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Design Procedures for Backbone Transport Networks with Shared Protection: Optimization-Based Models, Exact and Heuristic Algorithms, and Unavailability Computations

by

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Comments and criticisms from interested readers are cordially invited.

Abstract

This manuscript presents optimization-based design models and algorithms for reliable backbone transport networks. Due to their efficiency in providing data services and high data rates, dense wavelength division multiplexing (DWDM) networks became the technology of choice for these transport networks. DWDM networks achieve these high rates by enabling several wavelengths of light to be transmitted on a single strand of fiber. Optimization-based practical methods to design DWDM networks with both link-based and path-based shared protection schemes are introduced and compared in an empirical analysis. These design models address various restrictions inherit in DWDM technology including polarization mode dispersion and attenuation requirements. Furthermore, approximations are developed to determine unavailability for any given origin-destination pair in a DWDM network. Finally, path-based and link-based shared protection models are compared based on design costs and unavailability.

Keywords: integer programming applications, network provisioning, DWDM network design; Polarization Mode Dispersion, shared protection, availability

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1. Introduction

Fiber links in DWDM networks carry data in the form of optical signals with several wavelengths. Each wavelength can carry 40Gbps of data and continuous advances in optical transmission technologies promise even higher capacities. To make the fiber operational, equipment of several types are required. Optical amplifiers (As) are required at certain locations along a fiber link. As signals travel along fiber and through equipment, the background noise in the transmission tends to increase. Optical amplifiers do not distinguish between noise and the original signal and they boost the entire signal on a fiber link. Regenerators (Rs) are used to retrieve the original signal by a process called optical/electrical/optical (O/E/O) conversion. This O/E/O conversion requires not only regenerators but also optical amplifiers, wavelength division multiplexers (MUXs), and demultiplexers (DMUXs). These MUXs are used to combine the wavelengths into a single optical signal and similarly DMUXs unbundled the signal into individual wavelengths.

In this study, the traffic originates and terminates at the nodes. There are also equipment storage locations called *huts* along the fiber links and between the nodes. Amplifiers and equipment required for O/E/O conversion can be placed at these intermediate locations. Fiber between two locations (can be huts or nodes) with amplification at both ends is called a *span*. The maximum length of a span that can be traversed between two amplifiers is referred to as the *optical reach*. With a selected optical reach, there is a limit on the number of spans that can be traversed before an O/E/O conversion is required. These maximum number of spans (*max spans*) and the corresponding optical reach values used in this investigation are provided in Table 1.

Table 1 About Here

The maximum transmission distance for an optical fiber is also limited by dispersion. *Signal dispersion* is the spreading of optical light pulses as they travel along the fiber and vary depending on the characteristics and quality of the fiber. In this study, the polarization mode dispersion (PMD) value associated with each fiber link is used to determine the maximum transmission distance allowed prior to regeneration. The basic PMD restrictions are discussed in Birkan et al. (2006). Using these definitions, the *DWDM design problem* addressed in this investigation may be stated as follows:

> *Given dark fiber, a traffic demand matrix, and hut locations, the DWDM design problem consists of selection of working and protection paths, determining the size and location of equipment needed to light the fiber on these paths, while satisfying the traffic demand requirement and technological restrictions.*

In our first investigation, Birkan et al. (2006), integer programming models were developed for the basic design problem. These models provided routing and equipment needed for working paths, but provided no protection. A single fiber cut or an equipment failure can have a severe impact on the working traffic between several nodes. In a network with no protection, recovery from a failure may take several hours.

Our second investigation, Birkan et al. (2005), our models were expanded to include dedicated protection. That is, every working path has at least one dedicated backup path that can be used when failures occur on the working paths. This strategy is simple to implement and provides high reliability. Unfortunately, it is also the most expensive protection strategy since more than 50% of the hardware is idle most of the time.

The current investigation extends our successful models and algorithms to incorporate the more cost effective shared protection schemes. That is, backup equipment can be used to protect multiple demand pairs. Shared protection schemes come in two varieties: *link-based* and *path-based*. An extensive discussion of both may be found in Kennington et al. (2005). In link-based shared protection, detection of equipment failure or a fiber cut on a link results in traffic being rerouted around the failed link using a predetermined backup route. In path-based shared protection, every working path for each origin-destination pair has a node-disjoint backup path. For both shared protection methods, the fiber and equipment on the backup paths can be shared among several demand pairs. Shared protection schemes provide cost efficient means of protecting the working traffic by allowing the sharing of the backup capacity among several demand pairs.

In addition to shared protection, this investigation also addresses the issue of network reliability. Reliability of telecommunication networks is expressed in terms of unavailability values in service contracts. Unavailability is the probability of finding a system (such as a system composed of fiber and equipment that carries several wavelengths of traffic between two nodes) in a non-functional state at any given time. Since there may be numerous possible failure events, calculation of unavailability for complex networks with various equipment types can be very time consuming. In this study, practical approximation methods are introduced to determine unavailability for any given origin-destination pair in a DWDM transport network.

1.1 Survey of Literature

Several recent papers published in the area of DWDM technology focus on shared protection schemes and availability. Path-based and link-based shared protection design schemes for DWDM networks are discussed in detail in Grover (2004). Ho and Mouftah (2002) introduce the short leap shared protection scheme for DWDM mesh networks which is derived from the traditional link-based and path-based protection methods. Fiber links with heavy loads, high failure rates, and/or high average repair times are referred to as critical spans in Asthana et al. (2004). The authors propose and test an algorithm to protect these links with a minimum number of p-cycles. Modiano and Narula-Tam (2002) propose an ILP and a heuristic formulation for the NP-complete survivable routing problem. They compare their algorithms to various other methods such as shortest path routing and greedy routing algorithms.

Investigation by Yang et al. (2004) shows that for path-based shared protection the wavelengths and the resources allocated for O/E/O conversions in the backup paths can be shared by several demand pairs. They used greedy heuristic algorithms and integer programming to increase the sharing of expensive O/E/O conversions by (o,d) demand pairs. Ou et al. (2004) showed that adding a new demand pair with working and shared backup paths to an existing network is an NP-complete problem. Their approach to the problem of dynamic provisioning of DWDM networks with path-based shared protection uses a two-step optimization process. In the first stage, a heuristic algorithm finds two link disjoint paths for a new (o,d) demand pair within the current network. In the second stage, another algorithm tries to increase the sharing of resources for the new design.

