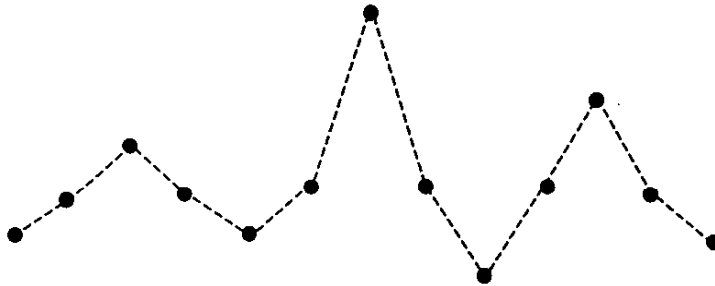


1. An ECG waveform, after digitizing has the following appearance:

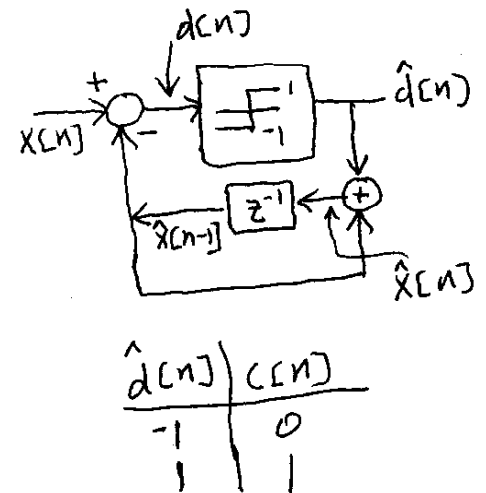


The corresponding samples are: $x[n] = \{0, 1, 2, 1, 0, 1, 3, 1, -1, 1, 2, 1, 0\}$. Find the code words $c[n]$ generated when this waveform is passed through a Delta modulator having $\alpha = 1$ and $\Delta = 1$. Assume zero is quantized to $+\Delta$. (10 pts)

$$\hat{d}[n] = d[n] + e[n], \quad e[n] = \hat{d}[n] - d[n], \quad \hat{x}[n] = \hat{x}[n-1] + \hat{d}[n]$$

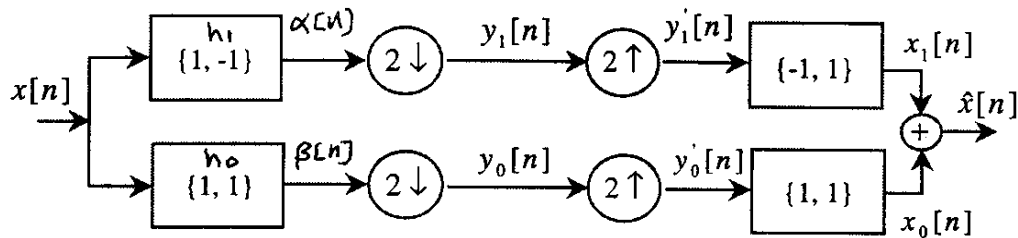
$$d[n] = x[n] - \hat{x}[n-1]$$

n	$x[n]$	$\hat{x}[n-1]$	$d[n]$	$\hat{d}[n]$	$\hat{x}[n]$	$c[n]$
0	0	0	0	1	1	1
1	1	1	0	1	2	1
2	2	2	0	-1	3	1
3	1	3	-2	-1	2	0
4	0	2	-2	-1	1	0
5	1	1	0	1	2	1
6	3	2	1	1	3	1
7	1	3	-2	-1	2	0
8	-1	2	-3	-1	1	0
9	1	1	0	1	2	1
10	2	2	0	1	3	1
11	1	3	-2	-1	2	0
12	0	2	-2	-1	1	0



¹ This exam was taken in accordance with the Southern Methodist University Honor Code

2. (a) Find the output of the following filter bank when the input is the waveform in problem 1. The notation here is $\{z[0], z[1]\}$ where z can be h_0, h_1, g_0 or g_1 . Please show all your work. (10 pts) (b) Do $h_0[n]$ and $h_1[n]$ satisfy the Smith-Barnwell constraint for perfect reconstruction? Why? (5 pts extra credit)



