Convolution of Signals in MATLAB

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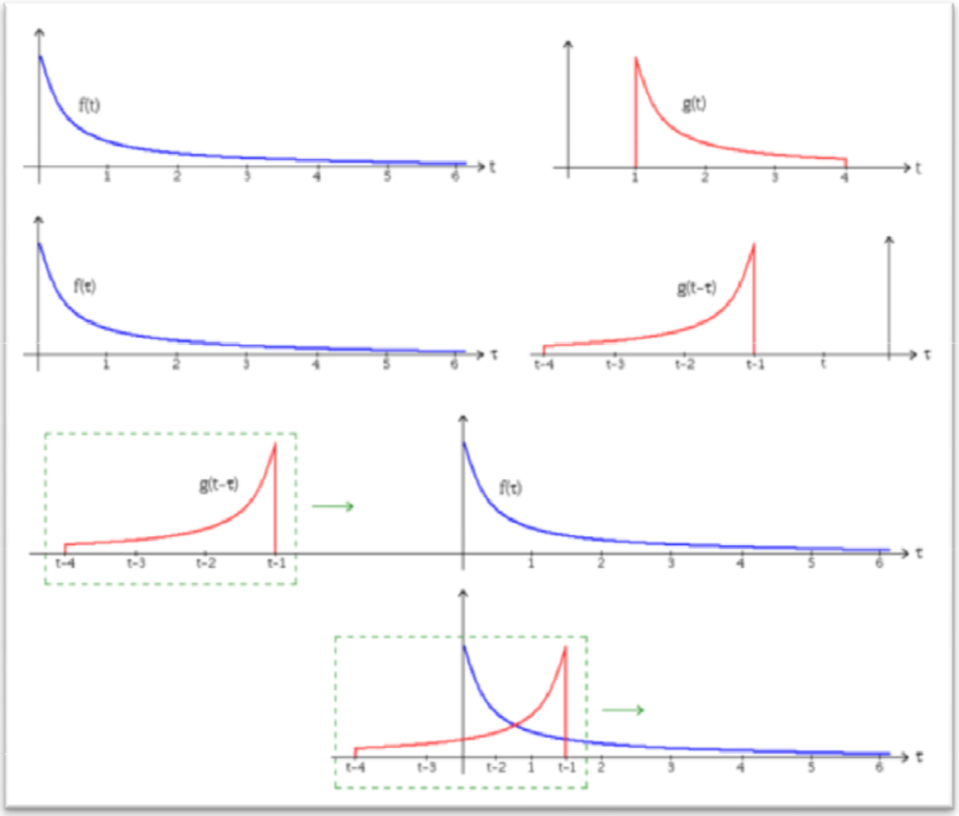
Review of Convolution

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(\tau - t)d\tau$$



Visual example copied from Wikipedia

Review of Convolution

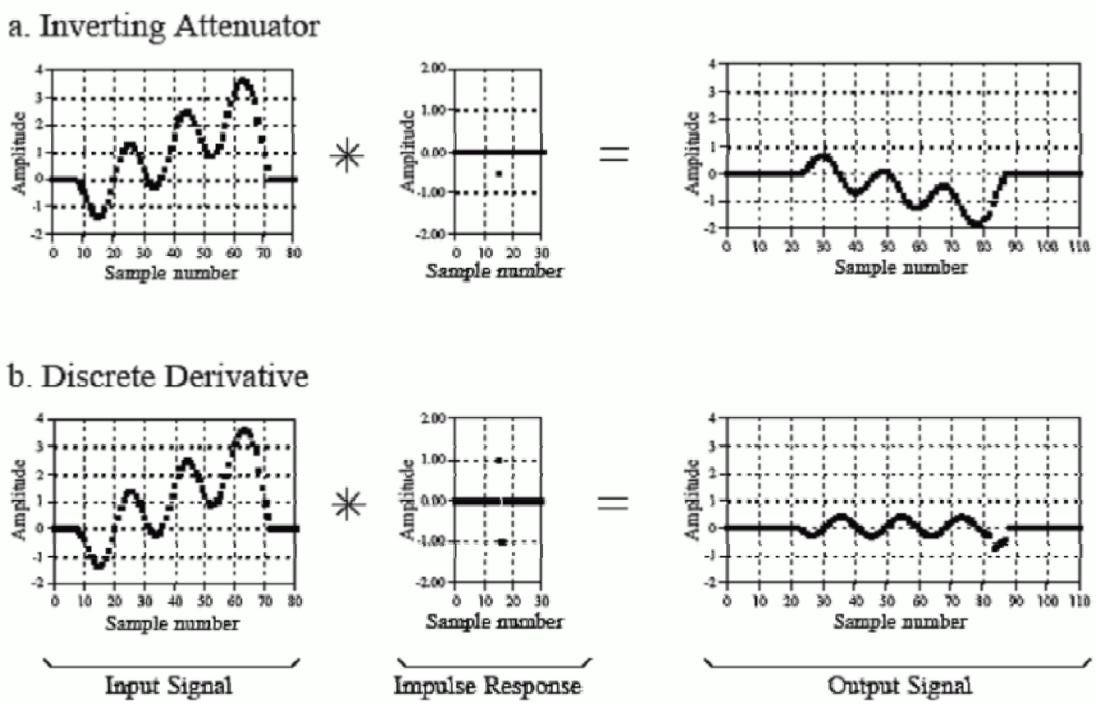
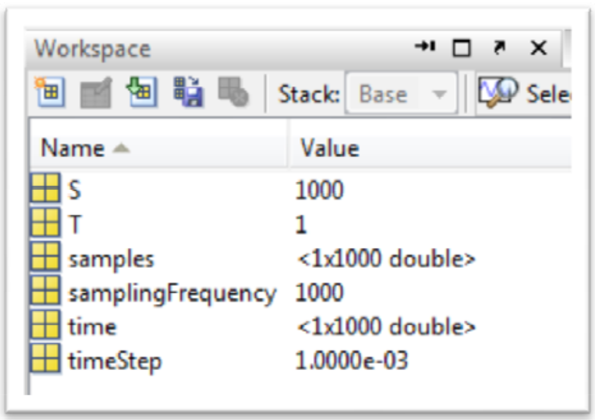


