BLIND DECONVOLUTION OF IMPULSIVE SIGNALS USING A MODIFIED SATO ALGORITHM

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ABSTRACT

Many blind deconvolution algorithms have been designed to extract digital communications signals corrupted by inter-symbol interference. Such algorithms generally fail when applied to signals with impulsive characteristics, such as acoustic signals. In this paper, we provide a theoretical analysis and explanation as to why Bussgang-type algorithms are generally unsuitable for deconvolving impulsive signals. We then propose a novel modification of one such algorithm, the Sato algorithm, to enable it to deconvolve such signals. Sufficient conditions on the source signal to guarantee local stability of the modified Sato algorithm about a deconvolving solution are derived. Computer simulations show the efficiency of the proposed approach as compared to the Shalvi-Weinstein algorithm for deconvolving impulsive signals.

1. INTRODUCTION

Blind deconvolution is a signal processing task that is important for several applications. In teleconferencing, blind deconvolution can be used to remove reverberation effects from speech signals for which no training signals are available [1]. In the general blind deconvolution task the source signal s_t is filtered by an unknown transmission channel, denoted by $h(z)$ with unknown impulse response h_t, $0 \leq t < \infty$. The received signal x_t, given by

$$x_t = \sum_{k=0}^{\infty} h_k s_{t-k}, \qquad (1)$$

contains mixtures of the source signal elements at numerous lags. The goal is to "undo" the mixing process by a linear filtering operation, denoted by $w(z)$. For simplicity, we shall approximate the deconvolution filter by the FIR model

$$u_t = \sum_{k=0}^{L} w_k x_{t-k}. \qquad (2)$$

The blind deconvolution task is solved if

$$u_t = c s_{t-d}, \qquad (3)$$

where c is an unknown gain and d is an unknown integer delay. In such situations, the original source signal is recovered in the deconvolution system output, except for perhaps an arbitrary gain and delay. To solve the blind deconvolution task, an iterative algorithm is applied to the coefficients $\{w_k\}$ to adjust them to the vicinity of a deconvolving solution. In the sequel, we shall employ a vector notation to represent certain filtering operations, such that $\boldsymbol{w}_t \triangleq [w_{0t}\ w_{1t}\ \cdots\ w_{Lt}]^T$ is the time-varying coefficient vector, $\boldsymbol{x}_t \triangleq [x_t\ x_{t-1}\ \cdots\ x_{t-L}]^T$ is the input signal vector, and

$$u_t = \boldsymbol{w}_t^T \boldsymbol{x}_t \qquad (4)$$

is the deconvolved output signal at time t.

The Sato algorithm was one of the first blind deconvolution procedures to appear in the signal processing literature [2]. This algorithm remains quite popular today, having become known as the decision-directed algorithm for blind equalization of BPSK signals [3]. The coefficient updates for the Sato algorithm are

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \mu(\gamma\, \mathrm{sign}(u_t) - u_t)\boldsymbol{x}_t, \qquad (5)$$

where $\mathrm{sign}(u_t)$ is one if u_t is positive, zero if u_t is zero, and (-1) if u_t is negative. The dispersion coefficient γ controls the output scaling and is computed as $\gamma \triangleq E\{s^2\}/E\{|s|\}$. The parameter μ controls the convergence performance of the algorithm and is normally chosen to be positive. The Sato algorithm can be viewed as one version of the Bussgang family of blind deconvolution algorithms that also includes the Godard and constant modulus algorithms (CMA) [4], [5]. Each of these algorithms can be written in the general form

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \mu(g_1(u_t) - g_2(u_t))\boldsymbol{x}_t, \qquad (6)$$

where $g_1(u)$ and $g_2(u)$ are nonlinearities with specific, usually monomial, forms.

It is well known that Bussgang-type adaptive algorithms perform well when deconvolving communication signals. For impulsive signals, such as many common acoustic signals in seismic and audio signal processing, such methods are known to fail, making them inappropriate for more general tasks such as acoustic dereverberation [1]. One possible remedy for this algorithm class has been suggested by Benveniste *et al.* [6]. This approach employs a prewhitening stage followed by a norm-constrained adaptation procedure. Such a solution complicates an otherwise simple adaptive procedure through additional processing stages. Few unconstrained approaches have been developed [7].

