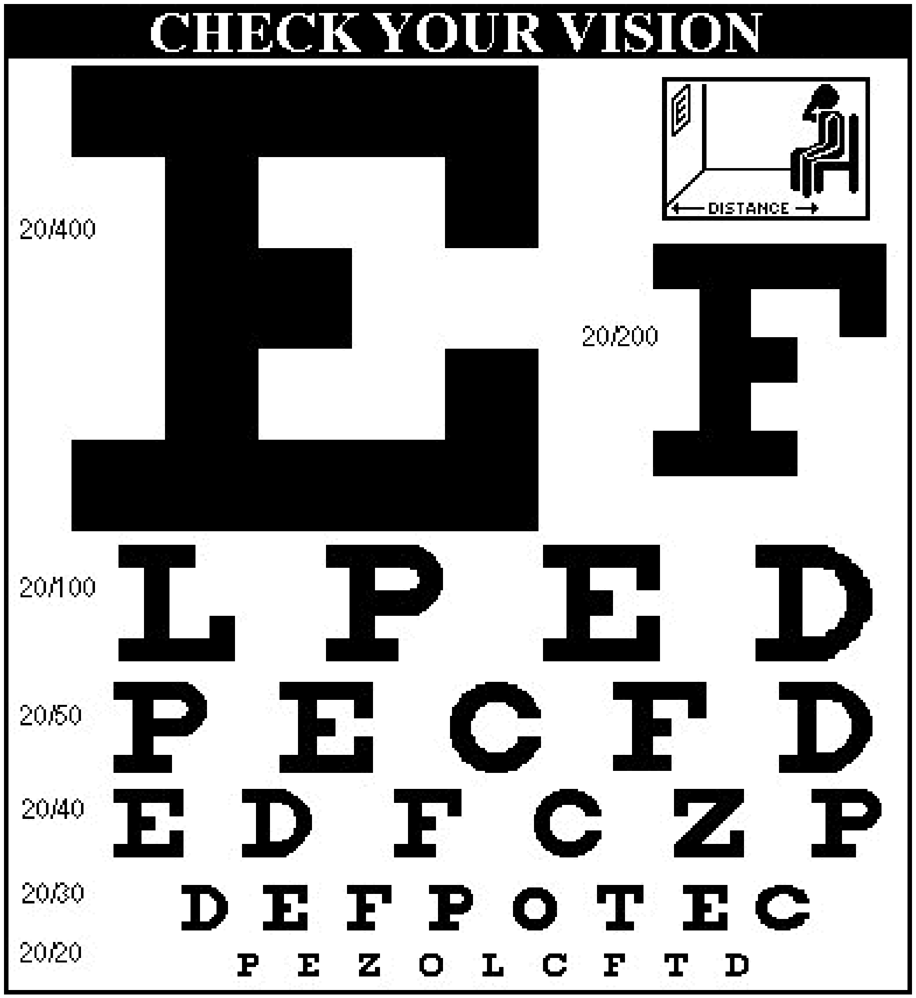
Introduction

We consider the detection of visual evoked potentials (VEP's) with the aim of developing a system for objective measurement of visual acuity. A matched subspace filter (MSF) is demonstrated to outperform a number of other evoked potential detectors. The MSF is suitable for detecting multi-harmonic VEP's, unlike the earlier single-Fourier component detectors. The MSF has also been shown to be a uniformly most powerful detector for unknown signals in a given subspace with unknown noise.

