Fixed-Point Complex ICA Algorithms for the Blind Separation of Sources Using Their Real Or Imaginary Components

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Abstract. The complex-valued signal model is useful for several practical applications, yet few algorithms for separating complex linear mixtures exist. This paper develops two algorithms for separating mixtures of independent complex-valued signals in which statistical independence of the real and imaginary components is assumed. The procedures extract sources assuming that the kurtoses of either the real or imaginary components are non-zero. Simulations indicate the efficacy of the methods in performing source separation for wireless communications models.

1 Introduction

The goal of blind source separation is to find an $(m \times m)$ matrix $\mathbf{B}$ such that

$$\mathbf{y}(k) = \mathbf{B}\mathbf{x}(k) \quad \text{and} \quad \mathbf{x}(k) = \mathbf{A}\mathbf{s}(k), \tag{1}$$

where $\mathbf{y}(k)$ contains estimates of the m sources in $\mathbf{s}(k)$, $\mathbf{A}$ is full rank, and $\mathbf{s}(k)$ is typically assumed to contain independent signals. This paper focuses on complex-valued source separation, in which all quantities in (1) are complex-valued. Few algorithms have been developed for complex ICA [1–5]. The complex FastICA procedure in [2] uses a circular contrast and may not perform well with mixtures containing non-circular sources such as real-valued BPSK signals.

In this paper, we consider algorithms for separating complex-valued signal mixtures using fourth-moment contrasts, in which the sources in $\mathbf{s}(k)$ are assumed to have independent real- and imaginary components. Such an assumption is quite reasonable in some applications, particularly in multiple-input, multiple-output (MIMO) wireless communications systems where higher-order modulation schemes are used. We develop two procedures that employ modified versions of the FastICA algorithm to extract each of the m complex sources based on the statistics of either their real or imaginary component. Simulations show the efficacy of the proposed methods for complex-valued source separation.

2 On Mixtures of Complex-Valued Signals

Without loss of generality, assume that the sources in $\mathbf{s}(k) = \mathbf{s}_R(k) + j\mathbf{s}_I(k)$ are zero-mean and strong-uncorrelated [6, 7], such that the source covariance and pseudo-covariance matrices are $E\{\mathbf{s}(k)\mathbf{s}^H(k)\} = \mathbf{I}$ and $E\{\mathbf{s}(k)\mathbf{s}^T(k)\} = \boldsymbol{\Lambda}$, and $\boldsymbol{\Lambda}$ is a diagonal matrix of real-valued circularity coefficients λ_i with $0 \leq \lambda_i \leq 1$.

Consider an algorithm that adjusts a single row $\mathbf{b} = [b_1 \ \cdots \ b_m]^T$ of $\mathbf{B}$ in (1) to extract a source $s_i(k)$ based on the statistics of its real or imaginary component $s_{R,i}(k)$ or $s_{I,i}(k)$. The output signal is $y(k) = y_R(k) + jy_I(k) = \mathbf{b}^T\mathbf{x}(k) = \mathbf{c}^T\mathbf{s}(k)$, where $\mathbf{c} = \mathbf{A}^T\mathbf{b} = \mathbf{c}_R + j\mathbf{c}_I$. Thus, $y_R(k) = \mathbf{c}_R^T\mathbf{s}_R(k) - \mathbf{c}_I^T\mathbf{s}_I(k)$ and $y_I(k) = \mathbf{c}_I^T\mathbf{s}_R(k) + \mathbf{c}_R^T\mathbf{s}_I(k)$. The normalized kurtoses of $s_{R,i}(k)$ and $s_{I,i}(k)$ are

$$\kappa_{R,i} = \frac{2E\{s_{R,i}^4(k)\}}{(1+\lambda_i)^2} - 3 \quad \text{and} \quad \kappa_{I,i} = \begin{cases} \dfrac{2E\{s_{I,i}^4(k)\}}{(1-\lambda_i)^2} - 3, & \text{if } 0 \leq \lambda_i < 1 \\ 0, & \text{if } \lambda_i = 1 \end{cases}. \quad (2)$$

The quantities $\kappa_{R,i}$ and $\kappa_{I,i}$ are related to the symmetric kurtosis of $s_i(k)$ as

$$\kappa_i = \frac{1}{4}\left[(1+\lambda_i)^2\kappa_{R,i} + (1-\lambda_i)^2\kappa_{I,i}\right]. \quad (3)$$