In our investigation, system reliability is measured by availability, the amount of time the system is in an operational state during a specific time period. Widely accepted methods to determine the availability of systems are discussed in Lewis (1987). To and Neusy (1994) employed these methods to assess the availability of simple link structures using series and parallel relationships for different protection schemes. Clouqueur and Grover (2002) outlined the assumptions that confirmed that these techniques are suitable for DWDM networks. Their investigation also depicts the advantages of mesh networks with span restoration over ring architectures when there are two link failures in a system. Connection between two demand pairs may involve several fiber optic links and different equipment types. A system decomposition is usually required to identify the relationships between these elements. Schupke (2005) proposed the inclusion of availability constraints in optimization-based network design algorithms to meet service guarantees.

In addition to the availability performance comparisons of various protection schemes, Arci et al. (2003) proposed methods to approximate availabilities for networks employing shared protection schemes. As the number of connections sharing restoration resources increase, the complexity of the availability equations increases. By excluding certain multiple failure scenarios where the connection in question is in an operational state, the availability analysis can be simplified. Impact of this simplification on the accuracy of the calculations is shown to be insignificant using realistic examples. These approximation techniques are adopted in this study as well as in Tornatore et al. (2005). Tornatore introduced heuristic optimization design algorithms for both dedicated and shared protection schemes that minimize transport network provisioning costs.

Fumagalli et al. (2002) used a heuristic algorithm based on simulated annealing to provision DWDM networks with path-based shared protection. The initial designs were fully fault-tolerant against single failure events. If a lower reliability degree was required, the algorithm searched for a lower cost configuration that achieved the requested reliability value. Mello et al. (2005a) approximated the connection availabilities for networks employing path-based shared protection schemes using Markov chains. To simplify the calculations the authors allowed at most two fiber link failures at any given time. The accuracy of this approach was verified by a simulation study. In Mello et al. (2005b) these approximation techniques were used along with a heuristic algorithm to provision DWDM networks with path-based shared protection. Zhang et al. (2003) proposed a method to provision the connection between demand pairs based on service guarantees defined in the service contract. They used integer programming based models for small network problems and several heuristic models for more complex problems. Challita et al. (2005) performed simulation studies on DWDM networks with no protection and dedicated protection to test how the reliability of components influence routing decisions.

1.2 Contributions

The first contribution of this investigation is a set of mathematical models and solution algorithms for the various versions of the DWDM design problem. The four versions considered in this investigation include (i) link-based protection and single path routing, (ii) path-based protection and single path routing, (iii) link-based protection and multi-path routing, and (iv) path-based protection and multi-path routing. Models and efficient heuristic algorithms are presented for all four cases.

The second contribution is a practical procedure for estimating the unavailability of the traffic between demand pairs for both shared protection schemes. Design models that can estimate the unavailability for each demand pair were then used to compare the reliability of path-based and link-based shared protection schemes.

The third contribution is an empirical analysis of the proposed procedures. Using realistic sized test networks, the empirical evidence demonstrates that the proposed procedures provide low-cost designs in a reasonable amount of computational time.

All design tools and test data are also available on the World Wide Web for downloading for verification and comparison purposes. The URL is http://engr.smu.edu/~jlk/publications.htm .

2. Link-Based Shared Protection

Link-based protection schemes were originally designed for the Automatic Protection Switching feature of ring based networks but can also be applied to mesh-based networks to increase availability (see Asthana et al. (2004), Grover (2004)). A network that employs link-based protection has the ability to detect failures on a working link and attempts to recover the traffic by switching to a standby path called a *backup* (*protection*) path. In this protection scheme, every link that carries working traffic in the network has a back-up path. The total protection capacity assigned to the links on these back-up paths should be greater than or equal to any working traffic that is assigned to the links that they are protecting. In other words, protection capacity on the links can be shared among multiple back-up paths, thus translating into savings in the provisioning costs for the service providers compared to other non-shared protection schemes such as dedicated protection. As illustrated in Figure 1, if link w1 carrying X units of traffic fails, the interrupted services can be restored by using the back-up capacity on links p1, p2, and p3. A failure on link w2 carrying Y units of traffic can also be restored by the back-up capacity on links p3, p4, and p5. In this example, back-up capacity on link p3 is shared among two back-up paths and the capacity of p3 should be at least $max(X, Y)$.

Figure 1 About Here

Link-based protection schemes can be successfully applied to DWDM optical networks that are provisioned on a per link basis, called *opaque* networks where an O/E/O conversion takes place at every terminal node. Link-based shared protection schemes are more difficult for all-optical networks where the network is designed on a per path basis with an objective of eliminating the number of expensive O/E/O conversions. All-optical network's intolerance to changes in working paths may lead to redundant or additional equipment and mitigate the cost savings that can be realized by this shared protection scheme. In the following sections, four different optimization-based models will be introduced for the link-based shared protection problem. The reason behind multiple models is to give the user a variety of options to choose from depending on their priorities such as cost vs. processing time or multiple paths per (o,d) demand pair vs. a single path per (o,d) demand pair. All models presented in this paper are based on an arc-path formulation.

The design process involves four stages. The first stage determines sets of working and protection paths. The working paths are for each (*o*,*d*) demand pair and the back-up paths are for each link. This is accomplished using a sequence of binary linear programs as described in Birkan et al. (2006). The second stage involves application of the hut selection algorithm for each (link, optical reach) combination as described in Birkan et al. (2006). This hut selection algorithm yields the minimum number of amplifier and regenerator locations for a given (link, optical reach) combination. Note that the exact size and type of amplifiers and regenerators can only be determined by knowing the traffic on the links. Thus, at this stage of the design process only the huts that will accommodate the equipment are known. The third stage eliminates redundant optical reach values for each link. The rules for elimination are:

1. If an optical reach value for a link requires the same number of huts for amplifier and regenerator placement on a link as another optical reach value then eliminate the smaller optical reach value.
2. If an optical reach value for a link requires a fewer number of huts for amplifier and regenerator placement on a link than another optical reach value then eliminate the one that requires more huts.

In the last stage of the design process a routing and provisioning optimization model is solved. Four different optimization models have been evaluated in this final stage.

2.1 Global Model for Link-Based Shared Protection

The global model may be described as follows: given a network topology and a set of demands, determine a minimum cost DWDM design with link-based shared protection.

All the other models described in the following sections are variations of the global model.