Snellen Chart

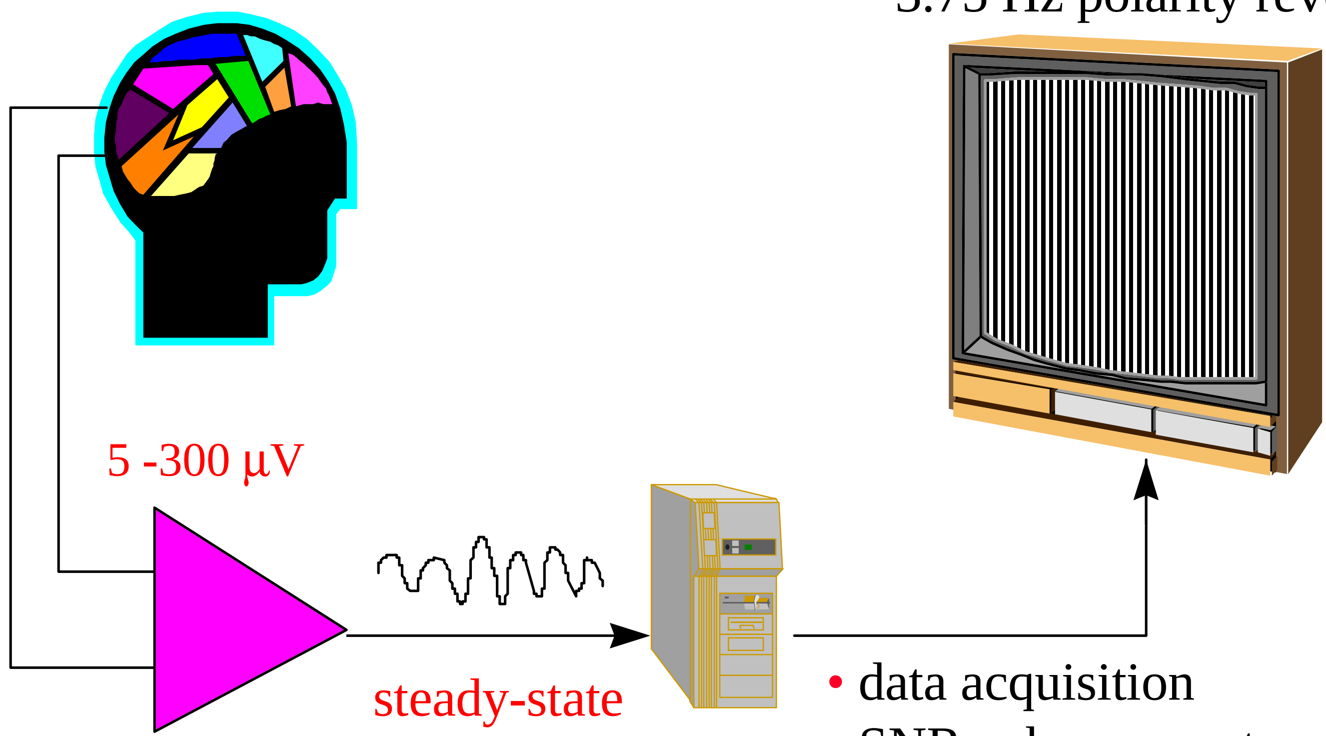


20/200 acuity implies:
visual system resolves
200/20=10 minutes of arc

Why objective acuity measurement?

- Snellen acuity measurements unreliable through first 3 years of life
- amblyopia can be reversed if treated early
- some patients have disturbed consciousness: mentally retarded, cerebral paulsey, head injury, alzheimer’s

Objective Measurement of Visual Acuity



- data acquisition
- SNR enhancement
- detection
- spatial frequency/contrast control

Overview of Detection Algorithms

Notation:

$$x = \begin{bmatrix} x_1^T & x_2^T & \cdots & x_L^T \end{bmatrix}^T \sim M \times 1$$

x_i: response to ith contrast reversal

Ensemble Average:

$$\bar{x} = \frac{1}{L}\sum_{k=1}^{L} x_k$$

Generalized T^2 Statistic (Picton et al., 1987)

DFT of i^th response:

$$X_i = \begin{bmatrix} 1 & \cos(2\pi f_k) & \cdots & \cos(2\pi(N-1)f_k) \\ 1 & \sin(2\pi f_k) & \cdots & \sin(2\pi(N-1)f_k) \end{bmatrix} x_i$$

sample mean:

$$\bar{X} = \frac{1}{L}\sum_{k=1}^{L} X_k$$

sample covariance:

$$C = \frac{1}{L}\sum_{k=1}^{L} (X_k - \bar{X})(X_k - \bar{X})^T$$

Generalized T^2 Statistic (cont.)

if X_i is $N(\mu, \Sigma)$

likelihood ratio test for:

$$H_0\!: \mu = 0 \qquad \text{vs.} \qquad H_1\!: \mu > 0$$

$$T^2 = L\overline{X}^T C^{-1} \overline{X}$$

$$\frac{L-2}{2(L-1)} T^2 \qquad \text{has density:} \qquad F(2, L-2)$$

Circular T^2 Statistic (Victor and Mast, 1991)