Theorem 1: *Under the above conditions, the real and imaginary components $y_R(k)$ and $y_I(k)$ of $y(k)$ have the following second moments and kurtoses:*

$$E\{y_R^2(k)\} = \frac{1}{2}\sum_{i=1}^{m}(1+\lambda_i)c_{R,i}^2 + (1-\lambda_i)c_{I,i}^2 \quad (4)$$

$$E\{y_I^2(k)\} = \frac{1}{2}\sum_{i=1}^{m}(1-\lambda_i)c_{R,i}^2 + (1+\lambda_i)c_{I,i}^2, \quad E\{y_R(k)y_I(k)\} = \sum_{i=1}^{m}\lambda_i c_{R,i}c_{I,i} \quad (5)$$

$$\begin{aligned}\kappa[y_R(k)] = &\sum_{i=1}^{m}\left[\frac{\kappa_{R,i}}{4}(1+\lambda_i)^2c_{R,i}^4 + \frac{\kappa_{I,i}}{4}(1-\lambda_i)^2c_{I,i}^4\right] \\ &+\frac{3}{2}\sum_{i=1}^{m}\left[E\{s_{R,i}^2(k)s_{I,i}^2(k)\} - \frac{1}{4}(1-\lambda_i^2)\right]c_{R,i}^2c_{I,i}^2 \\ &-4\sum_{i=1}^{m}c_{R,i}^3c_{I,i}E\{s_R^3(k)s_I(k)\} + c_{R,i}c_{I,i}^3E\{s_R(k)s_I^3(k)\} \end{aligned} \quad (6)$$

$$\begin{aligned}\kappa[y_I(k)] = &\sum_{i=1}^{m}\left[\frac{\kappa_{R,i}}{4}(1+\lambda_i)^2c_{I,i}^4 + \frac{\kappa_{I,i}}{4}(1-\lambda_i)^2c_{R,i}^4\right] \\ &+\frac{3}{2}\sum_{i=1}^{m}\left[E\{s_{R,i}^2(k)s_{I,i}^2(k)\} - \frac{1}{4}(1-\lambda_i^2)\right]c_{R,i}^2c_{I,i}^2 \\ &+4\sum_{i=1}^{m}c_{I,i}^3c_{R,i}E\{s_R^3(k)s_I(k)\} + c_{I,i}c_{R,i}^3E\{s_R(k)s_I^3(k)\} \end{aligned} \quad (7)$$

Corollary 1.1: Under the additional assumption that $s_{R,i}(k)$ and $s_{I,i}(k)$ are independent for all $1 \leq i \leq m$,

$$\kappa[y_R(k)] = \sum_{i=1}^{m}\left[\frac{\kappa_{R,i}}{4}(1+\lambda_i)^2c_{R,i}^4 + \frac{\kappa_{I,i}}{4}(1-\lambda_i)^2c_{I,i}^4\right] \quad (8)$$

$$\kappa[y_I(k)] = \sum_{i=1}^{m}\left[\frac{\kappa_{R,i}}{4}(1+\lambda_i)^2c_{I,i}^4 + \frac{\kappa_{I,i}}{4}(1-\lambda_i)^2c_{R,i}^4\right] \quad (9)$$

The fourth-order statistical structures of the real and imaginary components of linearly-mixed, statistically-independent strong-uncorrelated, and possibly non-circular complex sources is not as simple as in the real-valued case. If all $s_{R,i}(k)$ and $s_{I,i}(k)$ are jointly statistically-independent, however, (8) and (9) are similar in structure to the real-valued case, leading to the following theorem.

Theorem 2: *Consider the single-unit extraction criterion*

$$\mathcal{J}(\mathbf{b}) = \left| \frac{\kappa[y_R(k)]}{(E\{|y_R(k)|^2\})^2} \right| \tag{10}$$

where $y(k) = \mathbf{b}^T\mathbf{x}(k) = \mathbf{c}^T\mathbf{s}(k) = y_R(k) + jy_I(k)$. *Assume that* all *of the sources are statistically-independent with statistically-independent real and imaginary parts, and at least one of the sources has a real and/or imaginary part with* $\kappa_{R,i} \neq 0$ *and/or* $\kappa_{I,i} \neq 0$. *Then, maximization of* $\mathcal{J}(\mathbf{b})$ *over* $\mathbf{b}$ *under the constraint* $E\{y_R^2(k)\} = 1$ *yields one of the columns of* $\mathbf{A}^{-1}$ *for which* $\kappa_{R,i} \neq 0$ *or* $\kappa_{I,i} \neq 0$, *up to a complex scaling factor* $e^{j\pi p/2}$, *where* p *is an integer.*