In this paper, we provide a theoretical justification as to why Bussgang-type algorithms fail to deconvolve impulsive signals from their filtered measurements. We then provide a novel modification of the Sato algorithm to enable it to deconvolve impulsive signals. Our modified version is computationally simple, avoids signal prewhitening, and requires only multiplies and adds to implement. We determine sufficient statistical conditions to guarantee local stability of the algorithm about a deconvolving solution. Computer simulations indicate that the modified Sato algorithm is more efficient in deconvolving Laplacian-distributed source signals than the well known Shalvi-Weinstein algorithm.

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2. STABILITY OF BUSSGANG ALGORITHMS

We first study the local stability properties of Bussgang algorithms to determine their capabilities in extracting signals with various distributions. The procedure used for this analysis is the linearized ODE approach. For additional details on this method, see [8].

To analyze the local stability properties of Eq. (6), we express the deviations of the equalizer coefficients from their optimal values as

$$w_k = w_{k,\text{opt}} + \Delta_k. \tag{7}$$

The output of the equalizer

$$u_t = \sigma_u s_{t-d} + \sum_k \Delta_k x_{t-k} \tag{8}$$

is a delayed and scaled version of the original signal sequence perturbed by convolutive noise, where σ_u allows for a different output energy. We shall assume a two-sided infinitely long equalizer impulse response in order to circumvent problems with filter truncation. The perturbation vector $\boldsymbol{\Delta} \triangleq [\Delta_{-\infty}, \ldots, \Delta_0, \ldots, \Delta_\infty]^T$ now propagates similarly to Eq. (6) as

$$\begin{aligned}\boldsymbol{\Delta}_{t+1} = \boldsymbol{\Delta}_t + \mu\Big(& g_1\big(\sigma_u s_{t-d} + \sum_k \Delta_k x_{t-k}\big) \\ & - g_2\big(\sigma_u s_{t-d} + \sum_k \Delta_k x_{t-k}\big)\Big)\boldsymbol{x}_t,\end{aligned} \tag{9}$$

where we have used Eq. (8). We now approximate the nonlinearities $g_1(.)$ and $g_2(.)$ by their respective first-order Taylor series about $\sigma_u s_{t-d}$:

$$\begin{aligned}& g_{\{1,2\}}\big(\sigma_u s_{t-d} + \sum_k \Delta_k x_{t-k}\big) \\ & \approx g_{\{1,2\}}(\sigma_u s_{t-d}) + g'_{\{1,2\}}(\sigma_u s_{t-d}) \cdot \sum_k \Delta_k x_{t-k}.\end{aligned} \tag{10}$$

Using Eq. (10) in Eq. (9) and taking the expectation of both sides yields

$$\begin{aligned}E\{\boldsymbol{\Delta}_{t+1}\} &= E\{\boldsymbol{\Delta}_t\} + \mu E\left\{\big(g_1(\sigma_u s_{t-d}) - g_2(\sigma_u s_{t-d})\big)\boldsymbol{x}_t\right\} \\ &+ \mu E\left\{\big(g'_1(\sigma_u s_{t-d}) - g'_2(\sigma_u s_{t-d})\big)\sum_k \Delta_k x_{t-k}\boldsymbol{x}_t\right\}.\end{aligned} \tag{11}$$

At a stationary point of Eq. (6), we have

$$E\left\{\big(g_1(\sigma_u s_{t-d}) - g_2(\sigma_u s_{t-d})\big)\boldsymbol{x}_t\right\} = \boldsymbol{0}. \tag{12}$$

This condition also determines the output power, so that we are left with the last term in Eq. (11), the difference of the derivatives, as the driving term.

Employing the ODE method on the above update, we obtain

$$\frac{d\boldsymbol{\Delta}}{dt} = \mu \boldsymbol{A}\boldsymbol{\Delta}. \tag{13}$$

The elements of the transition matrix $\boldsymbol{A}$ are obtained by looking at

$$\frac{d\Delta_i}{dt} = \mu E\left\{\big(g'_1(\sigma_u s_{t-d}) - g'_2(\sigma_u s_{t-d})\big)\sum_k \Delta_k x_{t-k} x_{t-i}\right\}. \tag{14}$$

