Detection of Steady-State VEP's: Toward Electrophysiologic Measurement of Visual Acuity

Carlos E. Davila
Electrical Engineering Department
Southern Methodist University
Dallas, TX, USA

Collaborators and Acknowledgements

- Dick Srebro, University of Texas Southwestern Medical Center, Dallas, Texas, USA
- Ibrahim Ghaleb, Nortel, Dallas, Texas, USA
- Masoud Azmoodeh, Motorola, Fort Worth, Texas, USA
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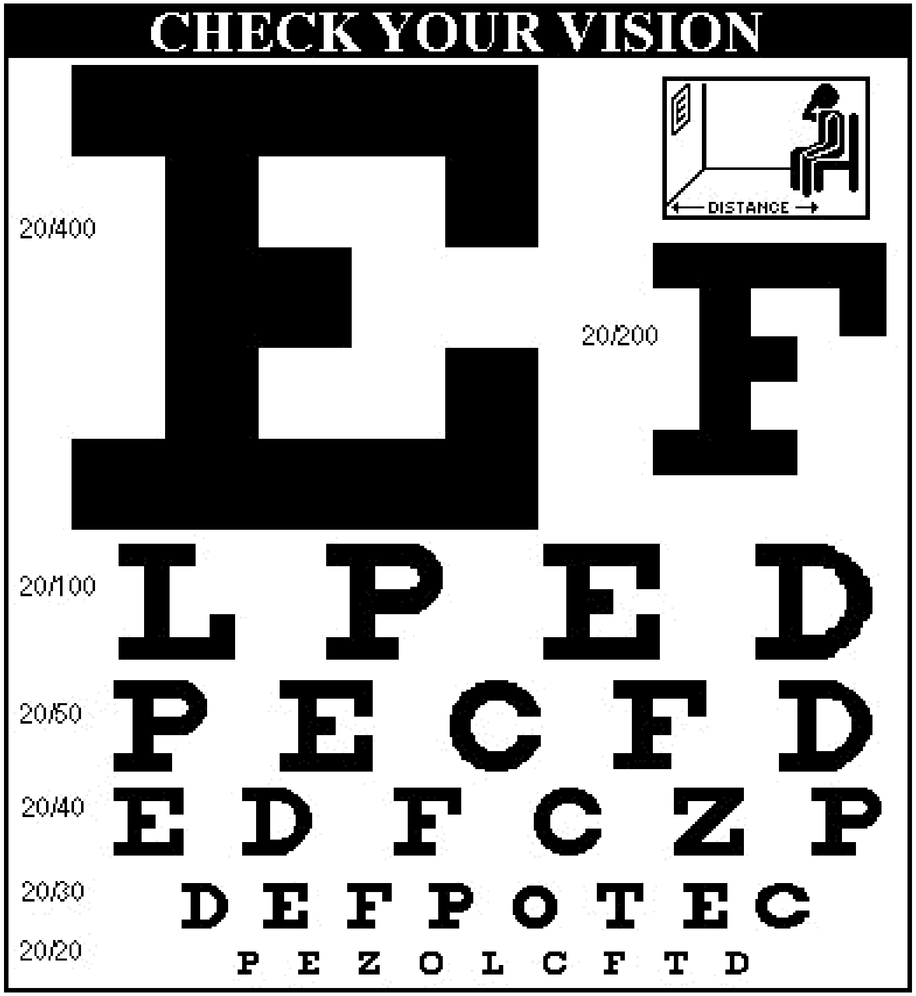
Overview

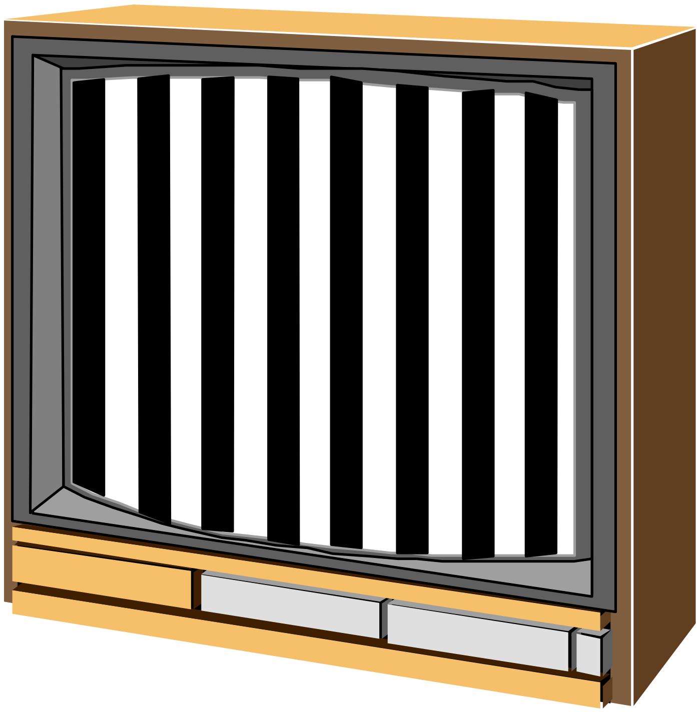
- Psychophysical acuity measurement
- Objective acuity measurement
- Overview of VEP detection algorithms
- Matched Subspace Filtering (MSF)
- Prewhitening
- Experimental Results
- Multichannel MSF Detection
- Experimental Results
- Conclusions

