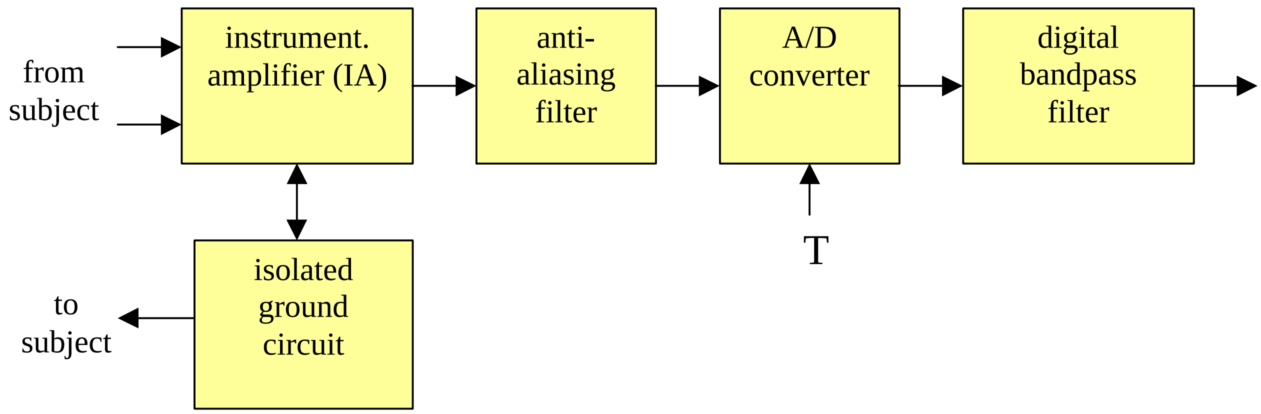
March 22, 1999 Name:____________________

Instructor: C. Davila

The exam concerns the following block diagram for measuring an EEG signal:



1. Design an instrumentation amplifier for measuring an EEG signal, the desired output should be 5 volts, typically (see Table 1 of Webster) and the IA should have high CMRR. Also design an isolated ground circuit that provides a grounding point to the subject while keeping him isolated from earth ground. Why is isolation from ground important?

2. Design an anti-aliasing (lowpass) 2-pole analog filter that cuts off at 30 Hz. Use a single operational amplifier. What is the gain of the filter at 0 Hz? What is the gain of the filter at 30 Hz?

3. Determine the sampling interval T such that the signal level at one half the sampling rate has been attenuated by 0.01 relative to the gain of the anti-aliasing filter at 0 Hz.

4. Determine what operation should be applied to the coefficients of an FIR lowpass filter to convert it to a bandpass filter. Hint: use the frequency shifting property of the Fourier Transform (i.e. multiplication of h[n] by $\exp(j\omega_0 n)$ produces a certain frequency shift in $H(e^{j\omega})$).

5. Use the result of the previous problem to design a 5-tap FIR bandpass filter that has a passband between 0.2π and 0.6π radians.

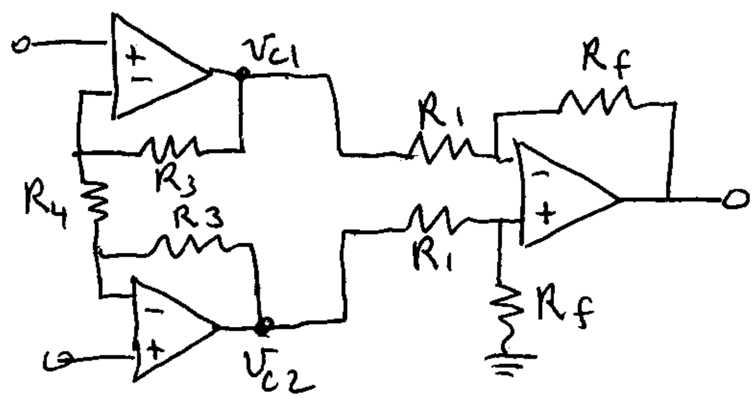
SOLUTIONS

1. FROM TABLE 1.1 : TYPICAL AMPLITUDE OF EEG IS 100 μV

WANT TO BOOST THIS TO 5V SO

$$A_d = \frac{5V}{100 \times 10^{-6} V} = 5 \times 10^4 .$$

WE MUST DESIGN OUR AMP TO HAVE A DIFFERENTIAL GAIN OF 50,000.