Proof: Define the $(2m)$-dimensional real-valued vector $\overline{\mathbf{c}}_{\mathcal{R}} = \sqrt{2}[\mathbf{c}_R^T[\mathbf{I} + \boldsymbol{\Lambda}]^{-1/2}\ \mathbf{c}_I^T[\mathbf{I} - \boldsymbol{\Lambda}]^{+/2}]^T$, with entries $\{\overline{c}_i\}$, where $\mathbf{N}^+$ denotes the pseudo-inverse of a square matrix $\mathbf{N}$. Let $\overline{\kappa}_i$ denote the $(2m)$-element sequence $\{\kappa_{R,1}, \ldots, \kappa_{R,m}, \kappa_{I,1}, \ldots, \kappa_{I,m}\}$. Substituting these relations into (8) and and (4) yields

$$\kappa_{\mathcal{R}}[y_R(k)] = \sum_{i=1}^{2m} \overline{\kappa}_i \overline{c}_i^4 \quad \text{and} \quad E\{y_R^2(k)\} = \sum_{i=1}^{2m} \overline{c}_i^2. \tag{11}$$

The relations in (11) are identical to those in the $2m$-dimensional real-valued separation case. Thus, constrained maximization of $\mathcal{J}(\mathbf{b})$ results in an extracted source with a non-zero-kurtosis real or imaginary component. The one non-zero coefficient of $\mathbf{b}^T\mathbf{A}$ equals $e^{j\pi p/2}$ because (i) absolute signs of the $\{\overline{c}_i\}$ do not matter, and (ii) the real or imaginary component of a source could be extracted.

3 FastICA Algorithms for Extracting a Single Source With Independent Real and Imaginary Components

We now develop fast-converging single-unit procedures to extract one source from mixtures of sources having independent real and imaginary components. Two methods are considered. The first algorithm relies on the strong-uncorrelating transform $\mathbf{G}$ that diagonalizes both the sample covariance and pseudo-covariance matrices $\mathbf{R}_{XX} = E\{\mathbf{x}(k)\mathbf{x}^H(k)\}$ and $\mathbf{P}_{XX} = E\{\mathbf{x}(k)\mathbf{x}^T(k)\}$, respectively [7]. Let $\boldsymbol{\Gamma} = \mathbf{G}\mathbf{A} = \boldsymbol{\Gamma}_R + j\boldsymbol{\Gamma}_I$ and $\mathbf{v}(k) = \mathbf{G}\mathbf{x}(k) = \mathbf{v}_R(k) + j\mathbf{v}_I(k)$. Then, $\mathbf{s}(k)$ and $\mathbf{v}(k)$ are related as

$$\begin{bmatrix} \mathbf{v}_R(k) \\ \mathbf{v}_I(k) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Gamma}_R & -\boldsymbol{\Gamma}_I \\ \boldsymbol{\Gamma}_I & \boldsymbol{\Gamma}_R \end{bmatrix} \begin{bmatrix} \mathbf{s}_R(k) \\ \mathbf{s}_I(k) \end{bmatrix}. \tag{12}$$

The matrix premultiplying $[\mathbf{s}_R^T(k)\ \mathbf{s}_I^T(k)]^T$ on the right-hand side of (12) is real-valued and orthogonal. Moreover, under strong-uncorrelation, $E\{\mathbf{v}_R(k)\mathbf{v}_R^T(k)\} =$

$\frac{1}{2}(\mathbf{I} + \widehat{\boldsymbol{\Lambda}})$, $E\{\mathbf{v}_I(k)\mathbf{v}_I^T(k)\} = \frac{1}{2}(\mathbf{I} - \widehat{\boldsymbol{\Lambda}})$, and $E\{\mathbf{v}_R(n)\mathbf{v}_I^T(n)\} = \mathbf{0}$, where $\widehat{\boldsymbol{\Lambda}} = \mathbf{G}\mathbf{P}_{XX}\mathbf{G}^T$ is diagonal. Since the elements of $\mathbf{v}_R(k)$ and $\mathbf{v}_I(k)$ are not unit variance as required by the real-valued FastICA algorithm, define

$$\underline{\mathbf{v}}(k) = \begin{bmatrix} \overline{\mathbf{v}}_R(k) \\ -\overline{\mathbf{v}}_I(k) \end{bmatrix} = \begin{bmatrix} \sqrt{2}(\mathbf{I} + \widehat{\boldsymbol{\Lambda}})^{-1/2}\mathbf{v}_R(k) \\ -\sqrt{2}(\mathbf{I} - \widehat{\boldsymbol{\Lambda}})^{+1/2}\mathbf{v}_I(k) \end{bmatrix}. \tag{13}$$