Snellen Chart

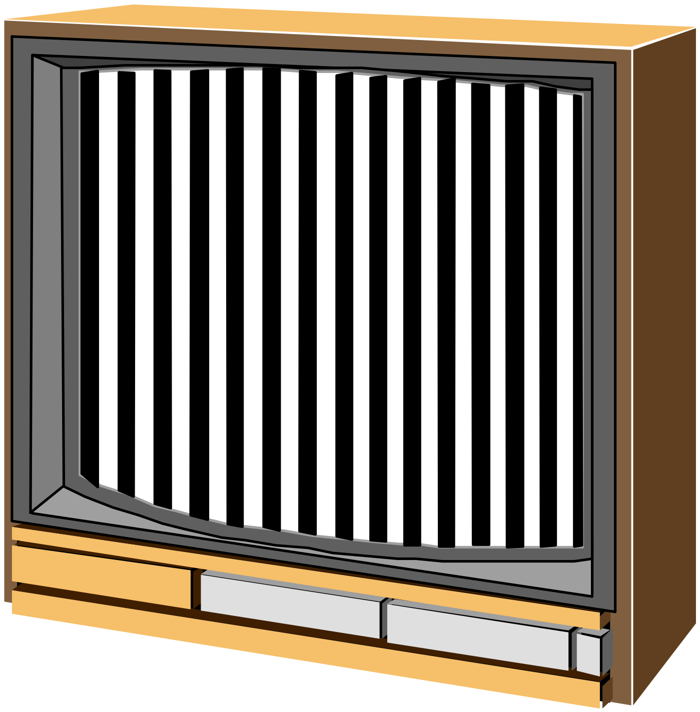


20/200 acuity implies: visual system resolves 200/20=10 minutes of arc

Contrast Grating



low spatial frequency grating



high spatial frequency grating

Psychophysical Grating Acuity

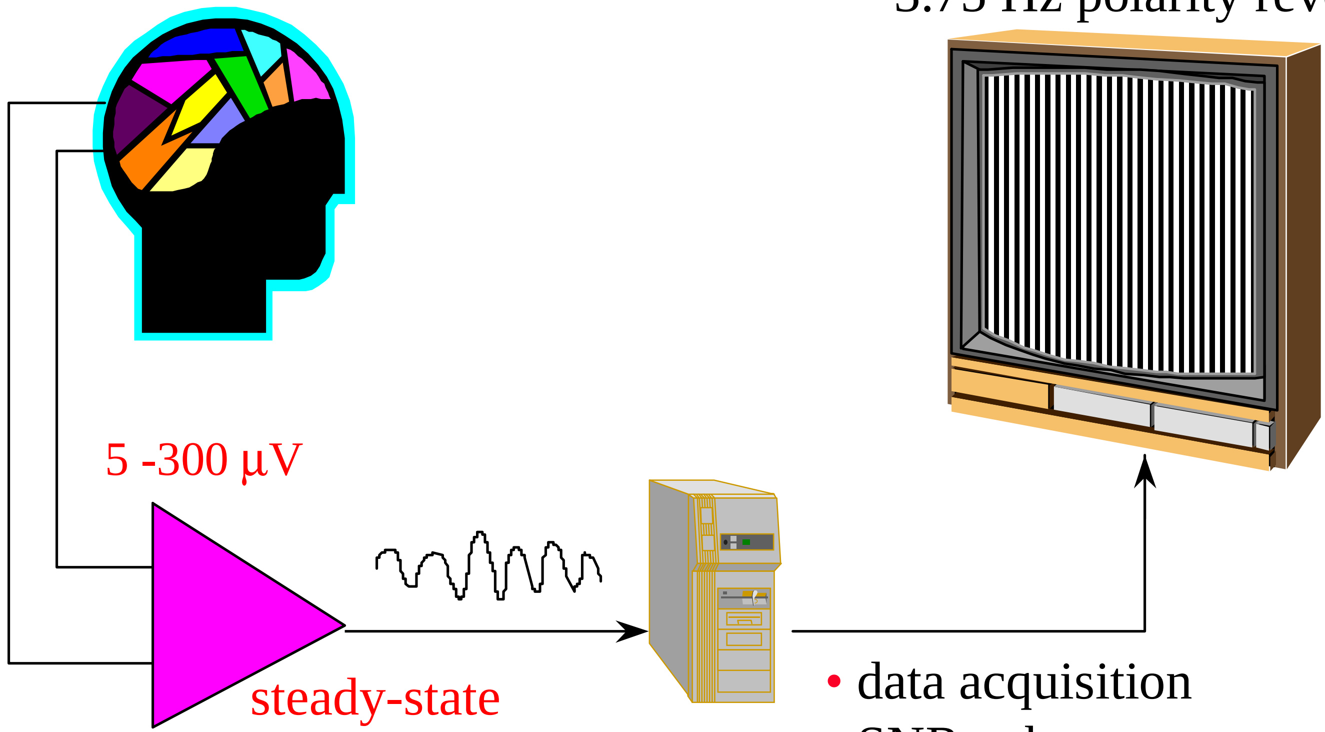
- Subject views a contrast grating as spatial frequency of grating is increased.
- When the bars can no longer be detected, subject presses a button.
- Spatial frequency at which subject no longer distinguishes separate bars is the grating acuity.
- Typical grating acuity: 28 cycles/degree (c/d)

Why objective acuity measurement?

- Snellen acuity measurements unreliable through first 3 years of life
- amblyopia can be reversed if treated early
- some patients have disturbed consciousness: mentally retarded, cerebral paulsey, head injury, alzheimer’s

Objective Measurement of Visual Acuity

sinusoidal grating stimulus
3.75 Hz polarity reversals



steady-state
visual evoked potential
(SSVEP)

- data acquisition
- SNR enhancement
- detection
- spatial frequency/contrast control

Overview of Detection Algorithms

Notation:

$$x = \begin{bmatrix} x_1^T & x_2^T & \cdots & x_L^T \end{bmatrix}^T \sim M \times 1$$

x_i: response to ith contrast reversal

Ensemble Average:

$$\bar{x} = \frac{1}{L} \sum_{k=1}^{L} x_k$$

Generalized T^2 Statistic (Picton et al., 1987)

DFT of ith response:

$$X_i = \begin{bmatrix} 1 & \cos(2\pi f_k) & \cdots & \cos(2\pi (N-1) f_k) \\ 1 & \sin(2\pi f_k) & \cdots & \sin(2\pi (N-1) f_k) \end{bmatrix} x_i$$

sample mean:

$$\bar{X} = \frac{1}{L} \sum_{k=1}^{L} X_k$$

sample covariance:

$$C = \frac{1}{L} \sum_{k=1}^{L} (X_k - \bar{X})(X_k - \bar{X})^T$$

Generalized T^2 Statistic (cont.)

if X_i is $N(\mu, \Sigma)$

likelihood ratio test for:

$$H_0\text{: } \mu = 0 \qquad \text{vs.} \qquad H_1\text{: } \mu > 0$$

$$T^2 = L\overline{X}^T C^{-1} \overline{X}$$

$\frac{L-2}{2(L-1)} T^2$ has density: $F(2, L-2)$

Circular T^2 Statistic (Victor and Mast, 1991)

DFT:

$$Y_i = \begin{bmatrix} 1 & \exp(-j2\pi f_k) & \cdots & \exp(-j2\pi(N-1)f_k) \end{bmatrix} x_i$$

$$\sim N(\mu, \Sigma)$$

$$H_0: \mu = 0 \quad \text{vs.} \quad H_1: \mu > 0$$

$$T_{circ}^2 = (L-1)\frac{(L-1)\left|\bar{Y}\right|^2}{\sum_{k=1}^{L}\left|Y_k - \bar{Y}\right|^2} \qquad \bar{Y} = \frac{1}{L}\sum_{k=1}^{L} Y_k$$

LT_{circ}^2 has density: $F(2,\ 2L-2)$

Rayleigh Phase Criterion (Kuwada et al, 1986)

$$R_p = \frac{1}{L}\sqrt{\left(\sum_{k=1}^{L}\cos\theta_k\right)^2 + \left(\sum_{k=1}^{L}\sin\theta_k\right)^2}$$

compared against a table of thresholds for a given false alarm rate.