FIGURE 6-4
Examples of signals being processed using convolution. Many signal processing tasks use very simple impulse responses. As shown in these examples, dramatic changes can be achieved with only a few nonzero points.

Smith, http://www.dspguide.com/ch6/2.htm

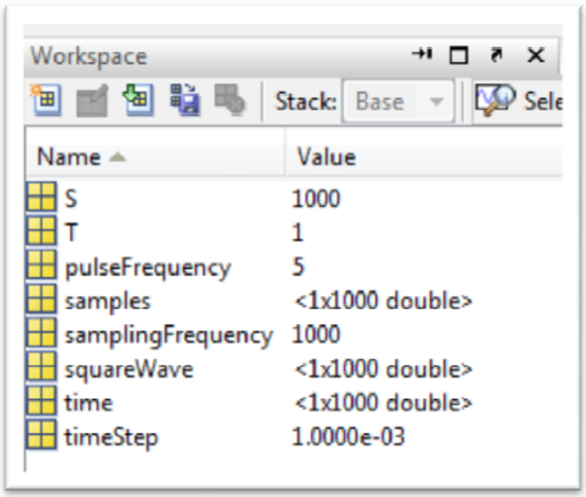
Generating Time Vectors in Matlab

```
%% Define time vector
samplingFrequency = 1000; %Hz
timeStep = 1/samplingFrequency; %sec
T = 1; %sec
S = T*samplingFrequency; %samples
samples = 1:S; %samples
time = samples*timeStep; %sec
```



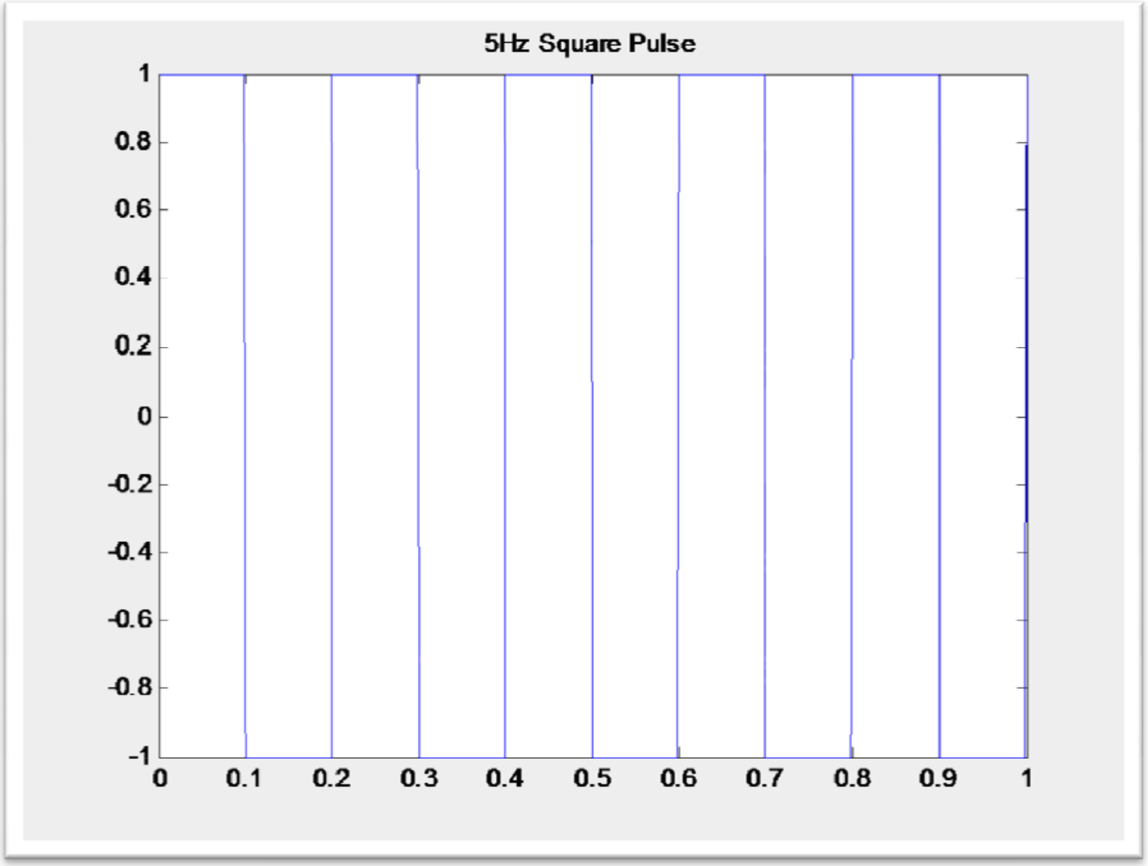
Generate Square Wave

```
%% Generate square wave
pulseFrequency = 5; %Hz
squareWave = square(time*pulseFrequency*2*pi);
```