(a)

$$X[n] = \{0, 1, 2, 1, 0, 1, 3, 1, -1, 1, 2, 1, 0\}$$

$$\alpha[n] = \{0, 1, 1, -1, -1, 1, 2, -2, -2, 2, 1, -1, -1, 0\}$$

$$y_1[n] = \{0, 1, -1, +2, -2, 1, -1\}$$

$$y_1'[n] = \{0, 0, 1, 0, -1, 0, 2, 0, -2, 0, 1, 0, -1, 0\}$$

$$x_1[n] = \{0, 0, -1, 1, 1, -1, -2, 2, 2, -2, -1, 1, 1, -1, 0\}$$

$$\beta[n] = \{0, 1, 3, 3, 1, 1, 4, 4, 0, 0, 3, 3, 1, 0\}$$

$$y_0[n] = \dots$$

$$y_0'[n] = \{0, 0, 3, 0, 1, 0, 4, 0, 0, 0, 3, 0, 1, 0\}$$

$$x_0[n] = \{0, 0, 3, 3, 1, 1, 4, 4, 0, 0, 3, 3, 1, 1, 0\}$$

$$\hat{X}[n] = \{0, 0, 2, 4, 2, 0, 2, 6, 2, -2, 2, 4, 2, 0, 0\}$$

$$\hat{X}[n] = 2X[n-1] \Rightarrow N=1$$

(b) SMITH-BARNWELL CONSTRAINT:

$$H_1(e^{j\omega}) = -H_0(-e^{-j\omega})e^{-j\omega}$$

$$H_0(e^{j\omega}) = 1 + e^{-j\omega}$$

$$H_0(e^{-j\omega}) = 1 + e^{j\omega}$$

$$H_0(-e^{-j\omega}) = 1 - e^{j\omega}$$

$$H_0(-e^{-j\omega})e^{-j\omega} = e^{-j\omega} - 1$$

$$-H_0(-e^{-j\omega})e^{-j\omega} = 1 - e^{-j\omega} = H_1(e^{j\omega}) \checkmark$$

SO THIS IS A PERFECT RECONSTRUCTION FILTER BANK

3. Design a syntactic detector capable of differentiating between normal sinus rhythm (NSR) and an abnormal rhythm. Assume that the waveform found in problem 1 is exemplary of NSR and reflects the sampling rate for this system. The detector should run continuously and if an abnormal rhythm is detected, then an alarm should sound. Use an approach similar to that used by Furno and Tompkins. (10 pts)

GRAMMAR SYMBOLS:

