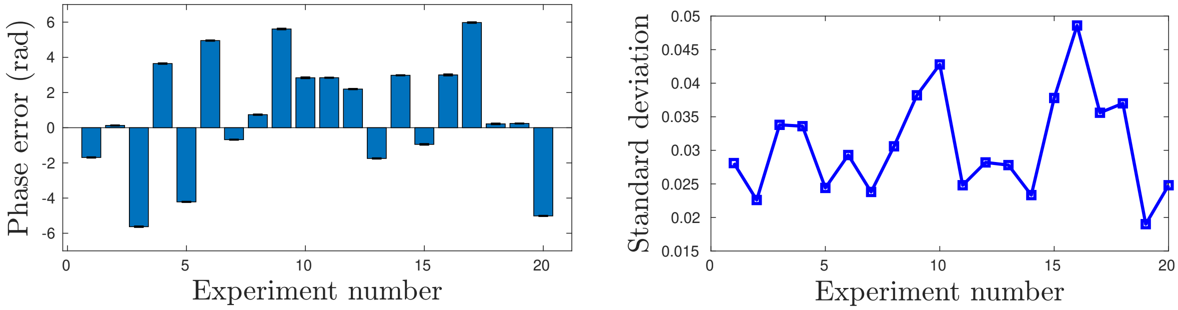


Fig. 3. (a) The received I/Q components of the two chains (denoted as A and B) within the same board. (b) The histogram of the phase offset between the two chains with a mean value of approximately 6 degrees.

experienced in a 1×2 system. This 2 dB loss is significant compared to the expected beamforming gain of 3 dB. More severe effects can be experienced if the phase offset takes on a value near $90°$. For example, at $\Delta\phi = -100°$ the beamformed signal will experience 10 dB reduction in its power, *defeating the purpose of the beamforming system and resulting in destructive combining*. Fortunately, and according to our measurements, a one-time calibration of the intra-board phase offset is sufficient for compensation as this offset is stable over time and consistent over different experiments. Specifically, the phase offset stays approximately constant up to 5 minutes with a standard deviation not exceeding $1.7°$.

Phase Error between two chains of different daughterboards (same USRP): Now, we investigate the phase offset between two chains of different boards within the same USRP. This means the two RF chains belong to two different LOs within the same USRP N310. We investigate stability over time and behavior over trials – the same investigation of the previous section. We conduct more than 20 experiments during which the phase offset between the two chains is measured and analyzed. Refer to Fig. 2 for illustration of RF1 and RF2 chains. The time duration of 20 experiments was 1 minute while for the 3 additional long experiments the duration ranged from 3 to 5 minutes. In total, 23 experiments were conducted. After processing and analyzing the results of the unwrapped phase difference between the two chains, we make the following conclusions:



(a) (b)

Fig. 4. (a) Mean and (b) standard deviation of the phase error between two RF chains of two different boards of the same USRP N310. Each of the 20 experiments lasted for 1 minute.

- In all experiments (short and long) there exists a random phase offset between the two chains. This phase offset is random per new experiment (no consistency) and fairly stable over time per experiment.
- Across all experiments, the standard deviation of the phase offset does not exceed 0.05 rad (2.86°) with an average value over all experiments of 0.0307 rad. The mean phase error between the two chains in 20 experiments is shown in Fig. 4(a), and the standard deviation is shown in Fig. 4(b). It is evident that the mean value of this phase error is random per new experiment and that its standard deviation is small. We will see in the next section how this is not the case when we measure the phase error between distributed nodes. To visualize the stability of this phase offset over time, let us look at Fig. 5, which illustrates the phase offset in experiment 21 which lasted for 3 minutes. It is clear that the phase offset remains approximately the same throughout the experiment with a mean and a standard deviation of $\Delta\phi = -4.5508$ rad and $\sigma_\phi = 0.0222$ rad, respectively.
- The distribution of the phase error over time follows a Normal distribution. An example can be seen in Fig. 6 where the measured phase offset – for a period of 3 seconds – is plotted along with its Normal distribution fit. We can see the close proximity of the fit to the actual measurements. The MAE between the Normal fit and the actual measurements was calculated and found to be in the 1×10^{-5} to 5×10^{-5} range. This finding is true for any investigated time duration starting from one second all the way to 3 minutes. A similar finding regarding the distribution of phase error between two chains of the same SDR (USRP B210) can be found in [2].

Remarks: From the above analysis we conclude that phase errors will always exist between different chains. Over a time duration of an experiment – spanning a few seconds to 5 minutes – this phase offset can be modeled as a Normally distributed random variable, following $\mathcal{N}(\overline{\Delta\phi}, \sigma_\phi)$. The mean , $\overline{\Delta\phi}$ is uniformly distributed over experiments as it can take on any value in the $[-2\pi, +2\pi]$ range. The standard deviation of this phase offset, σ_ϕ, is approximately constant over the tested time of up to 5 minutes and it does not exceed 3°. It is important to note that due to the phase offset stability over time, a one-time calibration is sufficient to compensate for this offset between chains of the same USRP.

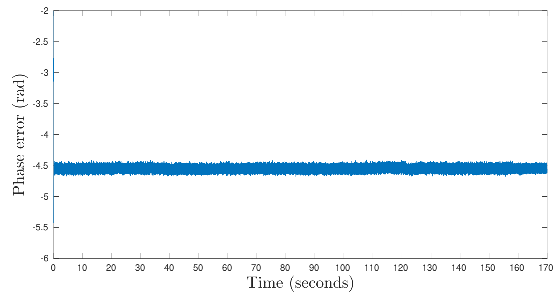


Fig. 5. Measured phase error between two chains over time. The mean and standard deviation of this phase offset do not change during the experiment.