Let *N* denote the set of nodes and *F* denote the set of links in the network. Let *D* denote the set of demand pairs (*o*,*d*) and W_{od} denote the candidate working paths that are available for demand pair (*o*,*d*). Let P_{od}^{w} be a binary variable which is 1 if path *w* is selected for demand pair (*o*,*d*); and 0, otherwise. The following constraints force the selection of exactly one working path for each (*o*,*d*) demand pair:

$$\sum_{w \in W_{od}} P_{od}^{w} = 1, \qquad \forall (o,d) \in D \tag{1}$$

Let B_{ij} denote the candidate back-up paths that are available for link (*i*,*j*) and let $\overline{P}_q$ be a binary variable which is 1 if back-up path *q* is selected for the working link (*i*,*j*); and 0, otherwise. The following constraints force the selection of exactly one back-up path for each link (*i*,*j*):

$$\sum_{q \in B_{ij}} \overline{P}_q = 1, \qquad \forall (i,j) \in F \tag{2}$$

Let Y_{ij}^{b} be a binary variable which is 1 if optical reach *b* is selected for link (*i*,*j*); and 0, otherwise. Let $\overline{R}$ denote the set of optical reach values listed in Table 1. The following constraints force the selection of exactly one optical reach value for each link:

$$\sum_{b \in \overline{R}} Y_{ij}^{b} = 1, \qquad \forall (i,j) \in F \tag{3}$$

Let t_{ij}^{b} ($\overline{t}_{ij}^{b}$) denote the total working (protection) traffic on link (*i*,*j*) using optical reach *b*. Let V_{ij} ($\overline{V}_{ij}$) denote the maximum possible working (protection) traffic on link (*i*,*j*). The candidate path and demand data can be used to determine these constants.

Constraints (4) and (5) force the working and protection traffic on link (*i*,*j*) to be 0 except for one optical reach value that is selected for that link.

$$t_{ij}^{b} \le V_{ij} Y_{ij}^{b}, \qquad \forall (i,j) \in F, \forall b \in \bar{R} \tag{4}$$

$$\bar{t}_{ij}^{b} \le \bar{V}_{ij} Y_{ij}^{b}, \qquad \forall (i,j) \in F, \forall b \in \bar{R} \tag{5}$$

Let $\tilde{W}_{ij}$ denote the working paths that use arc (*i*,*j*) and r_{od} denote the total demand in wavelengths for demand pair (*o*,*d*). The following set of constraints accumulates the working traffic on the links for a given optical reach value:

$$\sum_{b \in \bar{R}} t_{ij}^{b} = \sum_{(o,d) \in D} \sum_{w \in \tilde{W}_{ij} \cap W_{od}} r_{od} P_{od}^{w} + \sum_{(o,d) \in D} \sum_{w \in \tilde{W}_{ji} \cap W_{od}} r_{od} P_{od}^{w}, \qquad \forall (i,j) \in F \tag{6}$$

Let x_q denote the protection traffic on back-up path q and $\bar{B} = \bigcup_{(i,j) \in F} B_{ij}$ denotes the set of all back-up paths. The following set of constraints enable protection flow on path q only if back-up path q is selected for link (*i*,*j*):

$$x_q \le \Big(\sum_{(i,j) \in F \ni q \in B_{ij}} V_{ij} \Big) \bar{P}_q, \qquad \forall q \in \bar{B} \tag{7}$$

The following set of constraints force the protection traffic to meet the demand on the links:

$$\sum_{q \in B_{ij}} x_q = \sum_{b \in \bar{R}} t_{ij}^{b}, \qquad \forall (i,j) \in F \tag{8}$$

Let $\tilde{B}_{ij}$ denote the back-up paths that use arc (*i*,*j*). The following set of constraints converts the path flows into links flows for the protection traffic:

$$\sum_{q \in \tilde{B}_{ij} \cap B_{kl}} x_q \le \sum_{b \in \bar{R}} \bar{t}_{ij}^{b}, \qquad \forall (i,j) \in F, \forall (k,l) \in F \tag{9}$$

$$\sum_{q \in \tilde{B}_{ji} \cap B_{kl}} x_q \le \sum_{b \in \bar{R}} \bar{t}_{ij}^{b}, \qquad \forall (i,j) \in F, \forall (k,l) \in F \tag{10}$$

Let A_{ij}^{Sb}, A_{ij}^{Mb}, and A_{ij}^{Lb} be the integer variables that denote the number of small, medium and large amplifiers needed for a selected optical reach of *b* at each operational hut on link (*i*,*j*), respectively. Let *S*, *M*, and *L* be constants representing the number of wavelengths that can be processed by a small, medium, and large amplifier, respectively. If an optical reach for a given link imposes amplification at a hut on that link then the following set of constraints determine the size and number of amplifiers needed at that hut:

$$t_{ij}^{b} + \bar{t}_{ij}^{b} \le S\ A_{ij}^{Sb} + M\ A_{ij}^{Mb} + L\ A_{ij}^{Lb}, \qquad \forall (i,j) \in F, \forall b \in \bar{R} \tag{11}$$

Note that (11) also determines the number and size of MUX/DMUX equipment at the huts or nodes selected by the hut selection algorithm. The following set of constraints determines the required number of TEs at the nodes:

$$\sum_{b \in \bar{R}} \sum_{(i,j) \in F} (t_{ij}^{b} + \bar{t}_{ij}^{b}) = T_i, \qquad \forall i \in N \tag{12}$$

The constants O_{ij}^{Ab}, O_{ij}^{Rb}, and O_{ij}^{Mb} denote the number of amplifier, regenerator, and multiplexer/demultiplexer locations on link (*i*,*j*) if an optical reach of *b* is used, respectively. Then the objective function is to minimize the cost of provisioning the DWDM network and is given by

$$\begin{aligned} \text{minimize } & \underbrace{\sum_{(i,j)\in F} \sum_{b \in \bar{R}} (O_{ij}^{Ab}(C_{A^S} A_{ij}^{Sb} + C_{A^M} A_{ij}^{Mb} + C_{A^L} A_{ij}^{Lb}))}_{\text{Amplifiers}} + \underbrace{C_T \sum_{i \in N} T_i}_{\text{TEs}} \\ & \underbrace{+ C_R \sum_{(i,j)\in F} \sum_{b \in \bar{R}} (O_{ij}^{Rb}(t_{ij}^{b} + \bar{t}_{ij}^{b}))}_{\text{Regenerators}} + \underbrace{\sum_{(i,j)\in F} \sum_{b \in \bar{R}} (O_{ij}^{Mb}(C_{M^S} A_{ij}^{Sb} + C_{M^M} A_{ij}^{Mb} + C_{M^L} A_{ij}^{Lb}))}_{\text{MUX/DMUX}} \end{aligned} \tag{13}$$

where C_{A^S}, C_{A^M}, C_{A^L} are constants denoting the cost of small, medium, and large amplifiers, C_{M^S}, C_{M^M}, C_{M^L} are constants denoting the cost of small, medium, and large

MUX/DMUX equipment, C_T denotes the cost of TE equipment, and finally C_R denotes the cost of an R. A solution of the model (1) – (13) provides the network designer with demand routings and equipment needed to satisfy the demands.