Record Orthogonality Test by Permutation (ROTP) (Achim, 1995)

- Nonparametric
- Looks at power in ensemble averages obtained using all possible sign permutations of each single trial.
- If the power in the average corresponding to all "+" signs is in the top 5% of all ensemble average powers, a detection is made.

Psychophysical vs. Objective Acuity

- Poor agreement exists between psychophysical and objective acuity measurements:
  - D. Weiner, K. Wellish, J. Nelson, and M. Kupersmith, "Comparisons among Snellen, psychophysical, and evoked potential acuity determinations," *Am. J. Optom. Physiol. Opt.*, vol.62, pp. 669-679, 1985.
  - D. Allen, A. Norcia, and C. Tyler, "Comparative study of electrophysiological and psychophysical measurement of the contrast sensitivity function in humans," *Am. J. Optom. Physiol. Opt.*, vol.63, pp. 442-449, 1986.
- Most likely reason is inadequate detection performance.

Matched Subspace Filtering (Scharf, 1991)

Assmptions:

- Signal s lies in a known subspace.

$$s = \sum_{k=1}^{2N} \alpha_k s_k$$

- Exact form of signal (α_k) is unknown.
- Additive white Gaussian noise power is unknown.

Want a statistic which is uniformly most powerful under these three constraints.

Matched Subspace Filtering (cont.)

MSF Statistic:

$$f = \frac{M-2N}{2N} \frac{x^T P_s x}{x^T (I - P_s) x}$$

$$P_s = S\left(S^T S\right)^{-1} S^T \qquad S = \begin{bmatrix} s_1 & s_2 & \cdots & s_{2N} \end{bmatrix}$$

f has density $F(2N, M\text{-}2N)$

VEP Signal Model

The VEP consists of *N* even harmonics of the contrast reversal frequency:

$$s_k(n) = \cos(2\pi f_k n),\ k = 1,2,\ldots,N$$

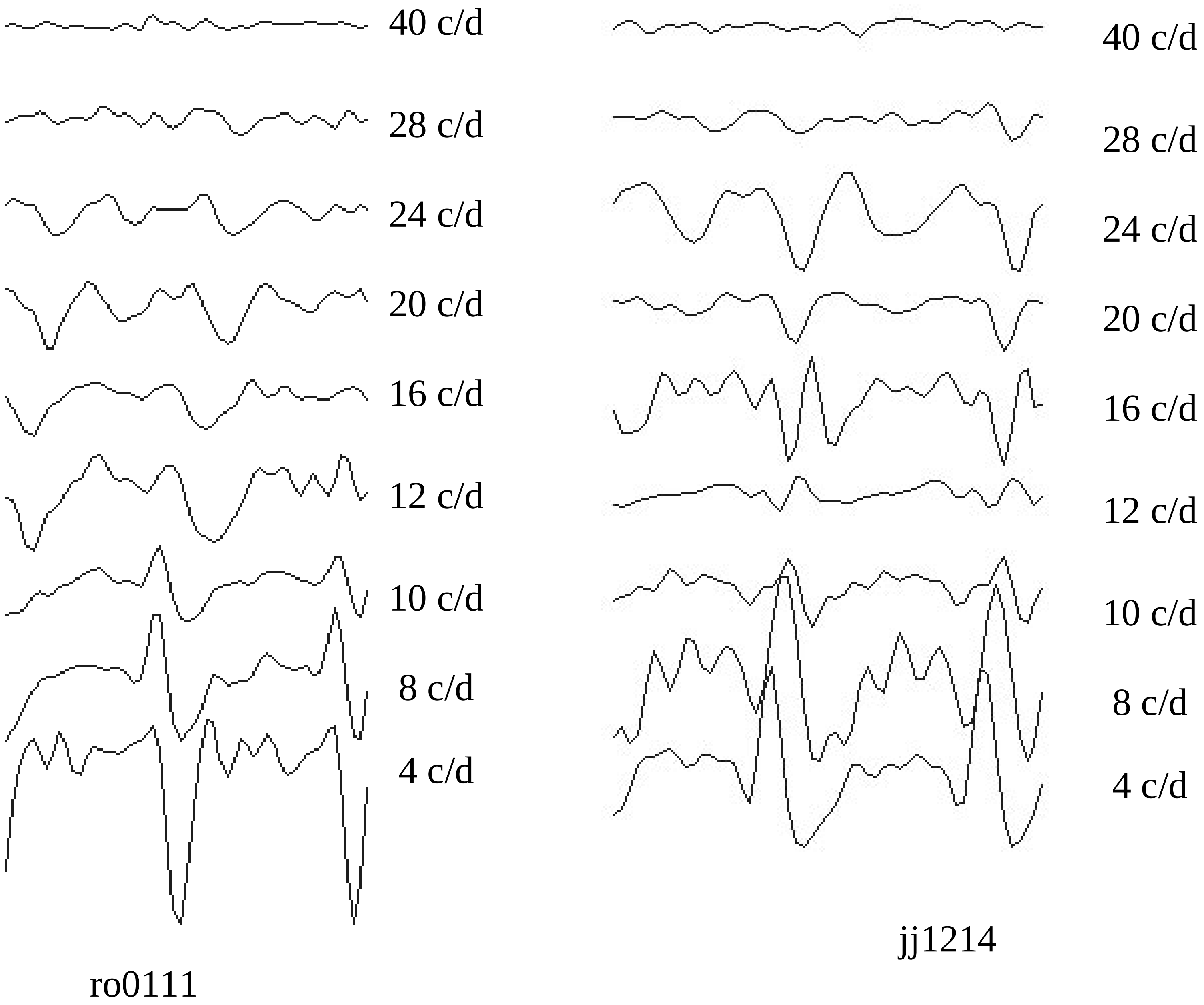
$$s_k(n) = \sin(2\pi f_{k-N} n),\ k = N+1, N+2,\ldots,2N$$

$$f_k = 2k \times f_{stim}, k = 1,2,\ldots,N$$

EEG noise is assumed to be an *AR(p)* process.