$$\underline{\mathbf{s}}(k) = \begin{bmatrix} \overline{\mathbf{s}}_R(k) \\ -\overline{\mathbf{s}}_I(k) \end{bmatrix} = \begin{bmatrix} \sqrt{2}(\mathbf{I} + \widehat{\boldsymbol{\Lambda}})^{-1/2}\mathbf{s}_R(k) \\ -\sqrt{2}(\mathbf{I} - \widehat{\boldsymbol{\Lambda}})^{+1/2}\mathbf{s}_I(k) \end{bmatrix}. \tag{14}$$

The scaling operations in (13)–(14) are not valid in the space of complex matrices. Despite this fact, the orthonormal mixing properties between the sources in $\mathbf{s}(k)$ and the prewhitened mixture $\mathbf{v}(k)$ are maintained with this scaling.

Theorem 3: *Let $\overline{\mathbf{v}}(k) = \overline{\mathbf{v}}_R(k) + j\overline{\mathbf{v}}_I(k)$ and $\overline{\mathbf{s}}(k) = \overline{\mathbf{s}}_R(k) + j\overline{\mathbf{s}}_I(k)$. Then, under strong-uncorrelation, the relationship between $\overline{\mathbf{v}}(k)$ and $\overline{\mathbf{s}}(k)$ is identical to that between $\mathbf{v}(k)$ and $\mathbf{s}(k)$, i.e.* $\overline{\mathbf{v}}(k) = \boldsymbol{\Gamma}\overline{\mathbf{s}}(k)$ with $\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H = \boldsymbol{\Gamma}^T\boldsymbol{\Gamma}^* = \mathbf{I}$.

Proof: The proof is obtained by considering the structure of linearly-mixed strong-uncorrelated random variables as described in [5, 7] and is omitted for brevity.

The above theorem allows us to proceed with the specification of the FastICA algorithm in this case, as $E\{\underline{\mathbf{s}}(k)\underline{\mathbf{s}}^T(k)\} = \mathbf{I}$. All of the identifiability, uniqueness, and separability results for complex-valued ICA are preserved [7].

Given the relationship $\mathbf{w}_t = \mathbf{w}_{R,t} + j\mathbf{w}_{I,t}$, let $\underline{\mathbf{w}}_t = [\mathbf{w}_{R,t}^T \;\; \mathbf{w}_{I,t}^T]^T$, and define the output of the single-unit extraction system as $\overline{y}_t(k) = \underline{\mathbf{w}}_t^T\underline{\mathbf{v}}(k)$. It can be easily shown that $\overline{y}_t(k) = \Re e[\mathbf{w}_t^T\overline{\mathbf{v}}(k)]$. Since $\underline{\mathbf{v}}(k)$ contains an orthogonally-mixed set of $(2m)$ independent, real-valued sources with zero means and unit variances, we can use the standard real-valued FastICA procedure with kurtosis contrast to adjust the coefficients in $\underline{\mathbf{w}}_t$ as

$$\widetilde{\underline{\mathbf{w}}}_t = \left(\frac{1}{N}\sum_{n=1}^{N}\overline{y}_t^3(n)\underline{\mathbf{v}}(n)\right) - 3\underline{\mathbf{w}}_t, \qquad \underline{\mathbf{w}}_{t+1} = \frac{\widetilde{\underline{\mathbf{w}}}_t}{\sqrt{\widetilde{\underline{\mathbf{w}}}_t^T\widetilde{\underline{\mathbf{w}}}_t}}. \tag{15}$$

As has been shown in [8] for real-valued mixtures, this algorithm is guaranteed to converge to an extracting solution, which for our data structure means that one of the real or imaginary components of $\mathbf{s}(k)$ is obtained in $\overline{y}_t(k)$ with unit-variance scaling and (possibly) a sign change.