DFT:

$$Y_i = \begin{bmatrix} 1 & \exp(-j2\pi f_k) & \cdots & \exp(-j2\pi(N-1)f_k) \end{bmatrix} x_i$$

$$\sim N(\mu, \Sigma)$$

$$H_0\colon \mu = 0 \ \text{ vs. } \ H_1\colon \mu > 0$$

$$T^2_{circ} = (L-1)\frac{(L-1)\left|\bar{Y}\right|^2}{\sum_{k=1}^{L}\left|Y_k - \bar{Y}\right|^2} \qquad \bar{Y} = \frac{1}{L}\sum_{k=1}^{L} Y_k$$

LT^2_{circ} has density: $F(2,\ 2L-2)$

Rayleigh Phase Criterion (Kuwada et al, 1986)

$$R_p = \frac{1}{L}\sqrt{\left(\sum_{k=1}^{L}\cos\theta_k\right)^2 + \left(\sum_{k=1}^{L}\sin\theta_k\right)^2}$$

compared against a table of thresholds for a given false alarm rate.

ROTP (Achim, 1995)

- Nonparametric
- Looks at power in ensemble averages obtained using all possible sign permutations of each single trial.
- If the power in the average corresponding to all "+" signs is in the top 5% of all ensemble average powers, a detection is made.

Matched Subspace Filtering (Scharf, 1991)

Assmptions:
- Signal s lies in a known subspace.

$$s = \sum_{k=1}^{2N} \alpha_k s_k$$

- Exact form of signal (α_k) is unknown.
- Additive white Gaussian noise power is unknown.

Want a statistic which is uniformly most powerful under these three constraints.

Matched Subspace Filtering (cont.)

MSF Statistic:

$$f = \frac{M-2N}{2N} \frac{x^T P_s x}{x^T (I - P_s) x}$$

$$P_s = S\left(S^T S\right)^{-1} S^T$$

f has density $F(2N, M\text{-}2N)$

VEP Signal Model

The VEP consists of *N* even harmonics of the contrast reversal frequency:

$$s_k(n) = \cos(2\pi f_k n),\ k = 1,2,\ldots,N$$

$$s_k(n) = \sin(2\pi f_{k-N} n),\ k = N+1, N+2,\ldots,2N$$

$$f_k = 2k \times f_{stim}, k = 1,2,\ldots,N$$

EEG noise is assumed to be an *AR(p)* process.

-must prewhiten

Whitening Filter Design

AR(p) model:

$$z(n) = -\sum_{k=1}^{p} a_k z(n-k) + u(n)$$

$u(n)$: Gaussian white noise

optimal whitening filter:

$$w = \begin{bmatrix} 1 & a_1 & a_2 & \cdots & a_p \end{bmatrix}^T$$

Yule-Walker Equations

$$R_{zz}\tilde{w} = b$$

$$R_{zz} = \begin{bmatrix} r_{zz}(0) & r_{zz}(-1) & \cdots & r_{zz}(-p) \\ r_{zz}(1) & r_{zz}(0) & \cdots & r_{zz}(-(p-1)) \\ \vdots & \vdots & \ddots & \vdots \\ r_{zz}(p) & r_{zz}(p-1) & \cdots & r_{zz}(0) \end{bmatrix}$$

$$\tilde{w} = \beta\begin{bmatrix}1 & a_1 & \cdots & a_p\end{bmatrix}^T \qquad b = \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}^T$$

$$\tilde{w} = \beta w \qquad \beta = \text{constant}$$

Perturbation due to Single Sinusoid

$$\left(R_{zz} + R_{ss}\right)\left(\tilde{w} + \delta\tilde{w}\right) = b$$

$$R_{ss} = \begin{bmatrix} 1 & \cos(2\pi f_0) & \cdots & \cos(2\pi f_0 p) \\ \cos(2\pi f_0) & 1 & \cdots & \cos(2\pi f_0 (p-1)) \\ \vdots & \vdots & \ddots & \vdots \\ \cos(2\pi f_0 p) & \cos(2\pi f_0 (p-1)) & \cdots & 1 \end{bmatrix}$$