-must prewhiten

Ensemble Averaged VEP's for two subjects



Whitening Filter Design

AR(p) model:

$$z(n) = -\sum_{k=1}^{p} a_k z(n-k) + u(n)$$

$u(n)$: Gaussian white noise

optimal whitening filter:

$$w = \begin{bmatrix} 1 & a_1 & a_2 & \cdots & a_p \end{bmatrix}^T$$

Yule-Walker Equations

$$R_{zz}\tilde{w} = b$$

$$R_{zz} = \begin{bmatrix} r_{zz}(0) & r_{zz}(-1) & \cdots & r_{zz}(-p) \\ r_{zz}(1) & r_{zz}(0) & \cdots & r_{zz}(-(p-1)) \\ \vdots & \vdots & \ddots & \vdots \\ r_{zz}(p) & r_{zz}(p-1) & \cdots & r_{zz}(0) \end{bmatrix}$$

$$\tilde{w} = \beta\begin{bmatrix}1 & a_1 & \cdots & a_p\end{bmatrix}^T \qquad b = \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}^T$$

$$\tilde{w} = \beta w \qquad \beta = \text{constant}$$

Perturbation due to Single Sinusoid

$$(R_{zz} + R_{ss})(\tilde{w} + \delta\tilde{w}) = b$$

$$R_{ss} = \begin{bmatrix} 1 & \cos(2\pi f_0) & \cdots & \cos(2\pi f_0 p) \\ \cos(2\pi f_0) & 1 & \cdots & \cos(2\pi f_0 (p-1)) \\ \vdots & \vdots & \ddots & \vdots \\ \cos(2\pi f_0 p) & \cos(2\pi f_0 (p-1)) & \cdots & 1 \end{bmatrix}$$

bias: $\delta\tilde{w} = \beta\delta w$

Perturbation Analysis

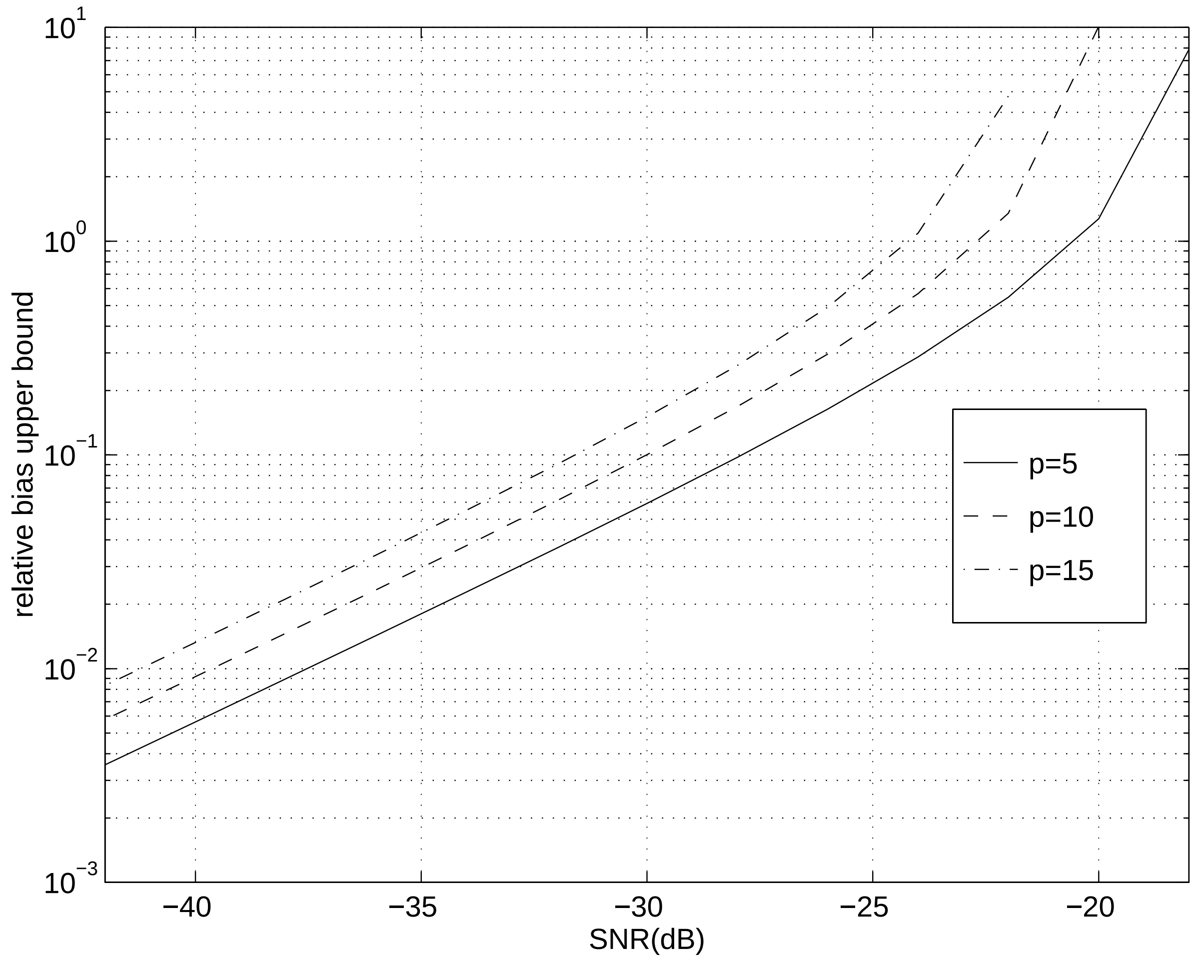
If: $\left\| R_{zz}^{-1} \right\| \left\| R_{ss} \right\| < 1$

then (Stewart, 1973) $$\frac{\left\| \delta \tilde{w} \right\|}{\left\| \tilde{w} \right\|} \leq \frac{\left\| R_{zz}^{-1} \right\| \left\| R_{ss} \right\|}{1 - \left\| R_{zz}^{-1} \right\| \left\| R_{ss} \right\|}$$