$$A_d = \frac{R_f}{R_1}\left(\frac{2R_3 + R_4}{R_4}\right)$$

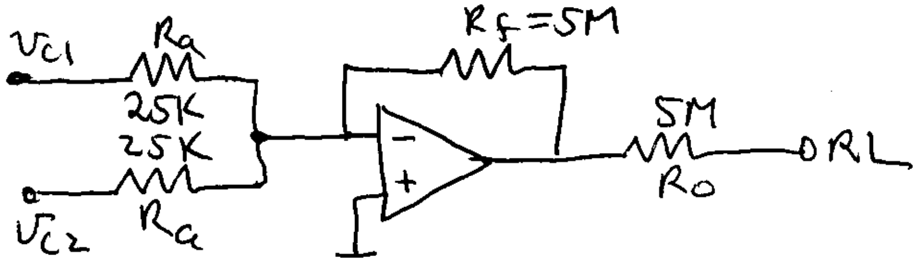
ANY COMBINATION WORKS

$R_3 = 22K\Omega$ $R_4 = 1\ K\Omega \Rightarrow \left(\frac{2R_3+R_4}{R_4}\right) = 45$

$R_f = 1000K\Omega$ $R_1 = 900\Omega$

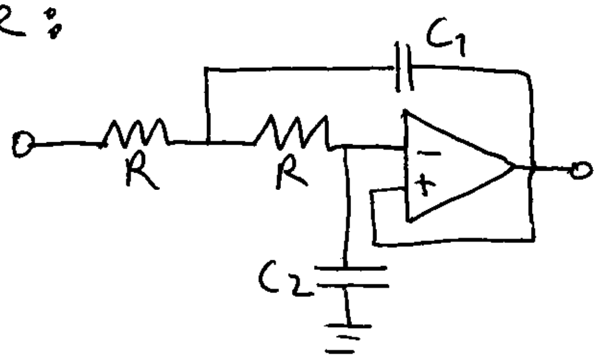
$= 1M\Omega$ WOULD DO IT (ASSUMING IDEAL OP AMPS)

ISOLATION: USE DRIVEN RIGHT LEG CIRCUIT:



ISOLATION IS IMPORTANT BECAUSE IT PREVENTS A PATIENT FROM PROVIDING A PATH TO GROUND FOR CURRENT.

2. ANTI-ALIASING FILTER IS A LOWPASS FILTER:



$$\Omega_c = 2\pi \times 30 = 60\pi$$

PICK $R = 10K\Omega$

$$C_1 = \frac{\sqrt{2}}{R\Omega_c} = \frac{\sqrt{2}}{10^4 \times 60\pi} \doteq 0.75\,\mu F$$

$$C_2 = \frac{1}{\sqrt{2}\,R\Omega_c} = \frac{1}{\sqrt{2} \times 10^4 \times 60\pi} \doteq 0.375\,\mu F$$

GAIN OF FILTER AT 0 Hz = 0dB OR 1
GAIN OF " " 30 Hz = −3dB OR $1/\sqrt{2}$

- THIS IS TRUE OF ALL BUTTERWORTH FILTERS.

3. WE NOW SEEK SAMPLING INTERVAL T SO THAT $|H(j\Omega_N)| = 0.01$ WHERE $\Omega_N = \frac{1}{2}$ SAMPLING FREQ.

$$= \frac{1}{2} \cdot \frac{2\pi}{T} = \frac{\pi}{T}$$

$$H(j\Omega_N) = \frac{1}{1 - \left(\frac{\Omega_N}{\Omega_c}\right)^2 + j\sqrt{2}\left(\frac{\Omega}{\Omega_c}\right)}$$

$$|H(j\Omega_N)|^2 = \frac{1}{1 + \left(\frac{\Omega_N}{\Omega_c}\right)^4} = 10^{-4}$$

$$10^4 = 1 + \left(\frac{\Omega_N}{\Omega_c}\right)^4$$

$$\frac{\Omega_N}{\Omega_c} = (10^4 - 1)^{1/4}$$

$$\Omega_N = \Omega_c (10^4 - 1)^{1/4} = \frac{\pi}{T}$$

$$T = \frac{\pi}{60\pi (10^4 - 1)^{1/4}} = 0.0016667$$

4. FREQ. SHIFTING PROPERTY:

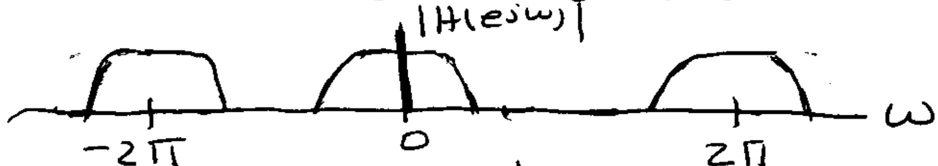
IF $\mathcal{F}\{h[n]\} = H(e^{j\omega})$