$$\text{bias:} \quad \delta\tilde{w} = \beta\delta w$$

Perturbation Analysis

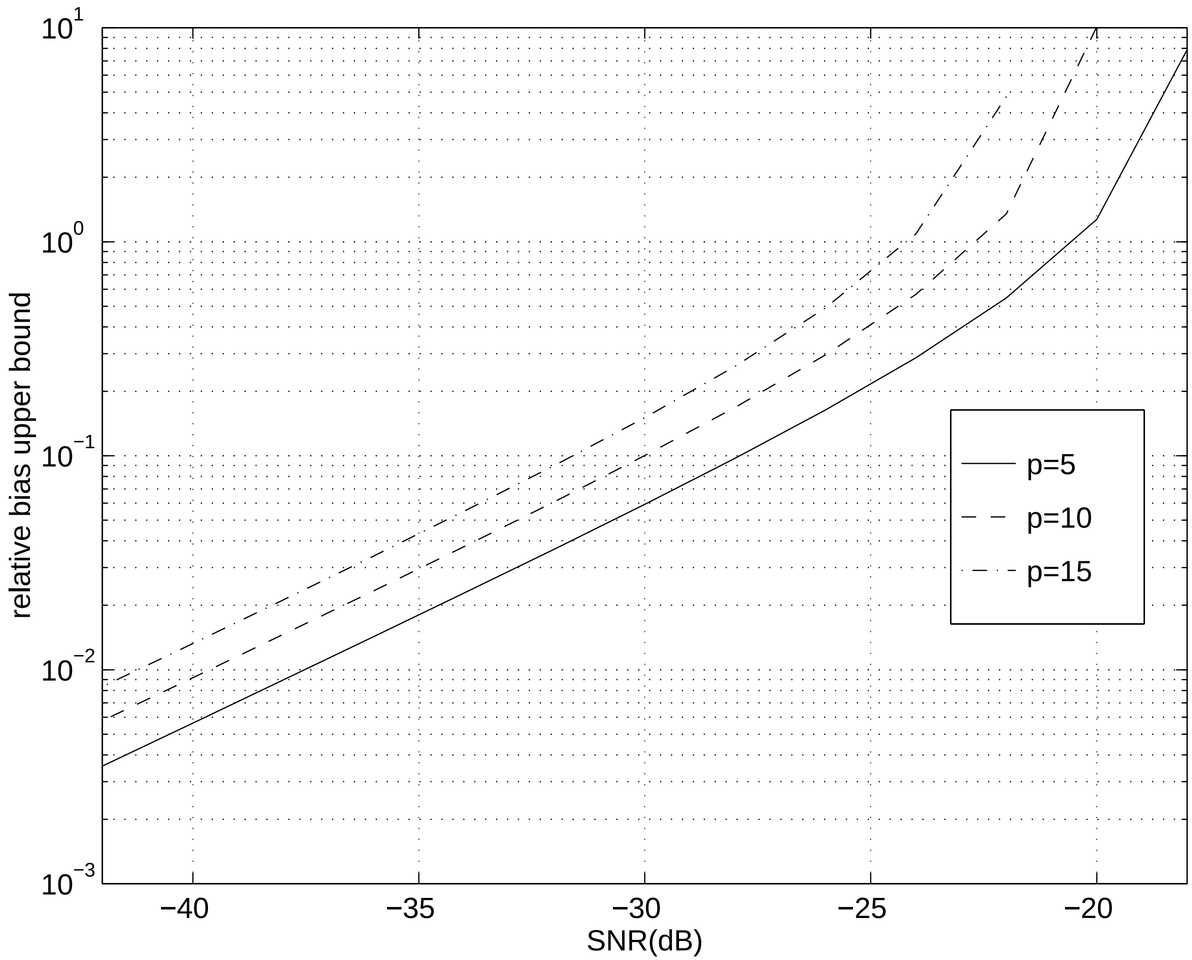
If: $$\left\|R_{zz}^{-1}\right\|\left\|R_{ss}\right\| < 1$$

then (Stewart, 1973) $$\frac{\left\|\delta\tilde{w}\right\|}{\left\|\tilde{w}\right\|} \le \frac{\left\|R_{zz}^{-1}\right\|\left\|R_{ss}\right\|}{1-\left\|R_{zz}^{-1}\right\|\left\|R_{ss}\right\|}$$