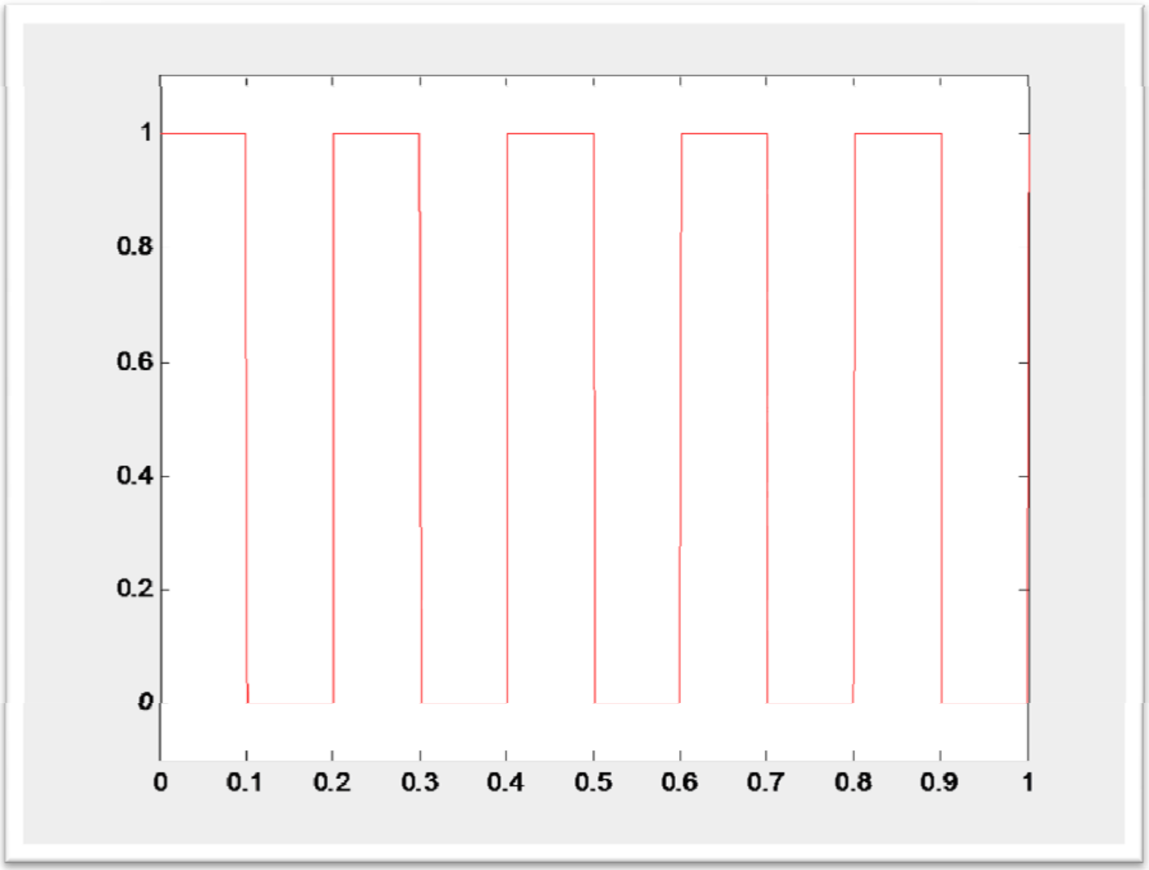
Plot Square Wave

```
%% Plot square wave
figure(1), plot(time,squareWave,'b')
title([num2str(pulseFrequency) 'Hz Square Pulse'])
```



Adjust Magnitude and Re-plot

```
%% Adjust amplitude
squareWavePos = (squareWave+1)/2;
close(1)
figure(1), plot(time,squareWavePos,'r')
set(gca,'YLim',[-0.1 1.1])
```



Nyquist Sampling Theorem

- If a continuous time signal has no frequency components above f_h, then it can be specified by a discrete time signal with a sampling frequency greater than twice f_h.

Define Nyquist Sampling Time Vector

```
%% Define Nyquist Sampling Time Vector
nsamplingFrequency = 2*pulseFrequency;
ntimeStep = 1/nsamplingFrequency; %sec
T = 1; %sec
nS = T*nsamplingFrequency; %samples
nsamples = 1:nS; %samples
ntime = nsamples*ntimeStep; %sec
```

Workspace | Current Folder

Stack: Base | Select data to plot

Name	Value
S	1000
T	1
nS	10
nsamples	[1,2,3,4,5,6,7,8,9,10]
nsamplingFreque...	10
ntime	[0.1000,0.2000,0.3000,0.4000,0.5000,0.6000,0.7000,0.8000,0.9000,1]
ntimeStep	0.1000
pulseFrequency	5
samples	<1x1000 double>
samplingFrequency	1000
squareWave	<1x1000 double>
squareWavePos	<1x1000 double>
time	<1x1000 double>
timeStep	1.0000e-03

Generate Nyquist Sampling Square Wave

```
%% Generate Nyquist Sampling Square Wave
nsquareWave = square(ntime*pulseFrequency*2*pi);
nsquareWavePos = (nsquareWave+1)/2;
```

