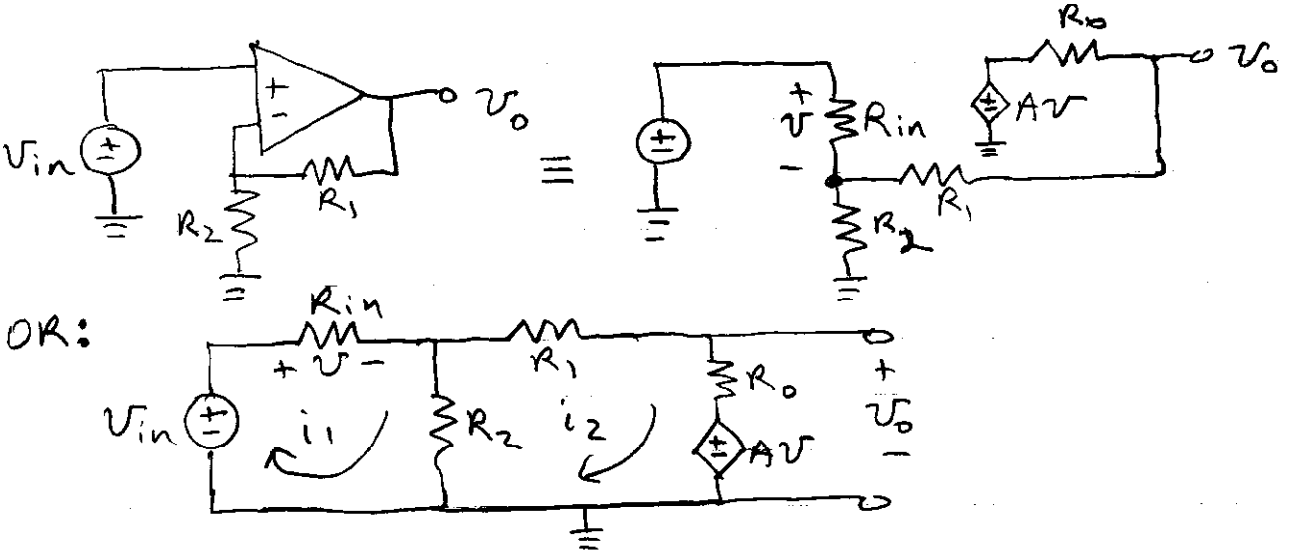
EE 5345 HW 1 – SOLUTIONS

1.



LOOP ANALYSIS:

① $V_{in} = i_1(R_{in}+R_2) - i_2R_2$

$0 = -i_1R_2 + i_2(R_2+R_1+R_o) + Ai_1R_{in}$

② $0 = i_1(AR_{in}-R_2) + i_2(R_2+R_1+R_o)$

$$i_1 = \frac{\begin{vmatrix} V_{in} & -R_2 \\ 0 & R_2+R_1+R_o \end{vmatrix}}{\begin{vmatrix} R_{in}+R_2 & -R_2 \\ AR_{in}-R_2 & R_2+R_1+R_o \end{vmatrix}}, \quad \beta = \frac{R_2}{R_1+R_2}$$

$$= \frac{V_{in}(R_2+R_1+R_o)}{(R_{in}+R_2)(R_2+R_1+R_o) + R_2(AR_{in}-R_2)}$$

$$Z_{in} = \frac{V_{in}}{i_1} = \frac{(R_{in}+R_2)(R_2+R_1+R_o) + R_2(AR_{in}-R_2)}{R_2+R_1+R_o}$$

$$= (R_{in}+R_2) + \frac{R_2(AR_{in}-R_2)}{R_2+R_1+R_o}$$

$$= R_{in}\left[1 + \frac{AR_2}{R_1+R_2+R_o}\right] + R_2 - \underbrace{\frac{R_2^2}{R_1+R_2+R_o}}_{\text{small compared to 1st term}}$$

$$\approx R_{in}(1+A\beta)$$

$$Z_{OUT} = \frac{V_{OC}}{i_{SC}}, \quad V_{OC} = i_2 R_O + AV$$
$$= i_2 R_O + A i_1 R_{in}$$

$$i_2 = \frac{\begin{vmatrix} R_{in}+R_2 & V_{in} \\ AR_{in}-R_2 & 0 \end{vmatrix}}{|\text{etc.}|}$$

$$= \frac{-V_{in}(AR_{in}-R_2)}{(R_{in}+R_2)(R_1+R_2+R_O)+R_2(AR_{in}-R_2)}$$

$$V_{OC} = \frac{-V_{in}R_O(AR_{in}-R_2)+AR_{in}V_{in}(R_1+R_2+R_O)}{(R_{in}+R_2)(R_1+R_2+R_O)+R_2(AR_{in}-R_2)}$$

$$= \frac{AR_{in}V_{in}(R_1+R_2)+V_{in}R_OR_2}{(R_{in}+R_2)(R_1+R_2+R_O)+R_2(AR_{in}-R_2)}$$