2.2 Relaxed Model for Link-Based Shared Protection

In order to reduce the complexity and running times of the global model, numerous strategies are considered. The relaxed model addresses this issue by solving the global model without the integrality requirement on the integer variables A_{ij}^{Sb}, A_{ij}^{Mb}, and A_{ij}^{Lb}. A post-optimization routine then converts the continuous values into actual amplifier requirements and also determines the cost. For example 2.25 large amplifiers are converted into two large and one small amplifiers. The relative costs for TE, A, R, and MUX/DMUX equipment in all appropriate sizes are listed in Table 2. The equipment costs are not the actual prices, but are selected so that the ratios are approximately correct. Since the large amplifiers have the lowest cost per wavelength ($2.5 per wavelength for the large amplifiers vs. $3.75 for the medium and $5 for the small), the relaxed model will use only the large amplifiers. The relaxed model coupled with the post-optimization rounding routine guaranties a feasible design. The only question is the possibility of additional cost compared to the global model.

Table 2 About Here

2.3 Path Reduction Heuristic for Link-Based Shared Protection

The routing and provisioning optimization in the global model considers $\sum_{(o,d)\in D} |W_{od}|$ working paths and $|\bar{B}|$ back-up paths. These paths also have multiple possible equipment configurations using different optical reach values. For the path reduction heuristic described in this section, we use a two-stage strategy that permits the use of multiple paths for both working and back-up traffic in the first stage. The second stage requires single paths for both working and protection; however, only paths selected in the first stage appear in the second stage model.

In the first stage, a modified version of the global model is run. Let $0 \le X_{od}^{w} \le 1$ be a variable that is greater than 0 if path *w* carries a portion of the demand between nodes *o* and *d* and $0 \le \bar{X}_q \le 1$ is a variable which is greater than 0 if back-up path q carriers a portion of the protection traffic. Instances of the two binary variables, P_{od}^{w} and $\bar{P}_q$, in the model are replaced by $P_{od}^{w} + X_{od}^{w}$ and $\bar{P}_q + \bar{X}_q$, respectively. Variables t_{ij}^{b} and $\bar{t}_{ij}^{b}$ should be set to integer so that the traffic on links can be easily converted into wavelengths for equipment requirements. Note that this step wasn't necessary for the global model since all the demand for a given (*o,d*) demand pair was serviced by a single path. Running this modified model may result in a solution with multiple working paths per (*o,d*) demand pair and multiple back-up paths for each link in the network. The second phase of the routing and provisioning model uses only the working and back-up paths selected in the first phase. In order to obtain a solution with a single path per (*o,d*) demand pair and a single back-up path for each link, the variables X_{od}^{w} and $\bar{X}_q$ are fixed to 0 in this stage.

As with the global model the output includes demand routings, protection routings, and equipment requirements.

2.4 Multiple-Path Model for Link-Based Shared Protection

The design costs can be reduced if multiple paths for both working and protection are permitted as defined in stage one above. The reason for lower costs is mainly due to the fact that network load can be more efficiently balanced across the multiple paths in a network with hardware that comes in modular sizes (such as 20, 40, or 80 wavelengths). Having multiple paths also has its disadvantages. If packets from the same connection are multiplexed across all the available paths, packets may arrive out of order since the multiple paths may not have exactly the same propagation delay (because of differences in length and/or number of intermediate routers and equipment traversed). Depending on what kind of protocol is used, these out of sequence data packets may lead to unnecessary additional re-transmission. Multiple paths can be achieved by relaxing the binary variables P_{od}^{w} and $\overline{P}_{q}$ and setting the variables t_{ij}^{b} and $\overline{t}_{ij}^{b}$ to positive integers as described in the first stage of the previous section. The output from the multiple-path model includes routings and equipment requirements on the selected paths.

3. Empirical Analysis of Link-Based Shared Protection

Three different test network topologies are used to evaluate the DWDM design models with link-based shared protection described in Section 2. The details about these test networks are provided in Table 3. The layout of these test networks can found in Birkan et al. (2006). All link-based shared protection models are implemented using the AMPL modeling language (Fourer et al. 2003) and are solved using the branch-and-bound optimizer in CPLEX http://www.cplex.com. All test runs are made on a Compaq AlphaServer DS20E with dual EV 6.7(21264A) 667 MHz processors and 4096 MB of RAM. Design costs are calculated using the relative equipment prices in Table 2.

Table 3 About Here

For each test network, 4 problems each with a different number of demand pairs are run with a maximum of 6 candidate working paths per (*o*,*d*) demand pair and a maximum of 6 candidate back-up paths per link. For the test problems, all the (*o*,*d*) demand pairs are randomly generated and the wavelengths assigned to these demand pairs are uniformly distributed over [10,80]. The number of demand pairs and the average total demand for these test problems can be found in Table 4. All the test problems are run with a CPLEX solver time limit of one hour and an optimality gap of 5%.

Table 4 About Here

The detailed results of the test problem runs for the global model are presented in Table 5. For the test problems using EU and US networks, the processing times required were at most 18.5 minutes and the reported design costs were at most 5% more than the lower limit reported by CPLEX. For example, the US480 problem with 250 (*o*,*d*) demand pairs required 7601 As, 9502 Rs, and 185,242 TEs. The total processing time required for the CPLEX solver was 6 minutes and 22 seconds. The total global design process required 8 minutes and 44 seconds. The cost of the design was $17,260,000 which was about 4.3% higher than $16,510,000, the lower limit reported by CPLEX. Test problems using the NA test network proved to be more challenging for the global model. All the test problems using the NA test network required more than the solver time limit of one hour. For example, the NA660 problem with 250 demand pairs required one hour of CPLEX time to determine a design of 16.07 million dollars and the reported optimality gap was about 8%.

Table 5 About Here

Table 6 presents the results of the relaxed model runs using the same test problems. For NA570 and NA610, both models were terminated after one hour of CPU time. For all other problem instances, the relaxed model required less time than the global model. The test problems using the EU and US test networks never required more than 4 minutes and 7 seconds. This is a nice improvement over the global model where some instances required up to 18.5 minutes for these test networks. For example the time required for NA680 was nine minutes for the relaxed model versus over 2 hours and 46 minutes for the global model with a time limit of twelve hours. Moreover, for this problem instance, the relaxed model was able to determine a lower cost design with a smaller optimality gap than the global model.