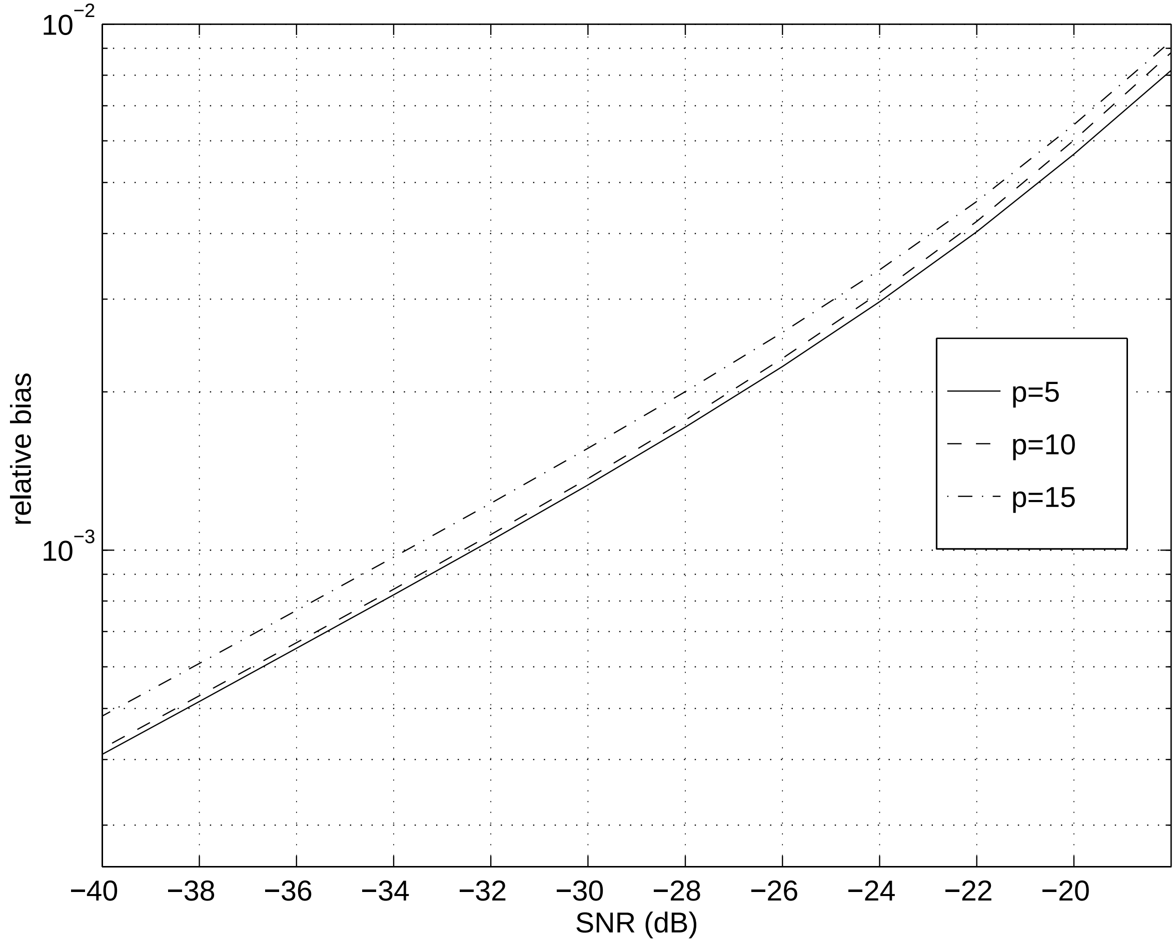
and if: $$\left\|R_{zz}^{-1}\right\|\left\|R_{ss}\right\| << 1$$

relative bias: $$\frac{\left\|\delta w\right\|}{\left\|w\right\|} \le \left\|R_{zz}^{-1}\right\|\left\|R_{ss}\right\|$$