Thus, matrix $\boldsymbol{A} \triangleq \boldsymbol{A}_1$ may be expressed as

$$[A_1]_{ik} = \zeta_1\Big[\sum_j h_{j+k-i}h_j\Big]_{ik} + (\zeta_0 - \zeta_1)\Big[h_{d-i}h_{d-k}\Big]_{ik}, \tag{15}$$

where the nonlinear moments are given by

$$\zeta_0 \triangleq E\left\{(g'_1(\sigma_u s) - g'_2(\sigma_u s))s^2\right\}, \tag{16}$$

$$\zeta_1 \triangleq E\left\{g'_1(\sigma_u s) - g'_2(\sigma_u s)\right\}. \tag{17}$$

The matrix $\boldsymbol{A}$ is both infinite dimensional and symmetric. Symmetric matrices have real eigenvalues. Thus, the corresponding ODE is locally stable if $\mu\boldsymbol{A}$ is negative definite. By defining the channel matrix

$$\boldsymbol{H} \triangleq \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \iddots \\ \cdots & h_d & h_{d+1} & h_{d+2} & \cdots \\ \cdots & h_{d-1} & h_d & h_{d+1} & \cdots \\ \cdots & h_{d-2} & h_{d-1} & h_d & \cdots \\ \iddots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{18}$$

and the channel vector $\bar{\boldsymbol{h}} \triangleq [\ldots, h_{d+1}, h_d, h_{d-1}, \ldots]^T$, respectively, we can formulate $\boldsymbol{A}$ as a sum of an autocorrelation matrix and an outer product:

$$\boldsymbol{A} = \zeta_1 \boldsymbol{H}^T\boldsymbol{H} + (\zeta_0 - \zeta_1)\bar{\boldsymbol{h}}\bar{\boldsymbol{h}}^T. \tag{19}$$

The bar in $\bar{\boldsymbol{h}}$ indicates the reverse order of its elements compared to the usual definition of a vector. As an infinite-dimensional matrix, $\boldsymbol{H}$ can be regarded as circulant with all its corresponding properties, and the eigenvalues of $\boldsymbol{H}$ are then just the DFT coefficients of a row of $\boldsymbol{H}$ [9]. Hence, if the channel transfer function has no spectral nulls, $\boldsymbol{H}$ is nonsingular and thus, $\boldsymbol{H}^T\boldsymbol{H}$ is positive definite. The matrix $\bar{\boldsymbol{h}}\bar{\boldsymbol{h}}^T$ is positive semidefinite with a rank of one. Sufficient conditions for $\mu\boldsymbol{A}$ to be negative definite are therefore

$$\mu\zeta_1 < 0, \tag{20}$$

$$\mu\zeta_0 \le \mu\zeta_1. \tag{21}$$

Most Bussgang-type algorithms can be written as a difference of two polynomials

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \mu(a\,\text{sign}(u_t)|u_t|^p - \text{sign}(u_t)|u_t|^q)\boldsymbol{x}_t, \tag{22}$$

which is simply a special form of Eq. (6).

Super-Gaussian signals are a class of distributions whose pdf is of the form

$$p_s(s) = \frac{\alpha}{2\beta\Gamma\left(\frac{1}{\alpha}\right)} e^{-\left(\frac{|s|}{\beta}\right)^\alpha}, \tag{23}$$

with $0 < \alpha < 2$. We now prove that for such distributions, e.g., Laplacian distributed signals ($\alpha = 1$), Bussgang-type algorithms fail to provide local stability. Our proof uses the following intermediate result.

Lemma: *For $0 < \alpha < 2$ and $p < q$ the ratio of Gamma functions behaves as*

$$\frac{\Gamma\left(\frac{p+2}{\alpha}\right)}{p\Gamma\left(\frac{p}{\alpha}\right)} < \frac{\Gamma\left(\frac{q+2}{\alpha}\right)}{q\Gamma\left(\frac{q}{\alpha}\right)}. \tag{24}$$

A proof of this lemma can be found in [10].

Theorem: *No blind Bussgang-type algorithm of the general form*

$$w_{t+1} = w_t + \mu(a\,\mathrm{sign}(u_t)|u_t|^p - \mathrm{sign}(u_t)|u_t|^q)x_t \qquad (25)$$

can be stabilized by any choice of p and q and any super-Gaussian distribution.