Table 6 About Here

The detailed results of the test problem runs for the path reduction heuristic may be found in Table 7. With this heuristic the running times never exceeded four minutes and the optimality gaps were within 10.2%. The designs for the EU test network took less than a minute. As illustrated in Figure 2, the average difference in optimality gaps for the global model and path reduction heuristic never exceeded 1.5%. On average, the relaxed model provided better solutions (lower cost) for the US and NA test networks than the other models. Figure 3 illustrates a comparison of the average processing times for the global, relaxed, and path reduction models. In terms of running times the global model required the most time and the path reduction heuristic proved to be the fastest of the three. For the same network, the average processing time for the heuristic was around 3 minutes. Overall, the path reduction heuristic achieved a significant decrease in running times over the global model at the price of a small increase in design costs. The relaxed model was superior to the global model both in terms of average costs and running times for the NA and US test networks. Table 8 presents the costs and the running times for the NA test cases with 200 and 250 demand pairs with twelve hours time limit. One instance of the global model used close to ten hours and another used the whole 12 hours to determine a solution within 5% of optimality.

Tables 7 & 8 and Figures 2 & 3 About Here

For the same 36 test problems, the results of the multiple-path model runs are given in Table 9. Multiple paths allowed significant reductions in equipment requirements and as expected it provided designs that are less expensive than the single path models. For all the test problem runs, the multiple-path model never required more than 2.5 minutes of processing time.

Table 9 About Here
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4. Path-Based Shared Protection

In a path-based shared protection scheme, every working light path has at least one node-disjoint backup path. In the event of a failure on the working path, traffic is rerouted over one or more node disjoint backup paths. The distinction between path-based and link-based schemes is that the path-based uses alternative routes from the origin to the destination of the demand pairs rather than simply taking an alternate route around the failed link. As a result, the path-based protection scheme protects against intermediate node failures where as link-based protection methods are vulnerable to node failures.

As with the link-based method, resource sharing takes place for the backup paths and the equipment allocated for restoration can be used by more than one protection path. An exception to this rule happens when different working paths use a common link and also the backup paths assigned to these working paths use common links. In this case, sharing is not allowed between these backup paths since a failure of a common working link requires the sum of all traffic demand to be routed on the common backup link. The example in Figure 4 illustrates this case. In this example there are 3 (*o,d*) demand pairs, d1 uses link (1,3), d2 uses links (2,1) and (1,3), and finally d3 uses link (1,2). As illustrated in the Figure 4, d1 and d2 not only use common working links, they also share backup links (2,4) and (4,3). Thus these two demand pairs cannot share resources on the backup paths. Demand pairs d2 and d3 also cannot share backup resources for the same reason. Even though demand pairs d1 and d3 have some common links on their backup paths, they can share resources because they do not have any common links on their working paths.

Figure 4 About Here

The DWDM design problem statement in Section 2 also applies to the path-based shared protection case and the design process involves the same four stages described in the link-based shared protection section. However, the binary linear program used for determining candidate paths in the first step and the routing and provisioning optimization model are replaced with the ones described below.

The candidate paths are determined by solving a sequence of binary linear programs that finds k-shortest distinct cycles for each (*o,d*) demand pair. Each one of these cycles for a given (*o,d*) demand pair can be split into two node disjoint paths that originate at node *o* and terminate at node *d*. The shorter of these two paths is then selected as the working path and the other path becomes the backup path. For more about this path generator model see Birkan et al. 2005. In the last stage of the design process a routing and provisioning optimization model is solved. Two different optimization models are proposed for this final stage.

4.1 Global Model for Path-Based Shared Protection

The global model may be described as follows: given a network topology and a set of demands, determine a minimum cost DWDM design with path-based shared protection. As with the global model for link-based shared protection, this model uses a single path for the working traffic and a single path for the protection traffic for each (*o,d*) demand pair.

Let Z_{od} denote the candidate cycles (working and backup path pairs) that are available for demand pair (*o*,*d*). Let H_z be a binary variable which is 1 if cycle z is selected for demand pair (*o*,*d*); and 0, otherwise. The following constraints force the selection of exactly one cycle for each (*o*,*d*) demand pair:

$$\sum_{z \in Z_{od}} H_z = 1, \qquad \forall (o,d) \in D \tag{14}$$

Let $\hat{W}_z$ denote the set of arcs in the working path of cycle z. The following set of constraints accumulates the working traffic on the links for a given optical reach value:

$$\sum_{b \in \bar{R}} t_{ij}^{b} = \sum_{(o,d) \in D} \sum_{z \in Z_{od} \ni (i,j) \in \hat{W}_z} r_{od} H_z + \sum_{(o,d) \in D} \sum_{z \in Z_{od} \ni (j,i) \in \hat{W}_z} r_{od} H_z, \qquad \forall (i,j) \in F \tag{15}$$

Let $\overline{x}_{ij}^{z}$ denote the protection traffic on the backup path of cycle z and let E denote the set of arcs in the network. That is, $E = \{(i,j),(j,i) : (i,j) \in F\}$. The following set of constraints converts the path flows into links flows for the protection traffic:

$$\sum_{z \in \{\tilde{W}_{ij} \cup \tilde{W}_{ji}\} \cap \tilde{B}_{ij}} \overline{x}_{ij}^{z} + \sum_{z \in \{\tilde{W}_{ij} \cup \tilde{W}_{ji}\} \cap \tilde{B}_{ji}} \overline{x}_{ji}^{z} \le \sum_{b \in \bar{R}} \overline{t}_{ij}^{b}, \qquad \forall (i,j) \in F, \forall (k,l) \in F \setminus \{(i,j)\} \tag{16}$$

The following set of constraints force the protection traffic to meet the demand on the links:

$$\overline{x}_{ij}^{z} = \sum_{(o,d) \in D \ni z \in Z_{od}} r_{od} H_z, \qquad \forall (i,j) \in E, \forall z \in \tilde{B}_{ij} \tag{17}$$

The path-based global model is (3)-(5) and (11)-(17).

4.2 Multiple-Path Model for Path-Based Shared Protection

A multi-path path-based shared protection model allows a point-to-point demand to be routed on multiple cycles. This model can be constructed by relaxing the binary variable

H^z thus allowing more than one (working, backup) path pair for each (*o*,*d*) demand pair. Also, in order to convert the traffic into wavelength requirements, variables t_{ij}^b and $\overline{t}_{ij}^b$ should be integral.