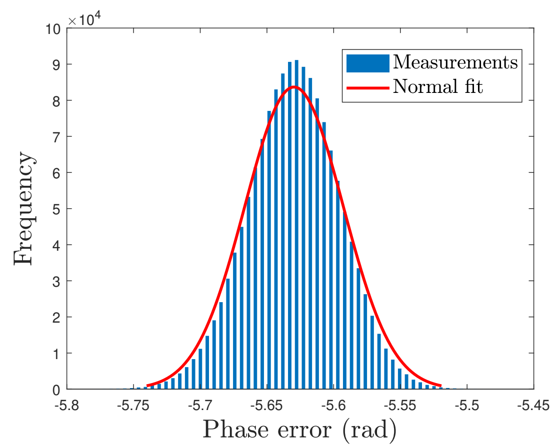
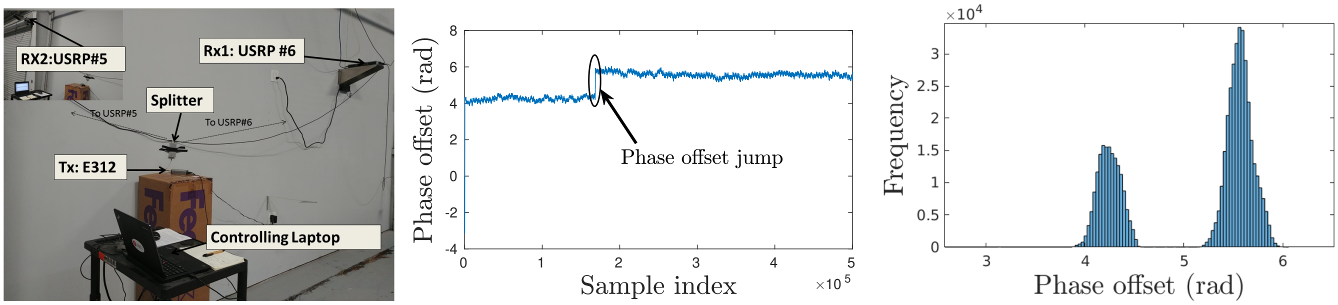


Fig. 6. Measurements and fitting of the phase offset between two different RF chains of two different boards within the same USRP N310. The measurements follow a Normal distribution for any time duration with a stable mean and standard deviation. Shown here is the offset for 3 seconds.

B. Phase Error in Distributed USRPs

In this section, we study the phase offset between distributed USRPs. Fig.7(a) shows the experiment setup. The two USRPs are geographically separated and are about 4 meters apart. We conducted 11 experiments, 10 of which lasted for 1 minute while the 11^{th} experiment lasted for 3 minutes. The mean and standard deviation of the phase error across all experiments are given in Table II. We make the following observations:

- Similar to the previous experiments, random phase offset between nodes exists. The mean of this phase offset is random per new experiment, as can be seen in Table II.
- In most experiments, this phase error is not stable over time. Phase drifts exist and cause the phase offset to jump



(a) (b) (c)

Fig. 7. (a) Measurement setup of phase offset for distributed USRPs. (b) Random phase offset jumps between two distributed USRPs. (c) The resultant distribution when phase offset jumps occur.

TABLE II
PHASE OFFSET BETWEEN TWO RF CHAINS OF DISTRIBUTED USRPS

Phase offset stat.	Exp.1	Exp.2	Exp.3	Exp.4	Exp.5	Exp.6	Exp.7	Exp.8	Exp.9	Exp.10	Exp.11
Mean phase offset (rad)	-0.7269	-1.7907	0.9819	2.0301	2.3659	-0.7867	-2.0351	-1.8836	1.3453	2.0642	-1.1128
Standard dev. (rad)	0.1468	1.3785	0.1551	1.4924	0.0907	0.1847	6.029	0.0975	0.132	0.0945	0.4793

to a new, and sometimes considerably different value. Consequently, the distribution of the phase offset will no longer follow the Normal distribution we observed in previous sections. Instead, the phase error will look like a multi-modal Normal distribution with multiple peaks depending on the number of the phase offset jumps. An example of this phase error jump and the resultant distribution are shown in Fig. 7.

- As a consequence of these phase drifts and random phase offset jumps, the statistical knowledge of the offset, such as its mean and standard deviation, can no longer be used in a one-time calibration as we suggested in the previous, centralized setups.

It is worth mentioning that similar drifts and phase offset jumps were observed in a distributed system that we have previously built using two USRP N210s that share the same clocking via a MIMO synchronization cable [15]. We refer the interested reader to [14] for more details.

V. CONCLUSION

We experimentally analyzed the behavior of the random phase offset that exists between two RF chains that share the same LO, two RF chains of different LOs but within the same USRP, and two RF chains of two geographically separated USRPs. We have shown that the stability and distribution of this phase error is strongly dependent on the chains it is calculated from. The mean and standard deviation of this phase error do not change over time if it is measured from two chains belonging to the same daughterboard. However, if this phase error is measured across two different daughterboards of the same USRP, and while its standard deviation is small (less than 3°), its mean is random per new experiment. Lastly, if this phase error is measured between two SDRs that belong to different/distributed USRPs, phase drifts occur and cause this phase offset to randomly jump to a new and different value many times over the course of an experiment.

Therefore, we suggest that a one-time calibration of this phase offset is only sufficient if the chains used belong to the same LO/daughterboard. In contrast, a continuous calibration of this phase error is needed if an SDR-based application is using multiple chains belonging to different/distributed SDRs, or different LOs within the same SDR.

VI. ACKNOWLEDGMENT

This work was supported by the NSF grant CNS-1909381.

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