THEN $\mathcal{F}\{e^{+j\omega_0 n} h[n]\} = \sum_{n=-\infty}^{\infty} h[n] e^{j\omega_0 n} e^{-j\omega n}$

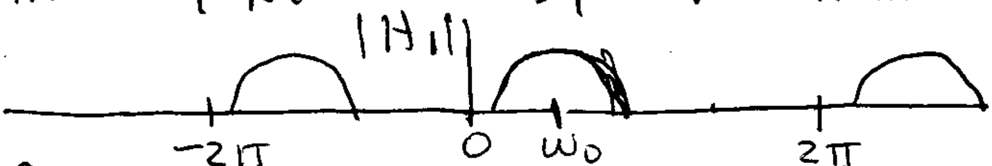
$$= \sum_{n=-\infty}^{\infty} h[n] e^{-j(\omega-\omega_0)n}$$

$$= H(e^{j(\omega-\omega_0)})$$

SO IF $H(e^{j\omega})$ IS LOW PASS:

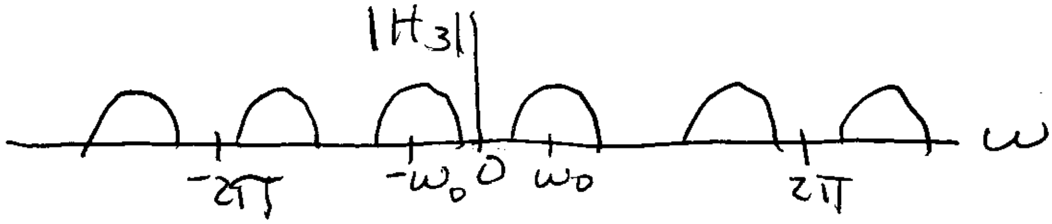


THEN $|H(e^{j(\omega-\omega_0)})|$ LOOKS LIKE:



THIS IS TECHNICALLY A BANDPASS FILTER BUT IT DOESN'T HAVE REAL COEFFICIENTS WHICH IS DESIREABLE, SO BETTER TO ALSO SHIFT BY $+\omega_0$ ($e^{-j\omega_0 n} h[n]$)

ADDING $H_1(e^{j\omega}) + H_2(e^{j\omega})$ GIVES:



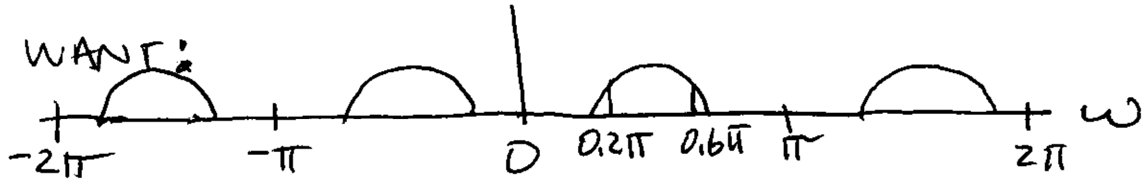
WHICH HAS THE FORM OF A BANDPASS FILTER AS SEEN ON SLIDE 139. IN TIME DOMAIN

$h_3[n] = h[n](e^{j\omega_0 n} + e^{-j\omega_0 n})$

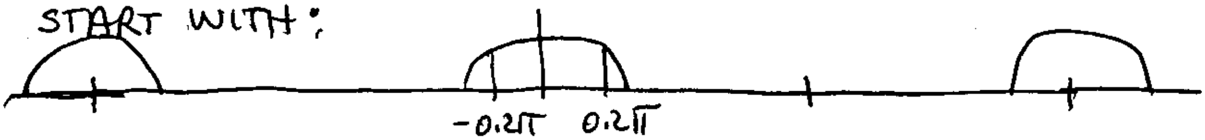
$= 2h[n]\cos\omega_0 n$

BP

5. WANT:



START WITH:



$\Rightarrow \quad \omega_0 = 0.4\pi$

NOW DESIGN LOWPASS FILTER:

$$h_{LP}[n] = \frac{\sin(n\,0.2\pi)}{\pi n} \qquad n = 0, \pm 1, \pm 2$$

$$h_{BP}[n] = 2h_{LP}[n]\cos(0.4\pi n)$$