BIGUP: $SLOPE > 1.5$

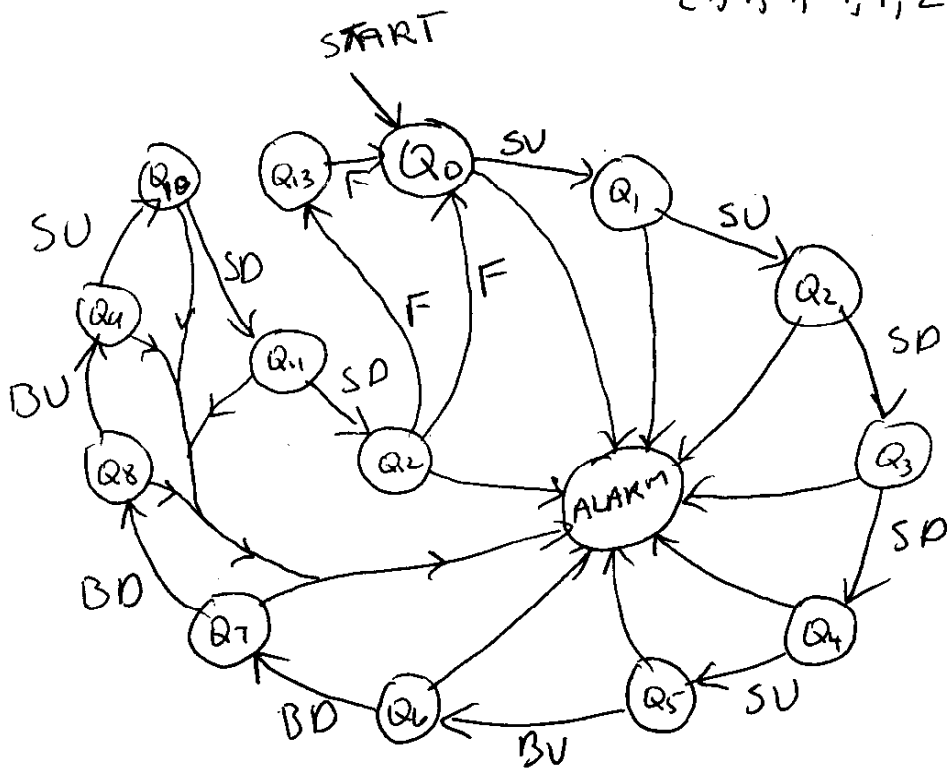
SMALLUP: $1.5 > SLOPE \geq 0.5$

FLAT: $0.5 > SLOPE \geq -0.5$

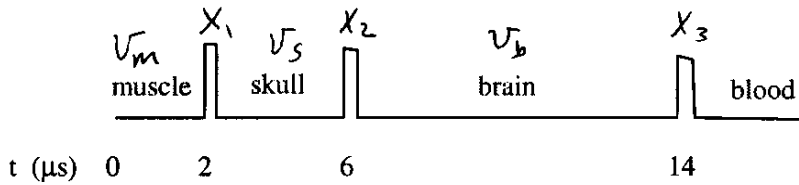
SMALLDOWN: $-0.5 > SLOPE \geq -1.5$

BIGDOWN: $-1.5 > SLOPE$

SLOPE FROM 1ST-DIFF {1, 1, -1, -1, 1, 2, -2, -2, 2, 1, -1, -1}



4. Consider the following A-mode ultrasound scan as displayed on an oscilloscope:



The tissue types are known and have been labeled. Determine the depth in meters for each tissue boundary. (10 pts)

$$2 \times 10^{-6} = 2 X_1 / v_m \quad \textcircled{1}$$

$$\begin{aligned} 6 \times 10^{-6} &= 2 X_1 / v_m + 2 (X_2 - X_1) / v_s \\ &= 2 X_1 (v_m^{-1} - v_s^{-1}) + 2 X_2 v_s^{-1} \quad \textcircled{2} \end{aligned}$$

$$\begin{aligned} 14 \times 10^{-6} &= 2 X_1 / v_m + 2 (X_2 - X_1) / v_s + 2 (X_3 - X_2) / v_b \\ &= 2 X_1 (v_m^{-1} - v_s^{-1}) + 2 X_2 (v_s^{-1} - v_b^{-1}) \\ &\quad + 2 X_3 v_b^{-1} \quad \textcircled{3} \end{aligned}$$

$$v_m = 1585 \text{ m/s}$$

$$v_s = 4080 \text{ m/s}$$

$$v_b = 1451 \text{ m/s}$$

$$\text{FROM } \textcircled{1}: X_1 = 2 \times 10^{-6} v_m / 2 = 2 \times 10^{-6} \times 1585 / 2 = \underline{\underline{1.6 \text{ mm}}}$$

$$\text{FROM } \textcircled{2}: \frac{6 \times 10^{-6} - 2 X_1 (v_m^{-1} - v_s^{-1})}{2 v_s^{-1}} = X_2 = \underline{\underline{9.7 \text{ mm}}}$$

$$\text{FROM } \textcircled{3}: \frac{14 \times 10^{-6} - 2 X_1 (v_m^{-1} - v_s^{-1}) - 2 X_2 (v_s^{-1} - v_b^{-1})}{2 v_b^{-1}} = X_3$$

$$X_3 = \underline{\underline{15 \text{ mm}}}$$