TO GET i_{SC}:

$i_{SC} = i_2 - i_3$

LOOP 1: $V_{in} = i_1(R_{in}+R_2) - i_2R_2$

LOOP 2: $0 = -i_1R_2 + i_2(R_1+R_2)$

LOOP 3: $0 = i_3R_O + AR_{in}i_1 = i_1AR_{in} + i_3R_O$

$$i_2 = \frac{\begin{vmatrix} (R_{in}+R_2) & V_{in} & 0 \\ -R_2 & 0 & 0 \\ AR_{in} & 0 & R_O \end{vmatrix}}{\quad} \doteq 0$$

$$= \frac{R_OR_2V_{in}}{R_O(R_{in}+R_2)(R_1+R_2)} \approx 0$$

$$i_3 = \frac{\begin{vmatrix} (R_{in}+R_2) & -R_2 & V_{in} \\ 0 & (R_1+R_2) & 0 \\ AR_{in} & 0 & 0 \end{vmatrix}}{\begin{vmatrix} (R_{in}+R_2) & -R_2 & 0 \\ 0 & (R_1+R_2) & 0 \\ AR_{in} & 0 & R_0 \end{vmatrix}}$$

$$= \frac{+V_{in} AR_{in}(R_1+R_2)}{R_0\left[(R_{in}+R_2)(R_1+R_2)\right]} = -i_{SC}$$

$$Z_{OUT} = \frac{V_{OC}}{i_{SC}} \doteq \frac{\left[AR_{in}V_{in}(R_1+R_2)+V_{in}R_0R_2\right]R_0(R_{in}+R_2)}{V_{in}AR_{in}\left[(R_{in}+R_2)(R_1+R_2+R_0)+R_2(AR_{in}-R_2)\right]}$$

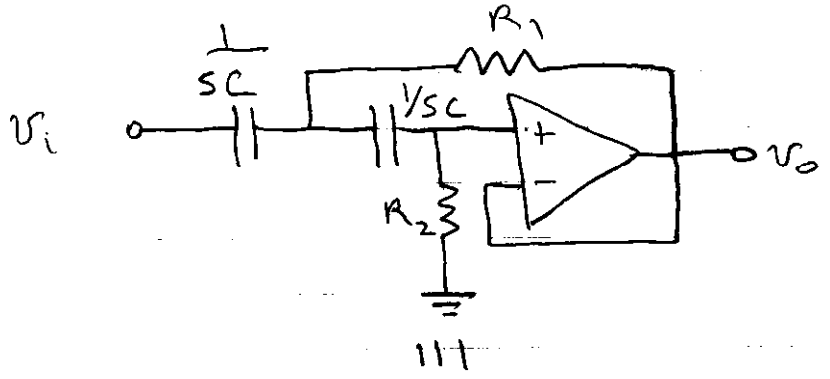
$$\doteq \frac{AR_{in}(R_1+R_2)(R_{in}+R_2)R_0}{AR_{in}(R_{in}+R_2)(R_1+R_2)+A^2R_{in}^2R_2}$$

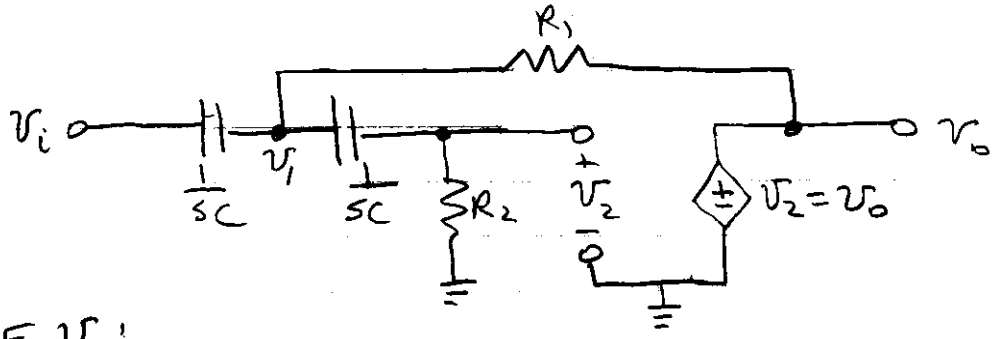
$$= \frac{R_0}{1+\dfrac{R_2AR_{in}}{(R_1+R_2)(R_{in}+R_2)}}, \quad R_2 \ll R_{in}$$

$$\approx \frac{R_0}{1+\beta A}$$

NOTE NON-INVERTING AMP ONLY APPROXIMATES IDEAL TYPE-I FEEDBACK RELATIONSHIPS FOR Z_{in} & Z_{OUT}. THIS IS DUE TO THE β NETWORK HAVING FINITE INPUT IMPEDANCE.