Proof: Due to Eqs. (20) and (21), the nonlinear moments ζ_0 and ζ_1 need to have the same sign. Then, stability can be controlled by proper choice of the sign of the step size μ. Without loss of generality, we assume that $p < q$, which makes $\zeta_0 < 0$. But due to the lemma stated above, we note that

$$\frac{p\Gamma\left(\frac{p}{\alpha}\right)}{\Gamma\left(\frac{p+2}{\alpha}\right)} - \frac{q\Gamma\left(\frac{q}{\alpha}\right)}{\Gamma\left(\frac{q+2}{\alpha}\right)} > 0, \qquad (26)$$

making $\zeta_1 > 0$, thus enforcing a different sign of ζ_1. □

3. ALGORITHM MODIFICATIONS

Given that Bussgang-type algorithms fail to deconvolve impulsive signals, a simple modification to such procedures that enables their use would be highly desirable. We propose to modify the Bussgang update in Eq. (6) by inserting terms proportional to the squared L_2-norm of the coefficient vector within the updates:

$$w_{t+1} = w_t + \mu(\|w_t\|^{\tilde{p}} g_1(u_t) - \|w_t\|^{\tilde{q}} g_2(u_t))x_t \qquad (27)$$

where $\|w\|^2 = w^T w$ and $\tilde{p}$ and $\tilde{q}$ are even-valued integers. The design goal is to choose $\tilde{p}$ and $\tilde{q}$ so that the corresponding algorithm is locally stable. Performing this design requires us to repeat our analysis procedure for Eq. (27).

By approximating

$$\|w + \Delta\|^{\tilde{p}} \approx \|w\|^{\tilde{p}} + \tilde{p}\|w\|^{\tilde{p}-2} \sum_k w_k \Delta_k, \qquad (28)$$

the first-order Taylor series approximation of the scalar part of the update term becomes

$$\begin{aligned}
&\|w\|^{\tilde{p}} g_1(\sigma_u s_{t-d} + \sum_i \Delta_i x_{t-i}) - \|w\|^{\tilde{q}} g_2(\sigma_u s_{t-d} + \sum_i \Delta_i x_{t-i}) \\
&\approx \|w\|^{\tilde{p}} g_1(\sigma_u s_{t-d}) + \|w\|^{\tilde{p}} g_1'(\sigma_u s_{t-d}) \sum_i \Delta_i x_{t-i} \\
&+ \tilde{p}\|w\|^{\tilde{p}-2} \sum_i \Delta_i w_i g_1(\sigma_u s_{t-d}) - \|w\|^{\tilde{q}} g_2(\sigma_u s_{t-d}) \\
&- \|w\|^{\tilde{q}} g_2'(\sigma_u s_{t-d}) \sum_i \Delta_i x_{t-i} - \tilde{q}\|w\|^{\tilde{q}-2} \sum_i \Delta_i w_i g_2(\sigma_u s_{t-d}).
\end{aligned} \qquad (29)$$

The additional factors $\|w\|^{\tilde{p}}$ and $\|w\|^{\tilde{q}}$ in Eq. (27) shift the stationary point of this update such that the output power σ_u^2 depends on $\tilde{p}$ and $\tilde{q}$. At the stationary point, we get

$$E\left\{\left(\|w\|^{\tilde{p}} g_1(\sigma_u s_{t-d}) - \|w\|^{\tilde{q}} g_2(\sigma_u s_{t-d})\right) x_t\right\} = 0, \qquad (30)$$

where the power of the output signal is given by

$$\sigma_u^2 = \|w * h\|^2 \sigma_s^2 = \|w * h\|^2. \qquad (31)$$

Eqs. (30) and (31) define a system of equations for the two unknown σ_u and $\|w\|^2$. Going through a similar analysis as before, we obtain the ODE

$$\frac{d\Delta}{dt} = \mu A \Delta = \mu(A_1 + A_2)\Delta, \qquad (32)$$

with

$$[A]_{ik} = \zeta_1 \Big[\sum_j h_{j+k-i} h_j\Big]_{ik} + (\zeta_0 - \zeta_1)\Big[h_{d-i} h_{d-k}\Big]_{ik} + \tilde{\zeta}_1 \Big[h_{d-i} w_k\Big]_{ik}. \qquad (33)$$