The algorithm in (15) requires the strong-uncorrelating transform, which requires specialized code to compute in the general case. It is possible to design a single-unit FastICA procedure to separate mixtures of complex-valued sources using only ordinary prewhitening. In this version, find any prewhitening matrix $\widehat{\mathbf{G}}$ satisfying $\widehat{\mathbf{G}}\mathbf{R}_{XX}\widehat{\mathbf{G}}^H = \mathbf{I}$, and set $\mathbf{v}(k) = \widehat{\mathbf{G}}\mathbf{x}(k)$. The pseudo-covariance matrix, which is not needed here, is not diagonal. The relationship between $\mathbf{v}(k)$ and $\mathbf{s}(k)$ is given in complex form by $\mathbf{v}(k) = \widehat{\boldsymbol{\Gamma}}\mathbf{s}(k)$, or in real form as in (12) with $\boldsymbol{\Gamma} = \widehat{\boldsymbol{\Gamma}}_R + j\widehat{\boldsymbol{\Gamma}}_I$. Define the $(2m)$-dimensional vectors of real-valued elements

$\underline{\mathbf{v}}(k)$ and $\underline{\mathbf{s}}(k)$ as

$$\underline{\mathbf{v}}(k) = \begin{bmatrix} \mathbf{v}_R(k) \\ -\mathbf{v}_I(k) \end{bmatrix} \text{ and } \underline{\mathbf{s}}(k) = \begin{bmatrix} \mathbf{s}_R(k) \\ -\mathbf{s}_I(k) \end{bmatrix} \tag{16}$$

Then, the sample autocorrelation matrix of $\mathbf{v}_R(k)$ is

$$\widehat{\mathbf{R}}_{VV} = \frac{1}{N}\sum_{n=1}^{N} \underline{\mathbf{v}}(n)\underline{\mathbf{v}}^T(n) \tag{17}$$

$$= \begin{bmatrix} \widehat{\boldsymbol{\Gamma}}_R & \widehat{\boldsymbol{\Gamma}}_I \\ -\widehat{\boldsymbol{\Gamma}}_I & \widehat{\boldsymbol{\Gamma}}_R \end{bmatrix} \left(\frac{1}{N}\sum_{n=1}^{N} \begin{bmatrix} \mathbf{s}_R(n)\mathbf{s}_R^T(n) & \mathbf{s}_R(n)\mathbf{s}_I^T(n) \\ \mathbf{s}_I(n)\mathbf{s}_R^T(n) & \mathbf{s}_I(n)\mathbf{s}_I^T(n) \end{bmatrix} \right) \begin{bmatrix} \widehat{\boldsymbol{\Gamma}}_R^T & -\widehat{\boldsymbol{\Gamma}}_I^T \\ \widehat{\boldsymbol{\Gamma}}_I^T & \widehat{\boldsymbol{\Gamma}}_R^T \end{bmatrix} \tag{18}$$

In the limit at $N \to \infty$, $\widehat{\mathbf{R}}_{VV}$ is not diagonal. Moreover, the powers in the $(2m)$ real-valued sources in $\underline{\mathbf{s}}(k)$ are not unity. Thus, a pair of fundamental assumptions about the FastICA procedure do not hold. Even so, we can derive a modified FastICA procedure to obtain one of the non-zero-kurtosis sources in $\underline{\mathbf{s}}(k)$; see [9] for a similar derivation of a different algorithm. Define $\underline{\mathbf{w}}_t$ as the system vector, and let $\underline{y}_t(k) = \underline{\mathbf{w}}_t^T\underline{\mathbf{v}}(k)$. Define

$$\underline{\mathbf{c}}_t = \begin{bmatrix} \mathbf{c}_{R,t} \\ -\mathbf{c}_{I,t} \end{bmatrix}, \; \underline{\widehat{\boldsymbol{\Gamma}}} = \begin{bmatrix} \widehat{\boldsymbol{\Gamma}}_R & \widehat{\boldsymbol{\Gamma}}_I \\ -\widehat{\boldsymbol{\Gamma}}_I & \widehat{\boldsymbol{\Gamma}}_R \end{bmatrix}, \quad \text{and} \quad \underline{\boldsymbol{\Lambda}}_S = \begin{bmatrix} \mathbf{I}+\boldsymbol{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}-\boldsymbol{\Lambda} \end{bmatrix}. \tag{19}$$

Then, we set $\underline{y}_t(k) = \underline{\mathbf{c}}_t^T\underline{\mathbf{s}}(k)$ and $\underline{\mathbf{c}}_t = \underline{\widehat{\boldsymbol{\Gamma}}}^T\underline{\mathbf{w}}_t$. Consider the fourth moment term

$$E\{|\underline{y}_t(k)|^4\} = \underline{\mathbf{c}}_t^T E\{\underline{\mathbf{s}}(k)\underline{\mathbf{s}}^T(k)\underline{\mathbf{c}}_t\underline{\mathbf{c}}_t^T\underline{\mathbf{s}}(k)\underline{\mathbf{s}}^T(k)\}\underline{\mathbf{c}}_t. \tag{20}$$