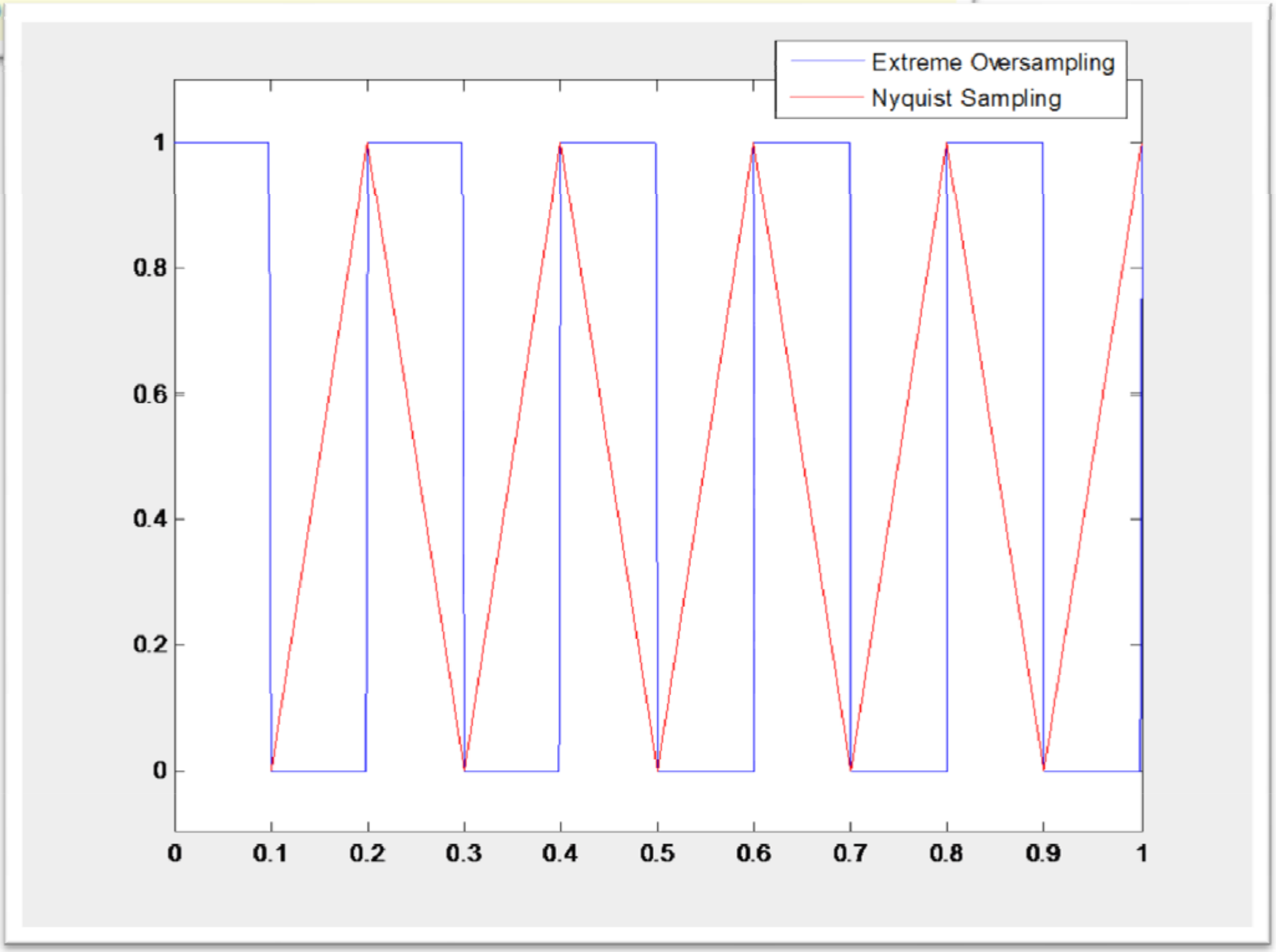
Workspace | Current Folder

Stack: Base | Select data to plot

Name	Value
S	1000
T	1
nS	10
nsamples	[1,2,3,4,5,6,7,8,9,10]
nsamplingFreque...	10
nsquareWave	[-1,1,-1,1,-1,1,-1,1,-1,1]
nsquareWavePos	[0,1,0,1,0,1,0,1,0,1]
ntime	[0.1000,0.2000,0.3000,0.4000,0.5000,0.6000,0.7000,0.8000,0.9000,1]
ntimeStep	0.1000
pulseFrequency	5
samples	<1x1000 double>
samplingFrequency	1000
squareWave	<1x1000 double>
squareWavePos	<1x1000 double>
time	<1x1000 double>
timeStep	1.0000e-03

Plot Nyquist Square Wave

```
%% Plot Nyquist Sampling Square Wave Compared to Oversampled Square Wave
figure(2), plot(time,squareWavePos,'b',ntime,nsquareWavePos,'r')
legend('Extreme Oversampling','Nyquist Sampling')
set(gca,'YLim',[-0.1 1.1])
```



Define Undersampled Time Vector

```
%% Define Undersampled Time Vector
usamplingFrequency = 1.7*pulseFrequency;
utimeStep = 1/usamplingFrequency; %sec
T = 1; %sec
uS = T*usamplingFrequency; %samples
usamples = 1:uS; %samples
utime = usamples*utimeStep; %sec
```

Workspace | Current Folder

Stack: Base | Select data to plot

Name	Value
S	1000
T	1
nS	10
nsamples	[1,2,3,4,5,6,7,8,9,10]
nsamplingFreque...	10
nsquareWave	[-1,1,-1,1,-1,1,-1,1,-1,1]
nsquareWavePos	[0,1,0,1,0,1,0,1,0,1]
ntime	[0.1000,0.2000,0.3000,0.4000,0.5000,0.6000,0.7000,0.8000,0.9000,1]
ntimeStep	0.1000
pulseFrequency	5
samples	<1x1000 double>
samplingFrequency	1000
squareWave	<1x1000 double>
squareWavePos	<1x1000 double>
time	<1x1000 double>
timeStep	1.0000e-03
uS	8.5000
usamples	[1,2,3,4,5,6,7,8]
usamplingFreque...	8.5000
utime	[0.1176,0.2353,0.3529,0.4706,0.5882,0.7059,0.8235,0.9412]
utimeStep	0.1176