The nonlinear moments in A_1 are now different from the ones given by Eqs. (16) and (17)

$$\zeta_0 \triangleq \|w\|^{\tilde{p}} E\{g_1'(\sigma_u s)s^2\} - \|w\|^{\tilde{q}} E\{g_2'(\sigma_u s)s^2\}, \qquad (34)$$

$$\zeta_1 \triangleq \|w\|^{\tilde{p}} E\{g_1'(\sigma_u s)\} - \|w\|^{\tilde{q}} E\{g_2'(\sigma_u s)\}. \qquad (35)$$

Furthermore, we have introduced

$$\tilde{\zeta}_1 \triangleq \tilde{p}\|w\|^{\tilde{p}-2} E\{g_1(\sigma_u s)s\} - \tilde{q}\|w\|^{\tilde{q}-2} E\{g_2(\sigma_u s)s\}. \qquad (36)$$

Although A_1 is still a symmetric matrix, A is no longer symmetric. Assigning the equalizer vector $w \triangleq [\ldots, w_{-1}, w_0, w_1, \ldots]^T$, we can write A as

$$A = \zeta_1 H^T H + (\zeta_0 - \zeta_1)\bar{h}\bar{h}^T + \tilde{\zeta}_1 \bar{h} w^T = \zeta_1 H^T H + \bar{h}\tilde{h}^T, \qquad (37)$$

where we have defined

$$\tilde{h} \triangleq (\zeta_0 - \zeta_1)\bar{h} + \tilde{\zeta}_1 w. \qquad (38)$$

Although the two last terms of Eq. (37) contain two outer products, the sum has still only one nonzero eigenvalue owing to the outer products sharing the same column vector. Hence, $\bar{h}\tilde{h}^T$ is either positive semidefinite or negative semidefinite with all but one eigenvalues equal to zero. Its definiteness depends on

$$\lambda = \mathrm{tr}\left(\bar{h}\tilde{h}^T\right) = \bar{h}^T \tilde{h}. \qquad (39)$$

The first part of Eq. (32) requires Eq. (20). For negative semidefiniteness of the second part of Eq. (32), we must ensure that

$$\mu \bar{h}^T \tilde{h} = \mu \bar{h}^T((\zeta_0 - \zeta_1)\bar{h} + \tilde{\zeta}_1 w \leq 0. \qquad (40)$$

Without loss of generality we can restrict the channel response to unit energy, thus $\bar{h}^T\bar{h} = 1$. We designate w_{opt} as the solution of the equalizer coefficients that perfectly inverts the channel $\bar{h}$, i.e., $\bar{h}^T w_{\mathrm{opt}} = 1$, so that the output power is $\sigma_u^2 = \|h * w_{\mathrm{opt}}\|^2 = 1$. The equalizer taps are adjusted to a scaled version of w_{opt},

$$w = \sigma_u w_{\mathrm{opt}}. \qquad (41)$$

Subsequently, we also have

$$\|w\|^m = \sigma_u^m \|w_{\mathrm{opt}}\|^m. \qquad (42)$$

Therefore, the convolution of the channel impulse response and the equalizer impulse response yields

$$\bar{h}^T w = \sigma_u. \qquad (43)$$

Hence, the 2nd condition for negative definiteness of μA is

$$\mu(\zeta_0 - \zeta_1 + \tilde{\zeta}_1 \sigma_u) \leq 0. \qquad (44)$$

In the following, we look at a specific design example for a modified Sato algorithm in more detail. More examples and modifications for general Bussgang-type algorithms are given in [10].

Suppose that the first stability condition of the Sato algorithm in Eq. (20), i.e., $\mu(2p_s(0)/E\{|s|\} - 1) \geq 0$, is not satisfied for a particular signal distribution and any positive value of μ. Clearly, we can satisfy this condition by choosing a negative μ. Such a choice causes the norm $\|\boldsymbol{w}_t\|$ to fail to converge, since the second condition, i.e., Eq. (21), is no longer satisfied. We can modify Sato's algorithm by appending the norm factor $\|\boldsymbol{w}\|^{\tilde{p}}$ to the update equation to get

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \mu(\|\boldsymbol{w}\|^{\tilde{p}}\gamma \operatorname{sign}(u_t) - u_t)\boldsymbol{x}_t. \tag{45}$$