5. Empirical Analysis of Path-Based Shared Protection

The test network topologies used in Section 3 are used to evaluate the DWDM design models with path-based shared protection. The same test problems described in the empirical analysis of the link-based protection models are also employed here to evaluate the single and multiple path models. All the test problems are run with a CPLEX solver time limit of 1 hour and an optimality gap of 5%.

The test problem runs for the global model were made with both 6 and 12 candidate cycles per (*o,d*) demand pair. The detailed results of these runs are presented in Tables 10 and 11. The EU test network problems with 6 candidate cycles per demand pair never required more than 2 minutes. For example the EU280 test problem with 125 demand pairs required 1 minute and 57 seconds and the cost of this design was $4,440,000. Increasing the candidate cycles to 12 lowered the design cost to $4,330,000 and increased the CPU time to 6 minutes. Problem NA670 with 250 demand pairs and 12 cycles was the only case where a feasible integer solution wasn’t available after 1 hour of processing time. The same problem required about 7 minutes with 6 candidate cycles and the solution was within 0.5% of the lower bound determined by the CPLEX solver.

Tables 10 and 11 About Here

The results of the test runs with the multi-path model are presented in Table 12. This model is run with 12 candidate cycles per (*o*,*d*) demand pair and achieved the least cost designs. The running times were comparable to the single-path global model with 6 candidate cycles. The impact of number of candidate cycles on cost and running times are illustrated in Figures 5 and 6. The 6 candidate cycle runs required the least amount of processing time. As the number of cycles increased from 6 to 12 the running times increased considerably especially for the global model. However, the gains in terms of lower costs weren't significant with these 12 cycle runs.

Table 12 and Figures 5 and 6 About Here

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6. Unavailability Analysis of Shared Protection Schemes

While the majority of DWDM equipment has relatively high reliabilities and built-in redundancy, DWDM networks are not free from service disruptions. According to the FCC, a 1000 mile fiber experiences an average of 3 cuts per year (see Grover 2004). Average repair time needed for fiber link repair (or replacement) is generally in the range of 12 to 14 hours (see To and Neusy 1994, El-Torky and Lafleur 2000, and Grover 2004).

Unavailability (*U*) is the probability of finding a restorable system in a non-functional state at any given time. Unavailability can also be expressed in units of minutes per year. For example an unavailability of 2 hours/year is 2/[(24)(365)] = 0.02283%. The failure rate (*r*) and the mean time to repair (*MTTR*) for a given component are required for the calculation of unavailability for that component. Mean time to failure (*MTTF*) can also be used in place of 1/r. Assuming failures occur independently of each other and the failure rates remain constant throughout a component's life, the unavailability of a single DWDM network component can be determined by the following equation (see Lewis 1987, To and Neusy 1994, Clouqueur and Grover 2002).

$$U = \frac{Downtime}{Uptime + Downtime} = \frac{MTTR}{MTTF + MTTR} = \frac{(r)(MTTR)}{1 + (r)(MTTR)} \tag{18}$$

$$\text{if } MTTF \;>> MTTR, \text{ then } U \approx (r)(MTTR) \tag{19}$$

Let A' denote *availability*, the complement of unavailability, then $A' = 1 - U$. Unavailability of a system with *K* components working in series (such as a link with Rs, As, and MUX/DMUXs or multiple links with various equipment forming a path) with unavailabilities U_k, $k=1, \ldots, K$ can be calculated using

$$U_{system} = 1 - \prod_{k=1}^{k=K} A'_k = 1 - \prod_{k=1}^{k=K} (1 - U_k) \tag{20}$$

Unavailability of a system in series can also be approximated by $U_{system} \approx \sum_{k=1}^{k=K} U_k$. The accuracy of this approximation is fairly high in particular with DWDM equipment where the unavailability of the equipment is low (see Grover 2004). If a system is composed of *K* redundant components where at least one of the units must be operational in order for the system to work, then the unavailability for the system is given by

$$U_{system} = \prod_{k=1}^{k=K} U_k \tag{21}$$

The failure rates and MTTR values for the DWDM equipment types considered in this study can be found in Table 13. Each link in the network is composed of DWDM equipment in various quantities working in a series configuration. Let *k* ,*l*, *m*, and *n* represent the number of As, Rs, TEs, and MUX/DMUXs, respectively on link (*i*,*j*) and let *d* denote the length of the fiber for link (*i*,*j*). The availability of link (*i*,*j*) can be calculated using:

$$A'_{link(i,j)} = \left(A'_{Amplifier}\right)^k \left(A'_{Regenerator}\right)^l \left(A'_{TE}\right)^m \left(A'_{MUX/DMUX}\right)^n \left(A'_{Fiber}\right)^d$$

Table 13 About Here

The unavailability calculation for an (*o*,*d*) demand pair with shared protection requires determining all the events (or scenarios) under which the (*o*,*d*) demand pair in question is in an operational state (see Arci et al. 2003). The example in Figure 7 illustrates a case where two node-disjoint working paths for two different demand pairs share resources on a backup link. The first (*o*,*d*) demand pair (1,2) uses link (1,2) for the working path and uses links p1, p2, and p3 for the backup path. The second (*o*,*d*) demand pair (5,6) uses link (5,6) for the working path and uses links p2, p4, and p5 for the backup path. The capacity of link p2 is sufficient to accommodate the failure of either working path, but not sufficient to accommodate the failure of both working paths. The list of events where demand pair (1,2) is in an operational state are shown in Table 14, where F represents a failed state and W represents a working state.