Generate Undersampled Square Wave

```
%% Generate Undersampled Square Wave
usquareWave = square(utime*pulseFrequency*2*pi);
usquareWavePos = (usquareWave+1)/
```

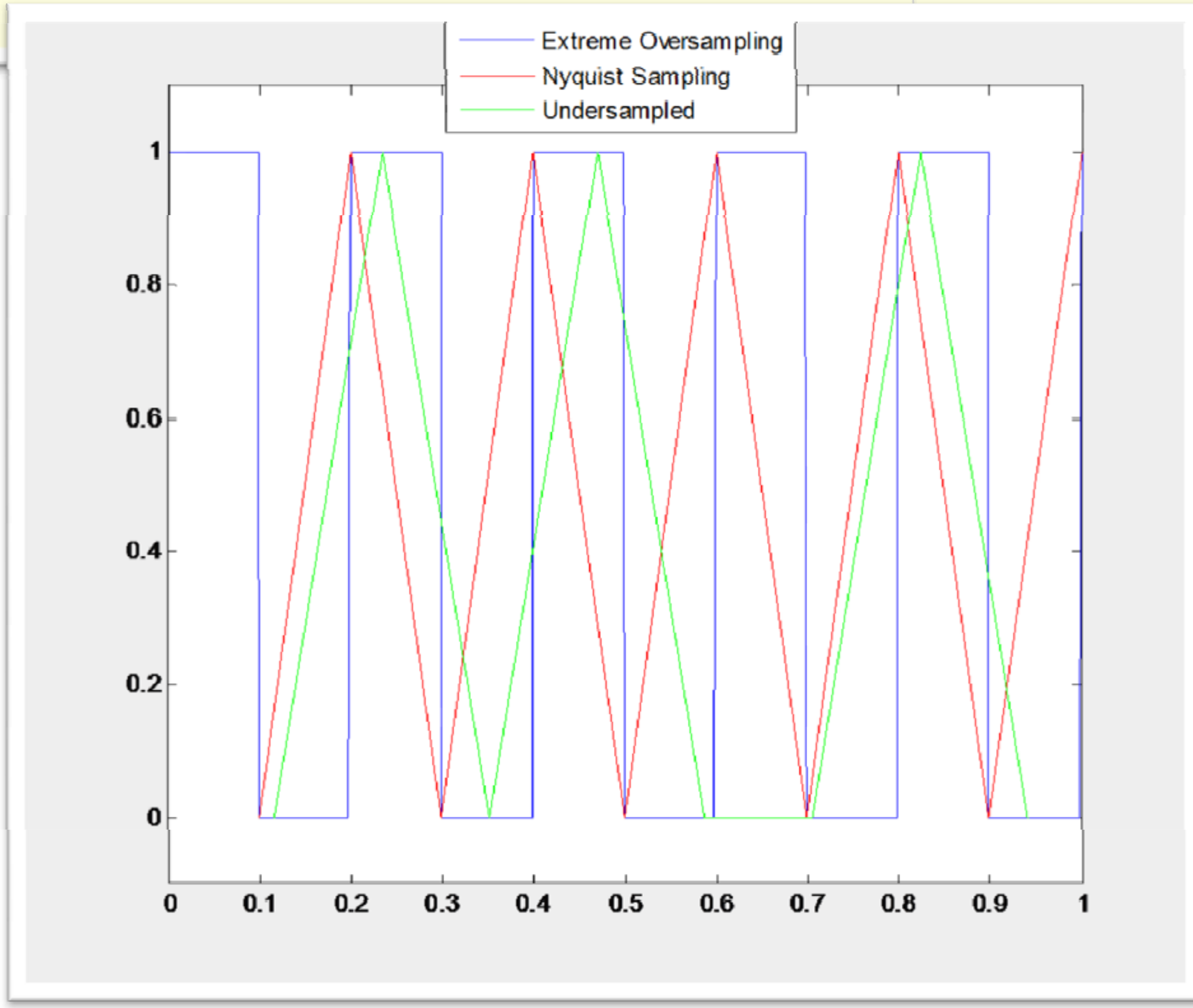
Workspace | Current Folder

Stack: Base | Select data to plot

Name	Value
S	1000
T	1
nS	10
nsamples	[1,2,3,4,5,6,7,8,9,10]
nsamplingFreque...	10
nsquareWave	[-1,1,-1,1,-1,1,-1,1,-1,1]
nsquareWavePos	[0,1,0,1,0,1,0,1,0,1]
ntime	[0.1000,0.2000,0.3000,0.4000,0.5000,0.6000,0.7000,0.8000,0.9000,1]
ntimeStep	0.1000
pulseFrequency	5
samples	<1x1000 double>
samplingFrequency	1000
squareWave	<1x1000 double>
squareWavePos	<1x1000 double>
time	<1x1000 double>
timeStep	1.0000e-03
uS	8.5000
usamples	[1,2,3,4,5,6,7,8]
usamplingFreque...	8.5000
usquareWave	[-1,1,-1,1,-1,-1,1,-1]
usquareWavePos	[0,1,0,1,0,0,1,0]
utime	[0.1176,0.2353,0.3529,0.4706,0.5882,0.7059,0.8235,0.9412]
utimeStep	0.1176