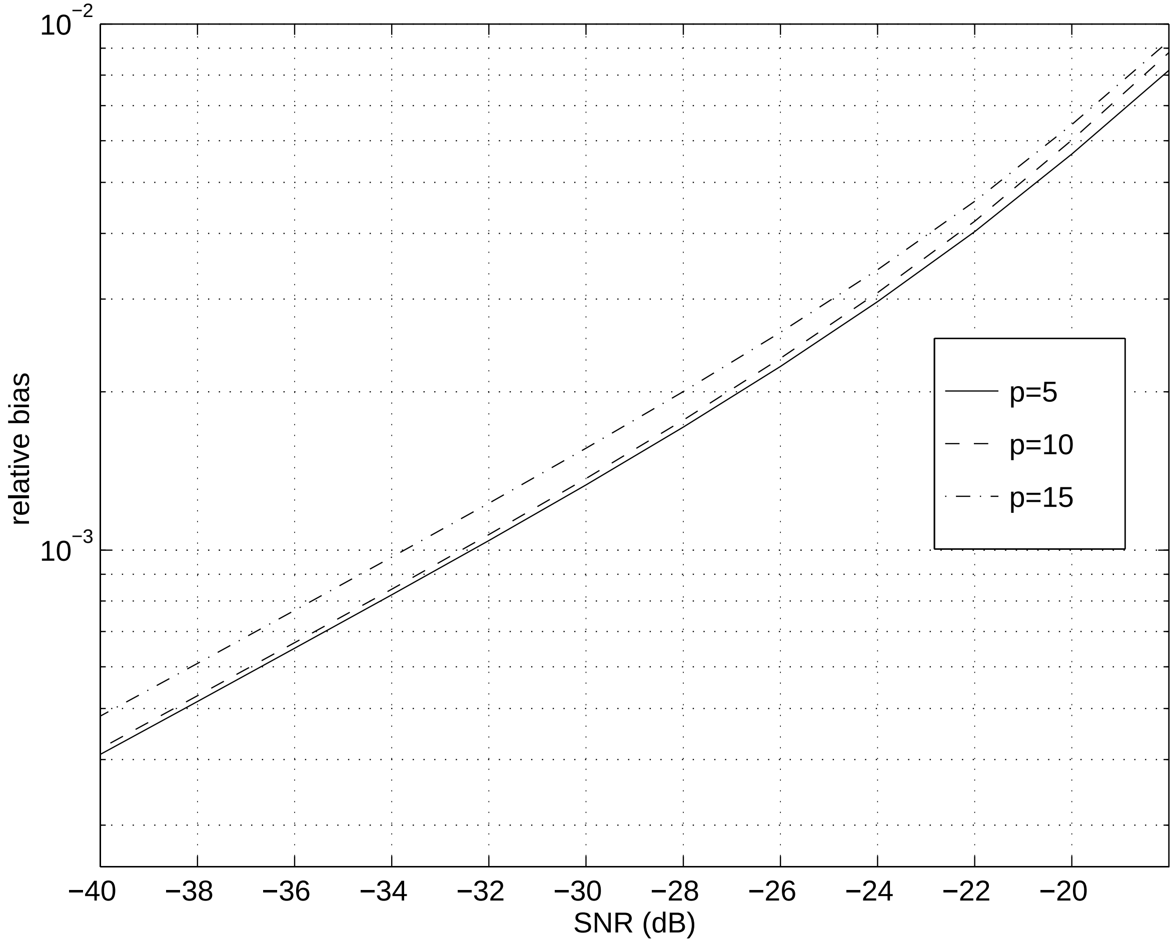
and if: $\left\| R_{zz}^{-1} \right\| \left\| R_{ss} \right\| << 1$

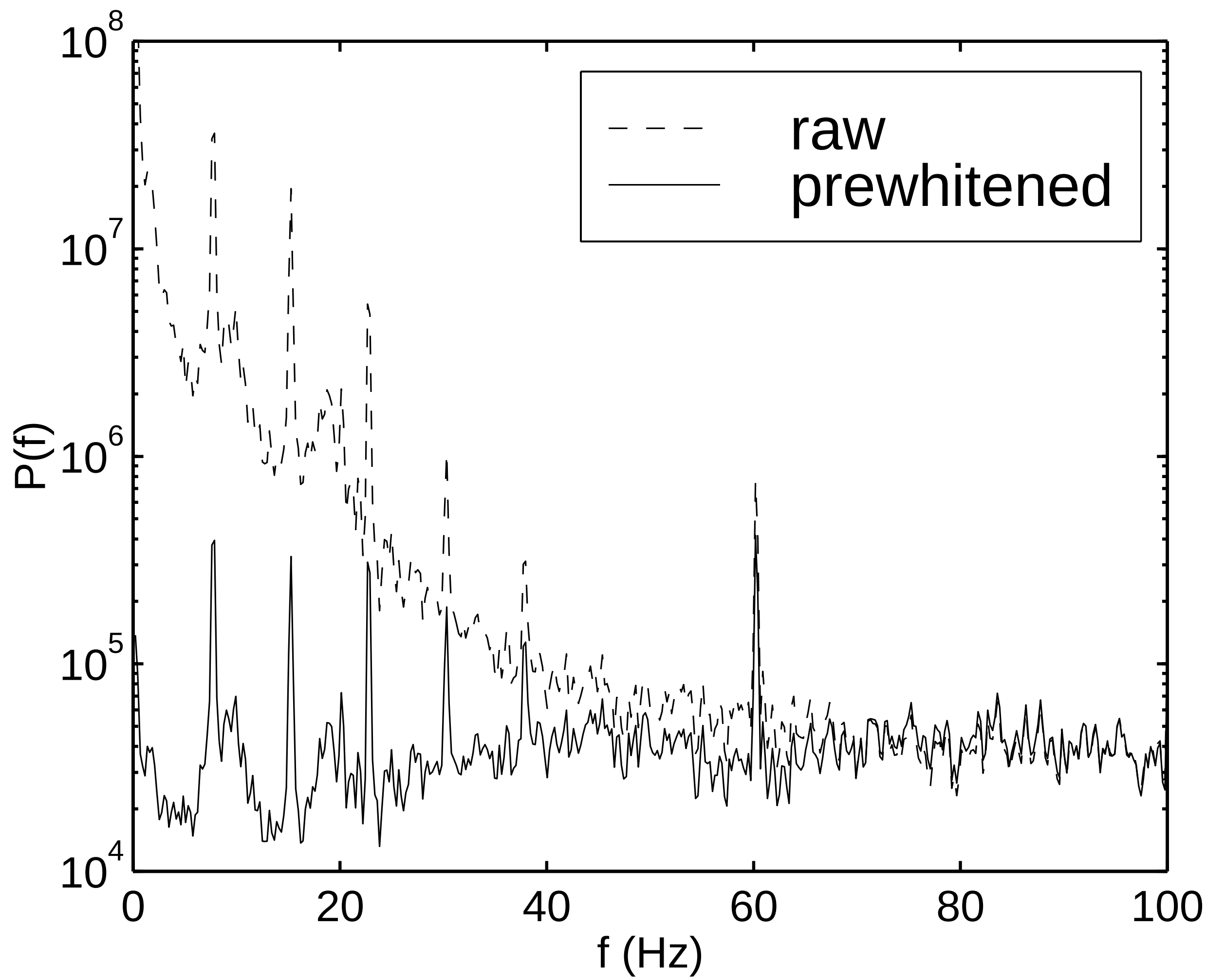
relative bias: $$\frac{\left\| \delta w \right\|}{\left\| w \right\|} \leq \left\| R_{zz}^{-1} \right\| \left\| R_{ss} \right\|$$