It is straightforward to show that

$$E\{\underline{\mathbf{s}}(k)\underline{\mathbf{s}}^T(k)\underline{\mathbf{c}}_t\underline{\mathbf{c}}_t^T\underline{\mathbf{s}}(k)\underline{\mathbf{s}}^T(k)\} = 2\underline{\boldsymbol{\Lambda}}_S\underline{\mathbf{c}}_t\underline{\mathbf{c}}_t^T\underline{\boldsymbol{\Lambda}}_S + \underline{\boldsymbol{\Lambda}}\underline{\mathbf{c}}_t^T\underline{\boldsymbol{\Lambda}}_S\underline{\mathbf{c}}_t + \underline{\mathbf{K}}\mathrm{diag}[\underline{\mathbf{c}}_t\underline{\mathbf{c}}_t^T], \tag{21}$$

such that the desired update is

$$\widetilde{\underline{\mathbf{c}}}_t = E\{\underline{\mathbf{s}}(k)\underline{\mathbf{s}}^T(k)\underline{\mathbf{c}}_t\underline{\mathbf{c}}_t^T\underline{\mathbf{s}}(k)\underline{\mathbf{s}}^T(k)\}\underline{\mathbf{c}}_t - 3\underline{\boldsymbol{\Lambda}}_S\underline{\mathbf{c}}_t\underline{\mathbf{c}}_t^T\underline{\boldsymbol{\Lambda}}_S\underline{\mathbf{c}}_t. \tag{22}$$

The power constraint changes to $E\{\underline{y}_t^2(k)\} = \underline{\mathbf{c}}_t^T\underline{\boldsymbol{\Lambda}}_S\underline{\mathbf{c}}_t = (1 \pm \lambda_i)$, and it is met when $\underline{\mathbf{c}}_t$ has only one non-zero unity-valued element, which is equivalent to $\underline{\mathbf{w}}_t^T\underline{\mathbf{w}}_t = 1$. Transforming back to the coordinates $\underline{\mathbf{w}}_t$, we obtain the update

$$\widetilde{\underline{\mathbf{w}}}_t = \left(\frac{1}{N}\sum_{n=1}^{N} \underline{y}_t^3(n)\underline{\mathbf{v}}(n)\right) - 3\widehat{\mathbf{R}}_{VV}\underline{\mathbf{w}}_t\left(\frac{1}{N}\sum_{n=1}^{N} \underline{y}^2(n)\right), \quad \underline{\mathbf{w}}_{t+1} = \frac{\widetilde{\underline{\mathbf{w}}}_t}{\sqrt{\widetilde{\underline{\mathbf{w}}}_t^T\widetilde{\underline{\mathbf{w}}}_t}} \tag{23}$$

4 Designing Multiple-Component Extraction Procedures

For either of our single-source extraction procedures in (15) or (23) , we now develop extensions that employ multiple parallel systems to extract each of the source components within the mixture. We exploit the structure of the complex prewhitened mixing system as indicated in (12) to make this task easier. Suppose that $\mathbf{w}_t = \mathbf{w}_{R,t} + j\mathbf{w}_{it}$ of either algorithm has converged such that $\Re e[\mathbf{w}_t^T \mathbf{v}(k)] = \underline{d}_{R,i} s_{R,i}(k)$ for some real-valued scalar $\underline{d}_{R,i}$. Then, $\Im m[\mathbf{w}_t^T \mathbf{v}(k)] = \underline{d}_{I,i} s_{I,i}(k)$ for some real-valued scalar $\underline{d}_{I,i}$. Similarly, if $\Re e[\mathbf{w}_t^T \mathbf{v}(k)] = \underline{d}_{I,i} s_{I,i}(k)$, then $\Im m[\mathbf{w}_t^T \mathbf{v}(k)] = \underline{d}_{R,i} s_{R,i}(k)$. In other words, extracting any real (or imaginary) component of a source in the mixture gives the corresponding imaginary (or real) component of that source via the complex conjugate of the complex-valued system output. We only need to run m single-unit real-valued extraction procedures and employ each extracted coefficient vector twice to deflate the signal space as sources are extracted. If Gram-Schmidt deflation is employed, the multi-source extension of the algorithm in (23) for the ith separation stage is

$$\underline{y}_{it}(k) = \underline{\mathbf{w}}_{it}^T \underline{\mathbf{v}}(k) \tag{24}$$