Figure 7 and Table 14 About Here

Since the events are mutually exclusive, the probability of finding the demand pair in an operational state is the sum of the individual event probabilities. Thus, the availability of demand pair (1,2) can be calculated using the following equation:

$$\begin{aligned} A'_{12} = {} & A'_{w1} + (1 - A'_{w1})\, A'_{w2} A'_{p1} A'_{p2} A'_{p3} + (1 - A'_{w1})\, (1 - A'_{w2})\, (1 - A'_{p4})\, A'_{p1} A'_{p2} A'_{p3} + \\ & (1 - A'_{w1})\, (1 - A'_{w2})\, (1 - A'_{p5})\, A'_{p1} A'_{p2} A'_{p3} A'_{p4} \end{aligned} \tag{22}$$

As discussed in Arci et al. (2003) considering all the failure events for more complex networks such as the transport network examples used in this study makes the problem intractable. The suggested method in their study approximates availabilities by only considering the events with single link failures which basically underestimates the demand pair availabilities. Using simulation, it is also demonstrated that these approximations converge to the actual values for DWDM networks with highly available components. Using this conservative approach, the availability of demand pair (1,2) can be written as follows:

$$A'_{12} \approx A'_{w1} + (1 - A'_{w1})\, A'_{w2} A'_{p1} A'_{p2} A'_{p3} \tag{23}$$

Also note that in order for the demand to be satisfied between nodes 1 and 2, the OXCs at the demand nodes must be operational at all times. When the traffic is switched to the backup path the OXCs at nodes 3 and 4 must also be in a working state. Let A'_{O_W} be the availability of the OXCs at the demand nodes and A'_{O_b} be the availability of the OXCs at the backup nodes for the demand pair (1,2). Taking into account the OXCs, the availability equation in (23) can be rewritten as follows:

$$A'_{12} \approx A'_{w1} A'_{O_W} + (1 - A'_{w1})\, A'_{w2} A'_{p1} A'_{p2} A'_{p3} A'_{O_W} A'_{O_B} \tag{24}$$

An updated version of Table 14 with possible node failure events are shown in Table 15. Note that this table summarizes all 1055 possible availability events for demand pair (1,2). For example, the second row represents 16 different failure events.

Table 15 About Here

Assuming each working path carries 40 wavelengths of traffic and each link consists of 1000 km of fiber, 11 amplifiers, 80 TEs, a MUX, and a DMUX, the unavailability estimate of demand pair (1,2) is 25.35 minutes. The details of this calculation for the realistic example illustrated in Figure 7 can be found in Table 16. The network in Figure 8 illustrates another case where each working path shares resources on their backup paths with two other demand pairs. Consider demand pair (7,8). This demand pair uses the link (7,8) for the working path and uses links p2, p5, p6, and p7 for the backup path. Resources (fiber and equipment) on links p2 and p5 are shared with the other demand pairs in the network. In order to determine the exact availability for demand pair (7,8) in Figure 8, a series of network flow problems were solved. These network flow problems tested every possible availability event for demand pair (7,8) on a special network derived from the network in Figure 8. A summary of these events are presented in Table 17. A total of 241 mutually exclusive events were found that allowed the demand between nodes 7 and 8 to be satisfied when link (7,8) fails (rows 2 through 18). The unavailability values determined using the approximate and the exact methods for the demand pairs discussed are listed in Table 18. Only the events in the first two rows of Table 17 are used in the approximate calculation for demand pair (7,8). However, the approximate method provided values very close to the exact values.

Tables 16, 17, 18 and Figure 8 About Here

Unavailability for DWDM networks employing path-based shared protection can be approximated as described above. In (24), the first term in the approximation assumes that the working path for the (*o*,*d*) demand pair in question is fully operational. The second term will cover all the events where this working path has failed and the backup path including all the other working paths that are sharing backup resources with the demand pair under consideration are operational. Determining unavailability for an (*o*,*d*) demand pair in a link-based shared protection setting requires more work since each link in the network has its own backup path. The pseudo-code for this procedure can be found in Figure 9.

Figure 9 About Here

To help evaluate the unavailability of DWDM transport networks with link-based and path-based shared protection schemes, six test cases are used. The minimum, average, and maximum unavailability values for the demand pairs in these test cases for the two protection schemes considered are listed in Table 19. Even though the path-based shared protection scheme protects against intermediate node failures, in almost all cases the average unavailabilities were lower for the link-based shared protection scheme.

There are two reasons for this phenomena. The first reason is related to the increased level of sharing in the path-based scheme. This implies that there are fewer resources available for protection. Hence, certain failures require that a subset of working paths must be operational, since their protection paths have shared resources that are being used to protect the designated failure. In the link-based case, we are only concerned with the availability of the individual links that are sharing backup resources with the demand pair in question. The second reason is the impact of the number of long paths that are sharing backup resources with the demand pair in question. Grover (2004) also warns of the impact of long working and backup paths on the availability of DWDM networks using shared protection schemes. Furthermore, in our analysis a node failure corresponds to the failure of an OXC. Since an OXC is a single element, it has a very high availability compared to a link which is prone to fiber cuts and equipment failures.

Table 19 About Here

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7. Summary and Conclusions

In this investigation, several optimization-based design models capable of provisioning DWDM networks with link-based and path-based shared protection are introduced. All the models described use an arc-path formulation and determine the minimum cost design along with the demand routings and equipment requirements for each configurable location in the network. In order to take advantage of the modularity of the DWDM equipment, some of models allow multiple routes per demand pair. All the models are compared based on running times, equipment requirements, and costs using several realistic test cases.

Unavailability values are widely used by telecommunication service providers to convey the reliability of their networks. The techniques to estimate the unavailability of a demand pair for both shared protection schemes are also examined in detail. Given a design, the unavailability of each demand pair can be approximated using these techniques.

Link-based and path-based shared protection schemes yield the benefit of reduced costs to restorable DWDM networks compared to dedicated protection schemes. Due to its efficiency in distributing backup capacity among multiple demand pairs, the path-based shared protection scheme proved to be the least cost alternative of the two especially for the single-path per (*o*,*d*) demand pair designs. Average design costs for the two shared protection schemes are presented in Table 20. Path-based restoration strategies also have the benefit of being impervious to intermediate node failures. However, link-based protection has slightly superior availability values.

Table 20 About Here

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Table 1. Optical Reach Values

Optical Reach	Maximum Spans	Optical Reach	Maximum Spans	Optical Reach	Maximum Spans
162	1	130	9	114	17
158	2	128	10	112	18
154	3	126	11	110	19
150	4	124	12	108	20
146	5	122	13	106	21
142	6	120	14	104	22
138	7	118	15	102	23
134	8	116	16	100	24

Table 2. Equipment Cost

Equipment	Max. Wavelengths	Cost/Unit
TE	1	75
R	1	130
A	20	100
A	40	150
A	80	200
MUX/DMUX	20	120
MUX/DMUX	40	180
MUX/DMUX	80	240

Table 3. Test Problem Descriptions

Name	Nodes	Links	Average Node Degree	Demand	DPMD	Total Huts	Distance Between Huts
EU	18	35	3.89	[10,80]	[0.1,1.0]	438	[5,90]
US	28	42	3.15	[10,80]	[0.1,1.5]	423	[3,134]
NA	36	67	3.69	[10,80]	[0.1,1.0]	705	[5,100]