2.





NODE v_1:

$$(v_i - v_1)sC = \frac{v_1 - v_2}{R_1} + (v_1 - v_2)sC$$

$$v_i sC = v_1\left(\frac{1}{R_1} + 2sC\right) + v_2\left(-\frac{1}{R_1} - sC\right) \quad (1)$$

NODE v_2:

$$(v_1 - v_2)sC = v_2/R_2$$

$$0 = v_1 sC + v_2(-sC - 1/R_2) \quad (2)$$

$$v_2 = \frac{\begin{vmatrix} (2sC + 1/R_1) & v_i sC \\ sC & 0 \end{vmatrix}}{\begin{vmatrix} (2sC + 1/R_1) & -(sC + 1/R_1) \\ sC & -(sC + 1/R_2) \end{vmatrix}} = v_o$$

$$= \frac{+ v_i s^2C^2}{+(2sC + 1/R_1)(sC + 1/R_2) - sC(sC + 1/R_1)}$$

$$\frac{v_o}{v_i} = \frac{s^2C^2}{-s^2C^2 - \frac{sC}{R_1} + 2s^2C^2 + \frac{sC}{R_1} + \frac{2sC}{R_2} + \frac{1}{R_1R_2}}$$

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{2}{SCR_2} + \frac{1}{S^2 R_1 R_2 C^2}} = H(S)$$

CHECK:

IF WE SUBSTITUTE:

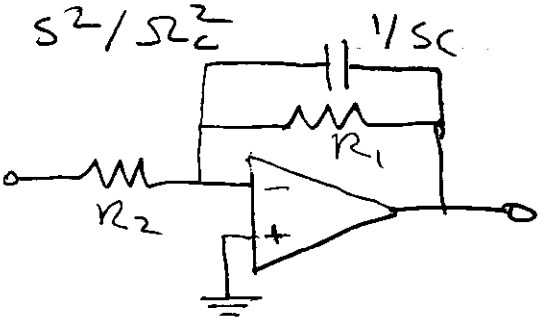
$$R_1 = \frac{1}{\sqrt{2} C \Omega_c}, \quad R_2 = \frac{\sqrt{2}}{C \Omega_c}$$

GET:

$$\frac{1}{1 + \frac{2}{SC\sqrt{2}/C\Omega_c} + \frac{1}{S^2 C^2 / C^2 \Omega_c^2}}$$

$$= \frac{1}{1 + \frac{\sqrt{2}}{(S/\Omega_c)} + \frac{1}{S^2/\Omega_c^2}} \checkmark$$

3. FIRST-ORDER BUTTERWORTH:



$$\frac{V_o(S)}{V_i(S)} = \frac{-\frac{R_1/SC}{(R_1 + 1/SC)}}{R_2}$$

$$= \frac{-R_1/R_2}{1 + SR_1C} = \frac{K}{1 + \left(\frac{S}{\Omega_c}\right)} = \frac{K}{B_1\left(\frac{S}{\Omega_c}\right)}$$

$\Omega_c = \frac{1}{R_1 C}$, $K = -R_1/R_2$, ↑ HAS FORM OF B_1

4.

R_f	R_g	R_1	R_2	(K)	CMRR (dB)
9.5	9.5	0.95	0.95		∞
9.5	9.5	0.95	1.05		40.42
9.5	9.5	1.05	0.95		40.42
9.5	9.5	1.05	1.05		∞
9.5	10.5	0.95	0.95		41.21
9.5	10.5	0.95	1.05		∞
9.5	10.5	1.05	0.95		34.79 ←
9.5	10.5	1.05	1.05		40.42
10.5	9.5	0.95	0.95		41.21
10.5	9.5	0.95	1.05		34.79 ←
10.5	9.5	1.05	0.95		∞
10.5	9.5	1.05	1.05		40.42
10.5	10.5	0.95	0.95		315.88
10.5	10.5	0.95	1.05		41.21
10.5	10.5	1.05	0.95		41.21
10.5	10.5	1.05	1.05		∞