$$\widetilde{\underline{\mathbf{w}}}_{it} = \left(\frac{1}{N}\sum_{n=1}^{N} \underline{y}_{it}^3(n)\underline{\mathbf{v}}(n)\right) - 3\left(\frac{1}{N}\sum_{n=1}^{N} \underline{y}_{it}^2(n)\right)\widehat{\mathbf{R}}_{VV}\underline{\mathbf{w}}_{it} \tag{25}$$

for $n = 1$ to $i-1$ do

$$\overline{\underline{\mathbf{w}}}_{it} = \widetilde{\underline{\mathbf{w}}}_{it} - \underline{\mathbf{w}}_n \underline{\mathbf{w}}_n^T \widetilde{\underline{\mathbf{w}}}_{it} \tag{26}$$

$$\overline{\overline{\underline{\mathbf{w}}}}_{it} = \overline{\underline{\mathbf{w}}}_{it} - \underline{\mathbf{m}}_n \underline{\mathbf{m}}_n^T \overline{\underline{\mathbf{w}}}_{it} \tag{27}$$

end

$$\underline{\mathbf{w}}_{i(t+1)} = \frac{\overline{\overline{\underline{\mathbf{w}}}}_{it}}{\sqrt{\overline{\overline{\underline{\mathbf{w}}}}_{it}^T \overline{\overline{\underline{\mathbf{w}}}}_{it}}} \tag{28}$$

where the vectors $\underline{\mathbf{w}}_n = [\mathbf{w}_{R,n}^T \quad \mathbf{w}_{I,n}^T]^T$ and $\underline{\mathbf{m}}_n = [-\mathbf{w}_{I,n}^T \quad \mathbf{w}_{R,n}^T]^T$ are the coefficient vectors from the previous extraction steps. After convergence of all units,

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_{R,1}^T + j\mathbf{w}_{I,1}^T \\ \vdots \\ \mathbf{w}_{R,m}^T + j\mathbf{w}_{I,m}^T \end{bmatrix}, \qquad \mathbf{y}(k) = \mathbf{W}\mathbf{v}(k). \tag{29}$$

The following theorem describes the separating capabilities of this algorithm.

Theorem 4: *Suppose* $\mathbf{x}(k)$ *contains a mixture of* m *complex-valued statistically-independent sources that all have statistically-independent real and imaginary parts in which all but one of the sources has either a real or an imaginary component with a non-zero kurtosis. Then, either of the algorithms in (15) or (23) combined with (26)–(29) extracts all complex-valued sources in* $\mathbf{s}(k)$.

Remarks: The above theorem allows for each source to have a zero-kurtosis real or imaginary part that could be Gaussian-distributed. Thus, our algorithms can extract several BPSK sources measured in Gaussian noise in a complex baseband representation of an array processing system in wireless communications. In addition, note that our techniques are more powerful than a general $(2m)$-dimensional FastICA procedure applied to a set of prewhitened signal mixtures generated from the real and imaginary parts of $\mathbf{x}(k)$. The latter procedure would require all but one of the $2m$ *total real and imaginary parts* of the complex sources to have a non-zero kurtosis.

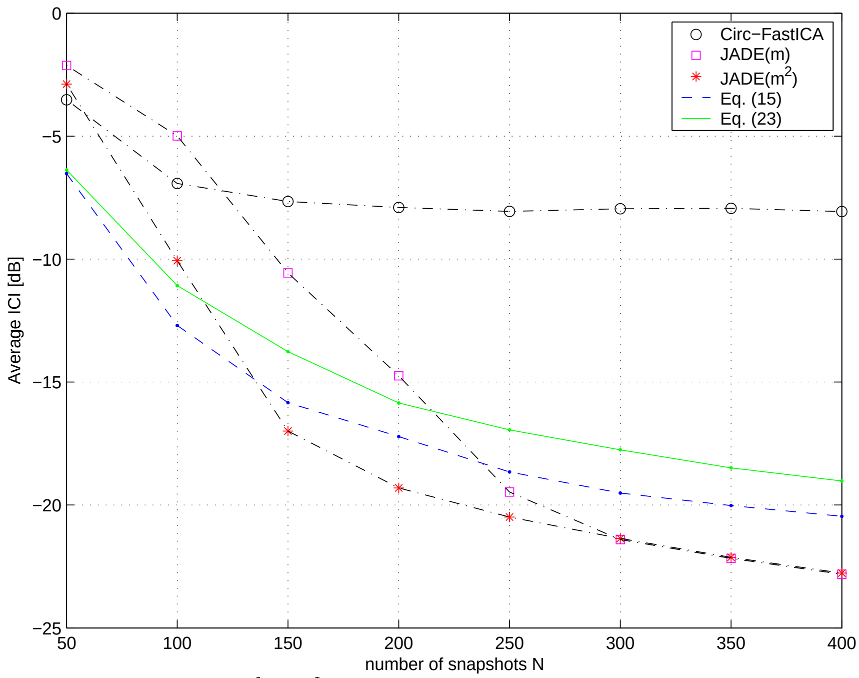

Fig. 1. Convergence of $E\{ICI_t\}$ for the various algorithms in a noiseless six-source separation task.