Relative Bias Bound vs. SNR



Estimated Relative Bias vs. SNR





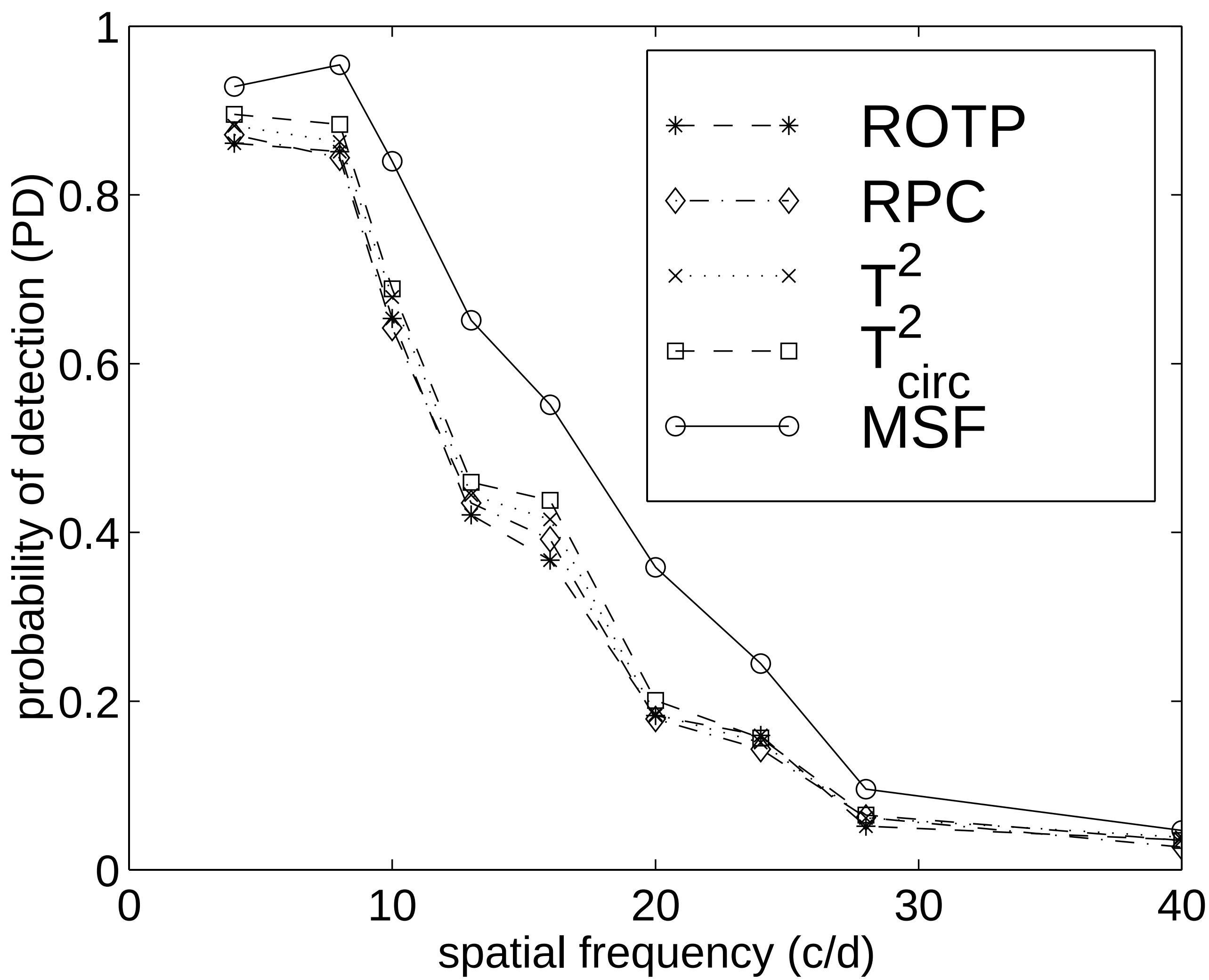
10^8
10^7
10^6
10^5
10^4
P(f)
0
20
40
60
80
100
f (Hz)
raw
prewhitened

Methods

- Vertical square wave gratings at 92% contrast.
- 5.5 degree circular field, luminance 30 foot lamberts.
- counterphase contrast reversal at 3.75 Hz, 7.5 reversals/s.
- spatial frequency varied from 4 c/d to 40 c/d, randomly.
- nineteen, 173-second runs at a fixed spatial frequency were obtained.
- EEG measured from O_z-C_z.

Data Analysis

- Each fixed spatial frequency run broken up into 25 M = 864-sample measurement vectors.
- Each measurement vector was filtered with a p = 15 whitening filter.
- Probability of detection (PD) was estimated for each spatial frequency for:
  - RPC
    T^2
    T^2_{circ}
  - ROTP
  - MSF (N = 4)