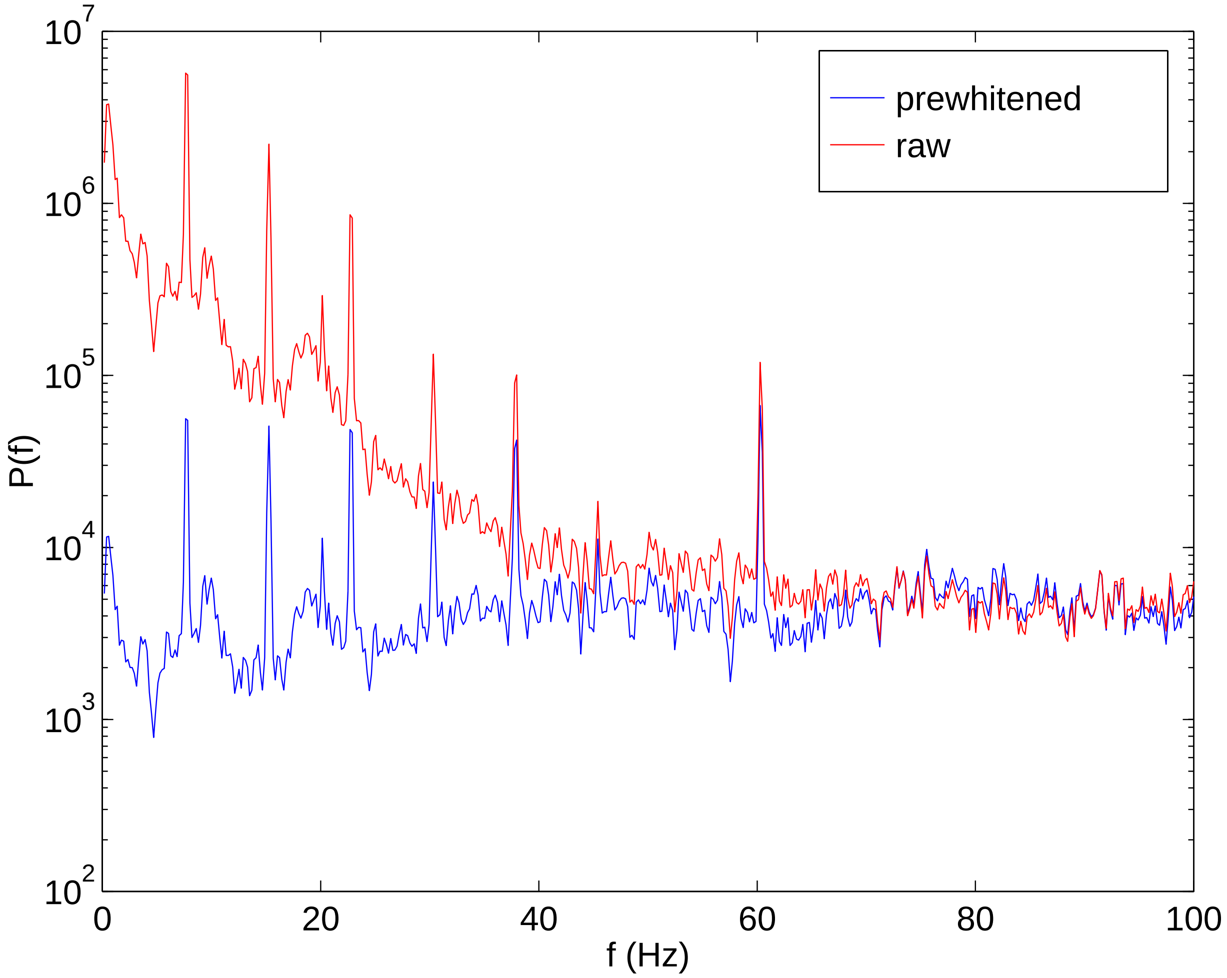
Relative Bias Bound vs. SNR



Estimated Relative Bias vs. SNR



Estimated PSD Before/After Prewhitening



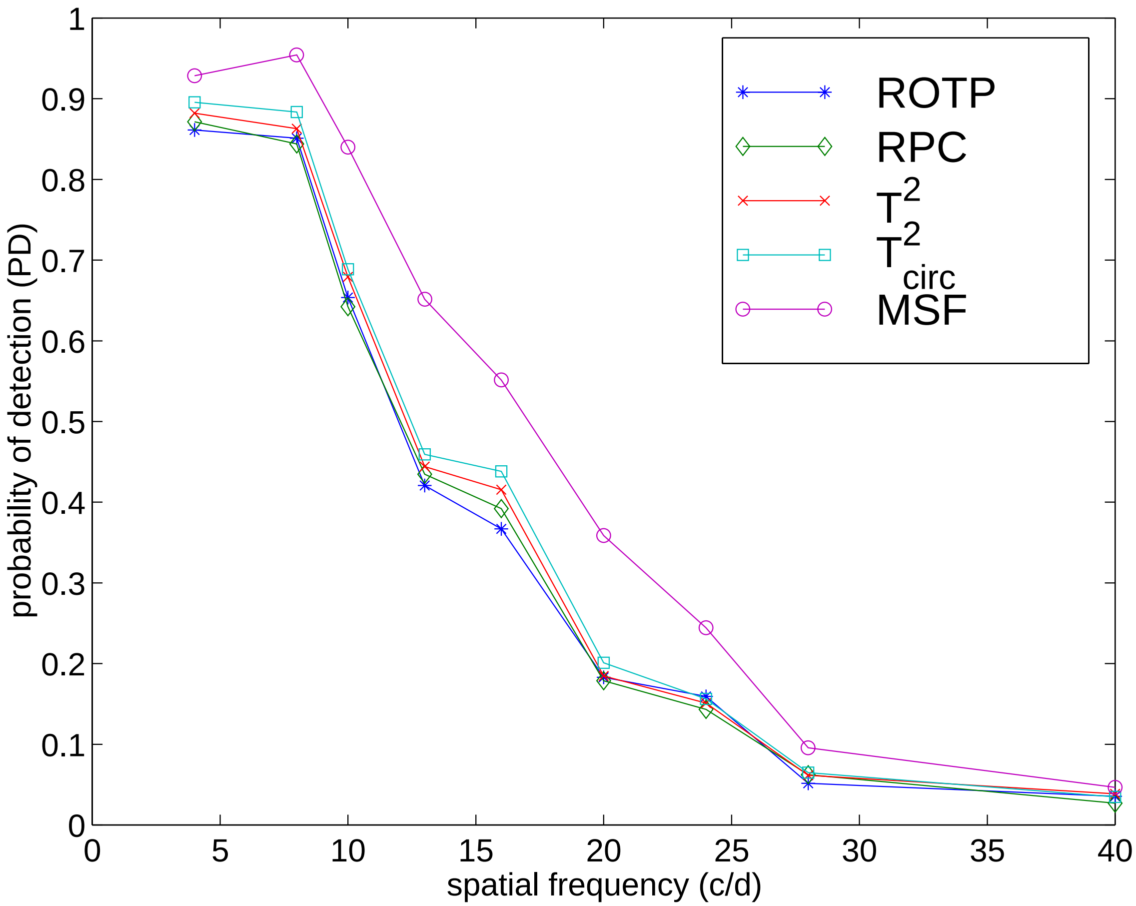
Methods

- Vertical square wave gratings at 92% contrast.
- 5.5 degree circular field, luminance 30 foot lamberts.
- counterphase contrast reversal at 3.75 Hz, 7.5 reversals/s.
- spatial frequency varied from 4 c/d to 40 c/d, randomly.
- nineteen, 173-second runs at a fixed spatial frequency were obtained.
- EEG measured from O_z-C_z.

Data Analysis

- Each fixed spatial frequency run broken up into 25 M = 864-sample measurement vectors.
- Each measurement vector was filtered with a p = 15 whitening filter.
- Probability of detection (PD) was estimated for each spatial frequency for:
  - RPC
  - T^2
  - T^2_{circ}
  - ROTP
  - MSF (N = 4)

Mean PD vs. Spatial Frequency



Conclusions

- Objective acuity measurement requires accurate and sensitive VEP detection.
- MSF detector looks at several harmonics of contrast reversal frequency, has better performance than previous detectors.
- Prewhitening does not affect signal component at low SNR.