Plot Undersampled

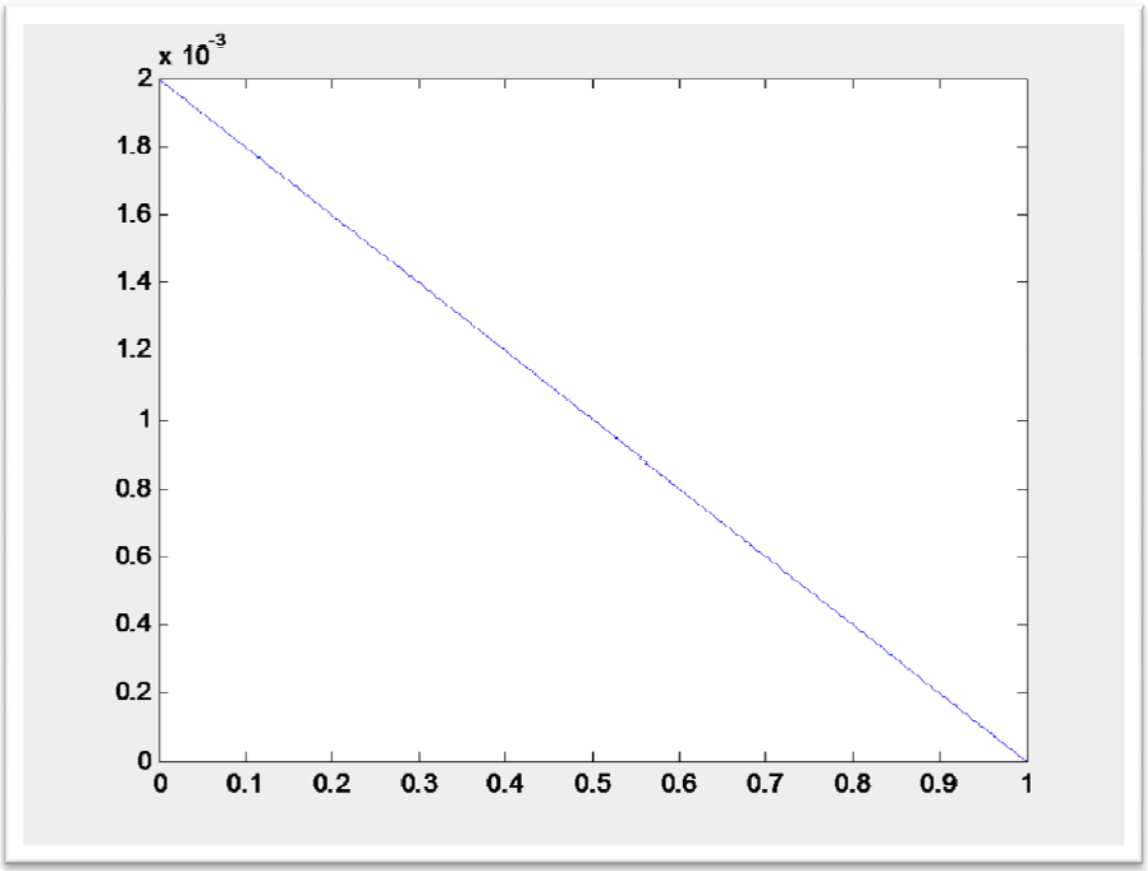
```
%% Plot Nyquist Sampling Square Wave Compared to Oversampled Square Wave
figure(2), plot(time,squareWavePos,'b',ntime,nsquareWavePos,'r',utime,usquareWavePos,'g')
legend('Extreme Oversampling','Nyquist Sampling','Undersampled')
set(gca,'YLim',[-0.1 1.1])
```