5 Simulations

We now explore the numerical performances of the proposed algorithms. All evaluations are performed on synthetic data using the MATLAB technical computing environment. Three BPSK and three 16-QAM sources were mixed using $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are random complex orthogonal and the complex diagonal elements of $\boldsymbol{\Sigma}$ have amplitudes in the range $[0.2, 1]$. The average inter-channel interference (ICI) is used to measure performance, as given by

$$ICI = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{\sum_{l=1}^{m}|c_{il}|^2 - \max_{1\leq k\leq m}|c_{ik}|^2}{\max_{1\leq k\leq m}|c_{ik}|^2}\right), \tag{30}$$

where $c_{il} = [\mathbf{W}\widehat{\mathbf{G}}\mathbf{A}]_{il}$. Shown in Figure 1 are the performances of the multi-unit versions of the algorithms in (15) and (23) along with those of two different versions of JADE using m and m^2 cumulant matrices [1], and the circular complex FastICA algorithm [2] with asymmetric deflation and $G(|y|^2) = 0.5|y|^2$. The proposed methods outperform the algorthm in [2], and they also perform better than JADE(m) for $N \leq 200$. Figure 2 shows a more-realistic situation in which an additional Gaussian was included in the $m = 7$-source mixtures and additive circular uncorrelated Gaussian noises with variances $\sigma_\nu^2 = 0.001$ was used as measurement interference. In this case, the proposed methods perform as well as or better than both JADE versions for $N \leq 500$ snapshots. Per-unit conver-

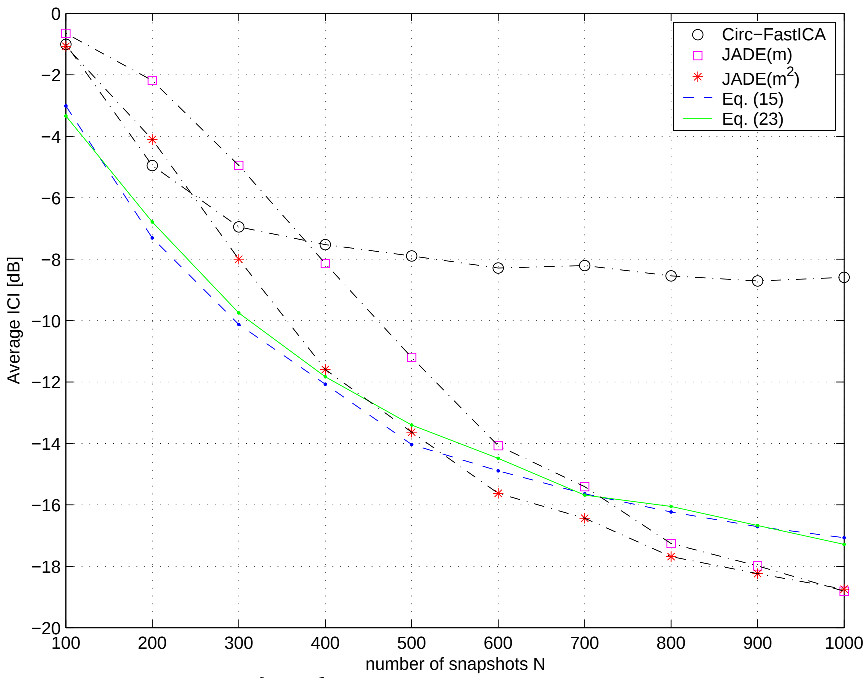

Fig. 2. Convergence of $E\{ICI_t\}$ for the various algorithms in a noisy seven-source separation task.

gence of the proposed algorithms is as fast as the original real-valued FastICA algorithm; only a few iterations of (15) and (23) are required at each stage.

6 Conclusions

In this paper, we have derived two novel algorithms for extracting independent sources from complex-valued mixtures using the fourth-moment properties of their real or imaginary components. The algorithms are computationally-simple and converge quickly. Simulations on mixtures of complex-valued signals typically found in wireless communications applications show the methods' efficacies.

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