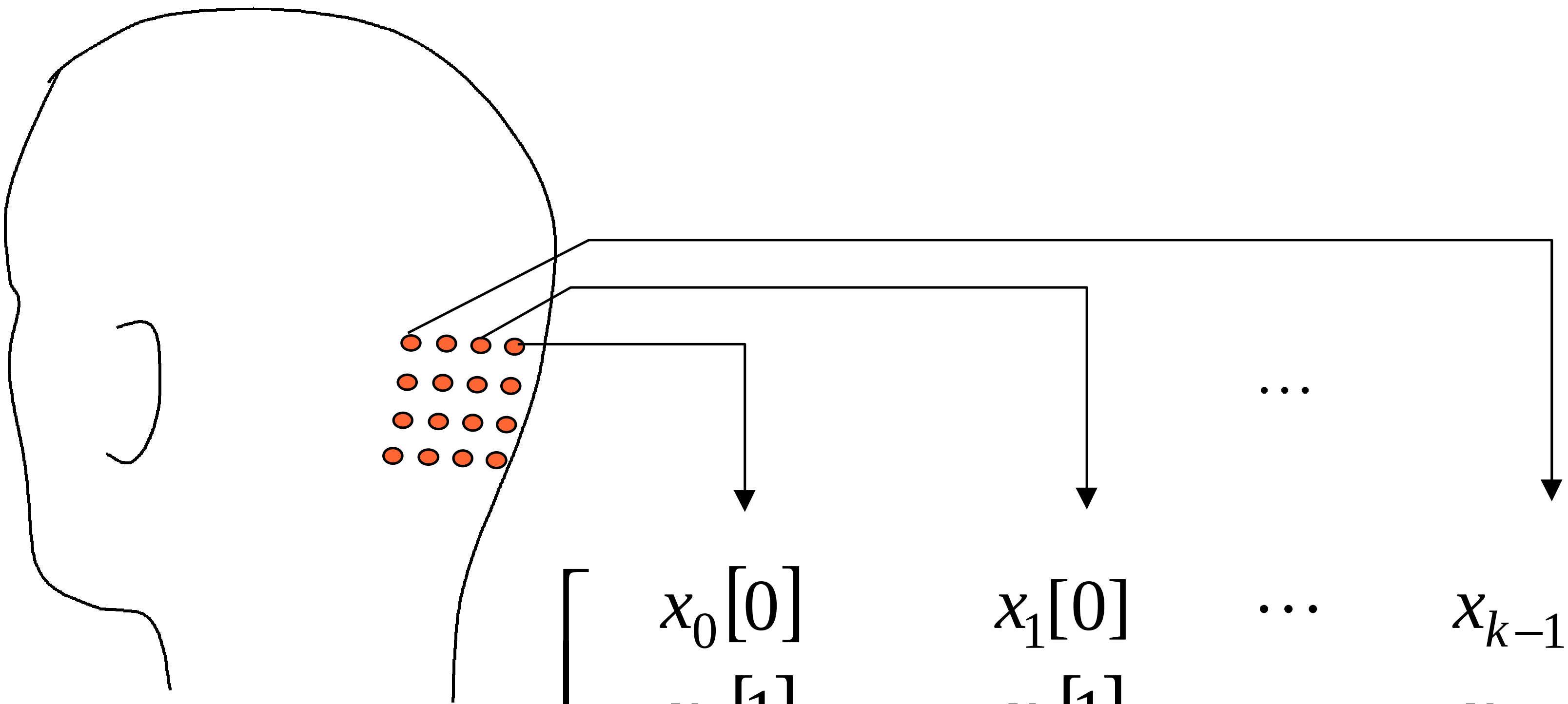


1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
probability of detection (PD)
spatial frequency (c/d)
ROTP
RPC
T^2
T^2_{circ}
MSF

Multichannel Detection

- This approach attempts to exploit any independence that may exist in measurements taken from different scalp locations
- We measure from k = 15 different scalp locations located over the posterior half of the head centered at O_z.
- Generalize the MSF statistic to multichannel measurements

Measurement Sites



$$X = \begin{bmatrix} x_0[0] & x_1[0] & \cdots & x_{k-1}[0] \\ x_0[1] & x_1[1] & \cdots & x_{k-1}[1] \\ \vdots & \vdots & \vdots & \vdots \\ x_0[M-1] & x_1[M-1] & \cdots & x_{k-1}[M-1] \end{bmatrix}$$

Multichannel MSF

We seek α to maximize SNR estimate:

$$\frac{\alpha^T X^T P_S X\alpha}{\alpha^T X^T (I - P_S) X\alpha}$$

Can solve this via solution of generalized eigenvalue problem:

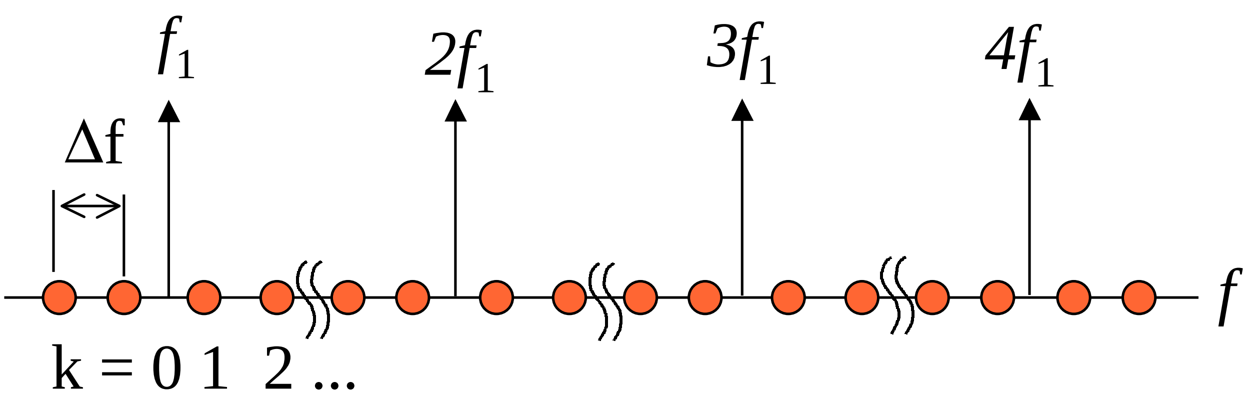
$$P_S X\alpha = \lambda (I - P_S) X\alpha$$

Multichannel MSF (cont.)

- our statistic is: $r = \sum_{n=1}^{2N} \lambda_n$
- $\lambda_1, \lambda_2, \ldots, \lambda_{2N}$ are the $2N$ largest eigenvalues
- density of r under H_0 is difficult to determine
- we use a nonparametric bootstrap approach to find detection threshold.

Bootstrap Algorithm

- Compute *r* values for frequencies that are slightly offset from those of stimulus harmonics at regular intervals, Δf.



Bootstrap Algorithm (cont.)