By determining the stationary point of Eq. (45) and using Eq. (42), we obtain

$$\sigma_u = \|\boldsymbol{w}_{\mathrm{opt}}\|^{\frac{\tilde{p}}{1-\tilde{p}}} \left(\gamma E\{|s|\}\right)^{\frac{1}{1-\tilde{p}}} \tag{46}$$

and the norm relation

$$\|\boldsymbol{w}\|^m = \|\boldsymbol{w}_{\mathrm{opt}}\|^{\frac{m}{1-\tilde{p}}} \left(\gamma E\{|s|\}\right)^{\frac{m}{1-\tilde{p}}}. \tag{47}$$

In order to test this modified algorithm for local stability, we need to compute the nonlinear moments. Using the definitions in the previous sections and the fact that the mode of a pdf is inversely proportional to its standard deviation, we get

$$\zeta_0 = -1, \tag{48}$$

$$\zeta_1 = \|\boldsymbol{w}\|^{\tilde{p}} 2\gamma p_u(0) - 1 = \frac{2p_s(0)}{E\{|s|\}} - 1, \tag{49}$$

$$\tilde{\zeta}_1 \sigma_u = \tilde{p}\|\boldsymbol{w}\|^{\tilde{p}-2}\gamma E\{|s|\}\,\sigma_u = \tilde{p}\|\boldsymbol{w}_{\mathrm{opt}}\|^{-2}. \tag{50}$$

For negative μ, the first stability condition is now met by super-Gaussian signals, while the stability condition in Eq. (44) can be written as

$$\|\boldsymbol{w}_{\mathrm{opt}}\|^2 \leq \tilde{p}\frac{E\{|s|\}}{2p_s(0)}. \tag{51}$$

These conditions allow for the deconvolution of super-Gaussian signals so long as the optimum coefficient vector is not overly long. It should be noted that these stability conditions are sufficient but not necessary due to our use of overly stringent conditions on the components of matrix $\boldsymbol{A}$.

4. COMPUTER SIMULATIONS

Computer simulations have been carried out to verify the stable behavior of the modified Sato algorithm for deconvolving a super-Gaussian, in this case Laplacian, signal. The channel model used for the computer simulations is a 7-tap FIR filter with impulse response $h(z) = 0.4 + z^{-1} - 0.7z^{-2} + 0.6z^{-3} + 0.3z^{-4} - 0.4z^{-5} + 0.1z^{-6}$. Moderate noise has been added resulting in a signal to noise ratio of SNR=30 dB. For a comparision of the modified Sato algorithm, we have implemented the Shalvi-Weinstein algorithm (SWA) [7], where both equalizer filters were 20 taps long. The exponents of the norms in the modified Sato algorithm were chosen as $\tilde{p} = 2$ and $\tilde{q} = 0$. Step sizes for each algorithm have been adjusted so that a steady-state ISI of 15 dB is achieved at convergence. One hundred different data realizations have been used to smooth the convergence curves. Fig. 1 shows the favorable behavior of the modified Sato algorithm compared to the SWA in this simulation. The slow convergence of SWA is indicative of the higher-order-statistics-based procedure used, whereas the Sato algorithm employs much more efficient estimation.

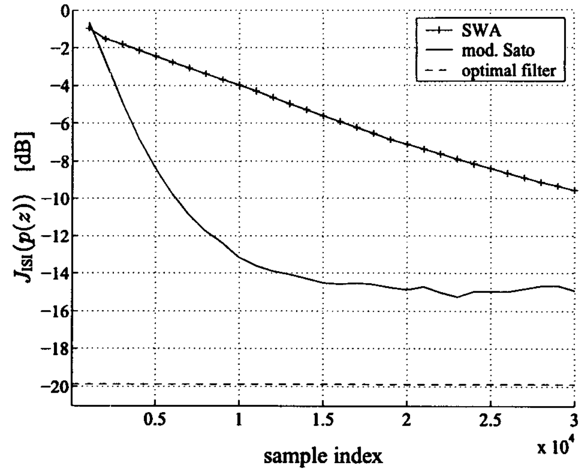

Fig. 1. Convergence comparison of the Shalvi-Weinstein algorithm (SWA) and the modified Sato algorithm.

5. CONCLUSION

It is well known that Bussgang-type algorithms fail to deconvolve signals with impulsive distributions. In this paper, we have proven why these algorithms fail for such distributions using local stability analysis. Our analysis provides a way of designing modified Bussgang algorithms to obtain locally stable and convergent behavior for various signal distributions. Simulations of a modified Sato algorithm confirm the efficacy of the modifications.

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