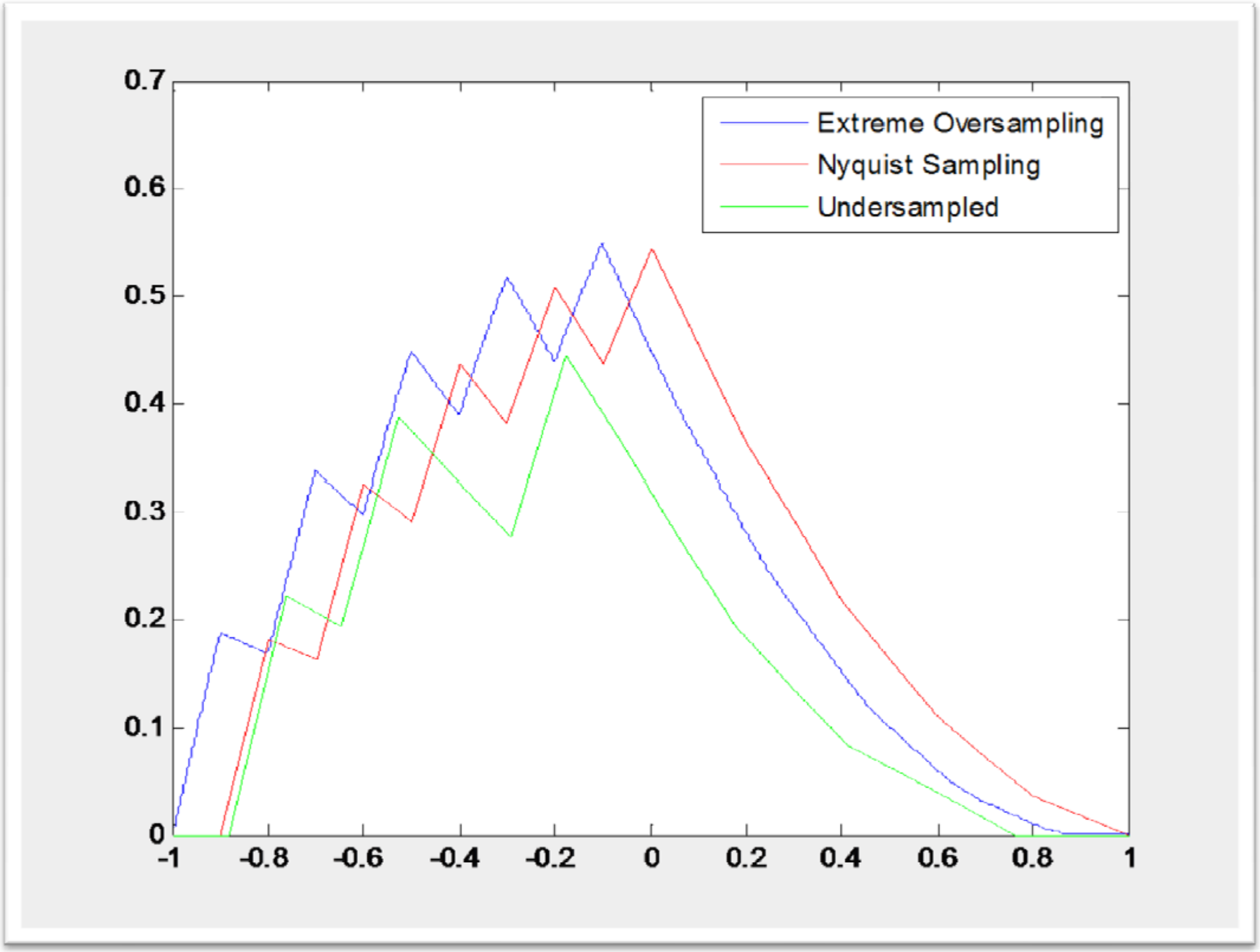
Convolution With Linear Decay Signal

```
%% Convolution with Linear Decay
%Generate Linear Decay Signals
lds = fliplr(time)/sum(time);
nlds = fliplr(ntime)/sum(ntime);
ulds = fliplr(utime)/sum(utime);

%Convolve Square Wave Signals with Linear Decay Signals
squareWaveLDS = conv(squareWavePos,lds);
nsquareWaveNLDS = conv(nsquareWavePos,nlds);
usquareWaveULDS = conv(usquareWavePos,ulds);

%Extend time vectors to match convolved signals
convtime = (-S:1:S)*timeStep;
nconvtime = (-nS:1:nS)*ntimeStep;
uconvtime = (-uS:1:uS)*utimeStep;

%Pad convolved signals with leading and trailing zero
squareWaveLDS = [0,squareWaveLDS,0];
nsquareWaveNLDS = [0,nsquareWaveNLDS,0];
usquareWaveULDS = [0,usquareWaveULDS,0];

figure(3), plot(convtime,squareWaveLDS,'b',nconvtime,nsquareWaveNLDS,'r',uconvtime,usquareWaveULDS,'g')
legend('Extreme Oversampling','Nyquist Sampling','Undersampled')
```