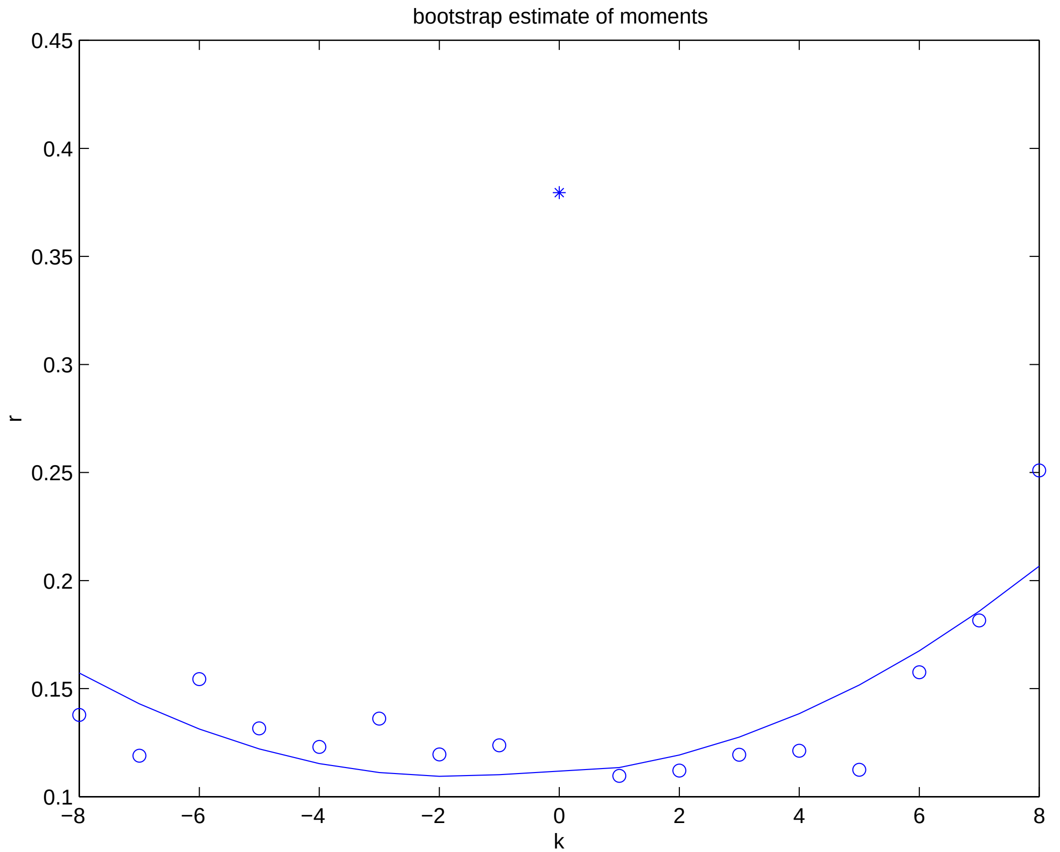
- These *r*-values are assumed to have a gamma distribution:

$$p(r) = \frac{1}{\Gamma(\eta)} \alpha^{\eta} r^{\eta-1} e^{-\alpha r} \qquad \alpha, \eta > 0$$

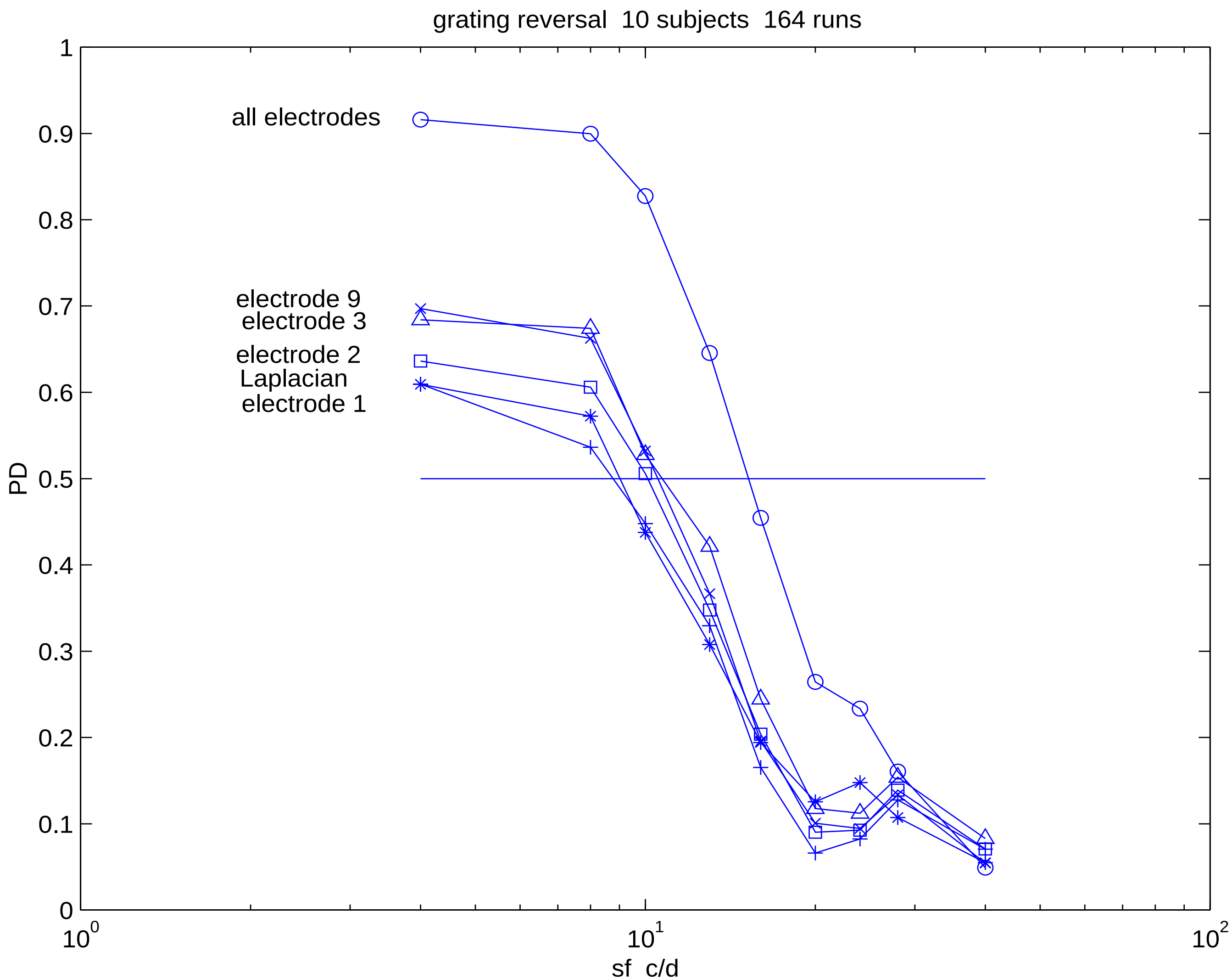
- first moment: η / α
- second moment: η / α^2
- can then determine density of H_0 *r*-values from estimating first and second moments of offset *r*-values.

Bootstrap Algorithm (cont.)

- First and second moments can be computed from a quadratic regression:



Multichannel-MSF: Results



Ongoing/Future Work

- Improved signal model in MSF detector.
- Eigenvalue-based detector (AIC, MDL, etc.)
- Spatial frequency control, and feedback system.
- Development of a commercial objective visual acuity measurement system for clinical use.

References

These slides and related papers can be downloaded in PDF format from:

www.seas.smu.edu/~cd/rpub.html

Conclusions

- Objective acuity measurement requires accurate and sensitive VEP detection.
- MSF detector looks at several harmonics of contrast reversal frequency, has better performance than other detectors.
- Prewhitening does not affect signal component at low SNR.
- Multichannel extension of MSF gives improved detection performance relative to single-channel MSF