Linear Decay Signal



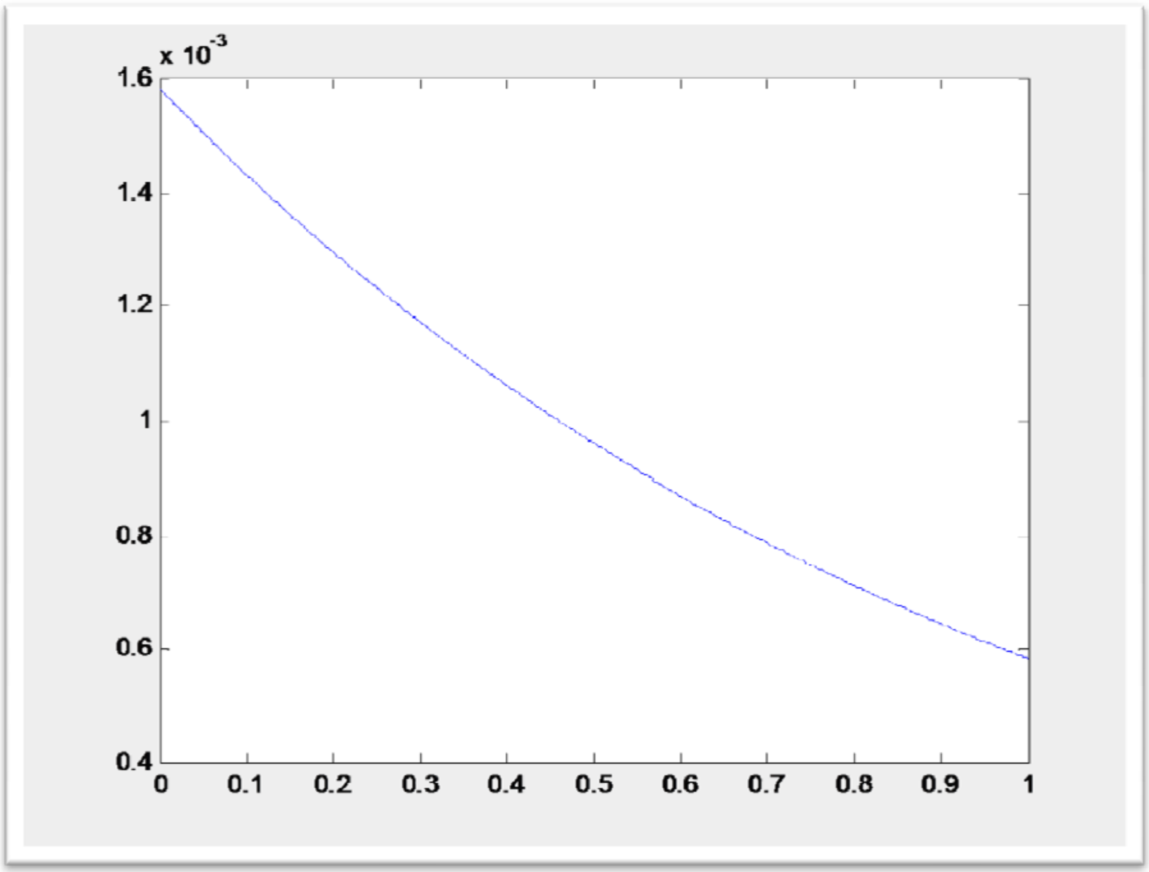
Plot of Convolutions



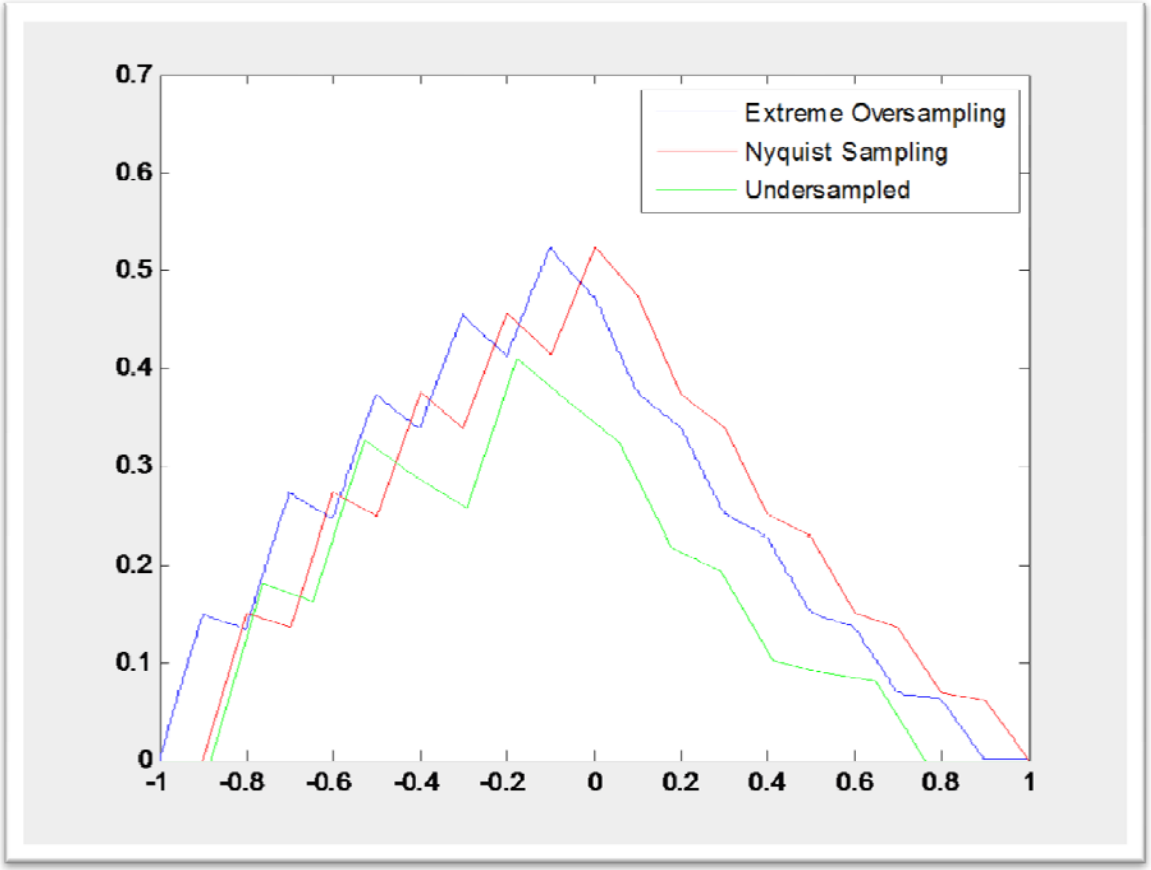
Convolution with Exponential Decay Signal

```
%% Convolution with Exponential Decay
%Generate Exponential Decay Signals
eds = exp(-time);
eds = eds/sum(eds);
neds = exp(-ntime);
neds = neds/sum(neds);
ueds = exp(-utime);
ueds = ueds/sum(ueds);

%Convolve Square Wave Signals with Exponential Decay Signals
squareWaveEDS = conv(squareWavePos,eds);
nsquareWaveNEDS = conv(nsquareWavePos,neds);
usquareWaveUEDS = conv(usquareWavePos,ueds);

%Pad convolved signals with leading and trailing zero
squareWaveEDS = [0,squareWaveEDS,0];
nsquareWaveNEDS = [0,nsquareWaveNEDS,0];
usquareWaveUEDS = [0,usquareWaveUEDS,0,0];

figure(4), plot(convtime,squareWaveEDS,'b',nconvtime,nsquareWaveNEDS,'r',uconvtime,usquareWaveUEDS,'g
legend('Extreme Oversampling','Nyquist Sampling','Undersampled')
```

Exponential Decay Signal



Plot of Convolutions



FFT of Signals in MATLAB

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Review of Fourier Transform

The Fourier Integral → $$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

DFT (Discrete Fourier Transform)

$$X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}, k = 1,2,\ldots, N$$