Table 4. Test Problems Characteristics

Problems	Number of Demand Pairs	Average Total Demand (λs)
EU110, EU120, EU130	50	2235
EU160, EU170, EU180	75	3339
EU210, EU220, EU230	100	4579
EU260, EU270, EU280	125	5697
US310, US320, US330	100	4379
US360, US370, US380	150	6615
US410, US420, US430	200	8755
US460, US470, US480	250	10926
NA510, NA520, NA530	100	4356
NA560, NA570, NA580	150	6622
NA610, NA620, NA630	200	8926
NA660, NA670, NA680	250	11449

Table 5. Global Model (Time Limit = 1 hour, MIP Gap = 5%)

Problem	As	Rs	TEs	CPLEX Time	Total Time	Cost (000,000)	Gap	Lower Bound
EU110	1247	1446	18320	0:02:17	0:02:37	1.87	4.8%	1.78
EU120	1294	1485	19682	0:18:08	0:18:29	1.99	4.0%	1.91
EU130	1316	1391	19220	0:07:08	0:07:28	1.95	5.1%	1.85
EU160	2128	2270	29004	0:04:18	0:04:48	2.99	5.0%	2.84
EU170	2217	2478	30842	0:03:26	0:03:55	3.18	5.0%	3.02
EU180	2164	2154	30652	0:06:06	0:06:36	3.11	4.8%	2.96
EU210	2802	3510	39944	0:03:20	0:03:58	4.15	5.1%	3.94
EU220	3078	3544	44600	0:02:54	0:03:32	4.56	3.9%	4.38
EU230	2741	3180	40678	0:01:51	0:02:28	4.15	4.8%	3.95
EU260	3474	3979	51184	0:11:05	0:11:56	5.21	5.0%	4.95
EU270	3640	3981	51979	0:04:43	0:05:34	5.31	4.0%	5.10
EU280	3588	4117	52836	0:03:00	0:03:51	5.39	5.0%	5.12
US310	3230	3266	77458	0:01:44	0:02:24	7.13	4.6%	6.80
US320	3200	3210	77386	0:01:13	0:01:52	7.11	5.1%	6.75
US330	3133	3344	76016	0:01:18	0:01:58	7.01	5.0%	6.66
US360	4413	5413	110856	0:01:00	0:02:18	10.27	5.1%	9.75
US370	4697	5537	112936	0:01:40	0:02:49	10.50	4.6%	10.02
US380	4552	7389	109726	0:03:49	0:05:08	10.47	5.0%	9.95
US410	6108	7751	146932	0:03:42	0:05:31	13.73	5.0%	13.05
US420	6162	7639	149566	0:05:14	0:07:03	13.93	5.0%	13.24
US430	5927	9326	147114	0:03:35	0:05:24	13.92	5.0%	13.23
US460	7658	9272	183742	0:04:34	0:06:55	17.12	4.1%	16.42
US470	7740	9498	187558	0:04:03	0:06:24	17.47	4.6%	16.67
US480	7601	9502	185242	0:06:22	0:08:44	17.26	4.3%	16.51
NA510	4122	3668	59748	1:00:00*	1:01:00	5.98	5.7%	5.64
NA520	4875	4724	70280	1:00:00*	1:01:08	7.08	6.8%	6.60
NA530	4230	4334	58478	1:00:00*	1:01:06	5.98	8.9%	5.45
NA560	6837	7100	97412	1:00:00*	1:01:34	9.90	5.9%	9.32
NA570	6287	6319	92994	1:00:00*	1:01:33	9.36	7.4%	8.67
NA580	6589	6873	95570	1:00:00*	1:01:35	9.69	6.2%	9.09
NA610	8921	9254	130632	1:00:00*	1:02:05	13.22	9.8%	11.93
NA620	8252	8396	124244	1:00:00*	1:02:03	12.45	8.4%	11.41
NA630	8737	8897	130628	1:00:00*	1:02:03	13.12	10.4%	11.75
NA660	11000	10608	159578	1:00:00*	1:01:34	16.07	8.2%	14.76
NA670	10804	11299	159092	1:00:00*	1:02:35	16.09	6.5%	15.04
NA680	10742	10941	158764	1:00:00*	1:02:35	16.01	6.6%	14.95

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* Terminated due to time limit of one hour

Table 6. Relaxed Model (Time Limit = 1 hour, MIP Gap = 5%)

Problem	As	Rs	TEs	CPLEX Time	Total Time	Cost (000,000)	Gap	Lower Bound
EU110	1399	1499	18304	0:02:12	0:02:33	1.90	6.3%	1.78
EU120	1445	1445	19834	0:03:04	0:03:24	2.03	5.9%	1.91
EU130	1437	1443	19054	0:01:18	0:01:39	1.96	5.6%	1.85
EU160	2178	2284	28847	0:03:12	0:03:40	2.99	5.0%	2.84
EU170	2328	2405	30792	0:00:43	0:01:12	3.18	5.0%	3.02
EU180	2272	2286	30304	0:01:34	0:02:03	3.12	5.1%	2.96
EU210	2886	3185	40538	0:00:59	0:01:37	4.16	5.3%	3.94
EU220	3184	3547	44680	0:00:51	0:01:28	4.59	4.6%	4.38
EU230	2859	3179	40332	0:01:02	0:01:40	4.14	4.6%	3.95
EU260	3486	4110	49892	0:00:44	0:01:35	5.13	3.5%	4.95
EU270	3736	4153	51188	0:00:53	0:01:34	5.29	3.6%	5.31
EU280	3671	4156	52326	0:00:30	0:01:21	5.37	4.7%	5.12
US310	3227	3266	76550	0:00:24	0:01:04	7.06	3.7%	6.80
US320	3178	3334	76538	0:00:08	0:00:48	7.06	4.4%	6.75
US330	3105	3254	76108	0:00:44	0:01:24	7.00	4.9%	6.66
US360	4652	5415	110972	0:00:45	0:02:04	10.32	5.5%	9.75
US370	4693	5583	113144	0:00:34	0:01:53	10.52	4.8%	10.02
US380	4583	5821	111684	0:01:32	0:02:31	10.41	4.4%	9.95
US410	6039	7587	146184	0:00:29	0:02:07	13.64	4.3%	13.05
US420	6192	7515	149194	0:01:62	0:03:21	13.89	4.7%	13.24
US430	6103	7652	148652	0:01:43	0:03:11	13.85	4.5%	13.23
US460	7651	9228	182642	0:00:23	0:02:44	17.02	3.5%	16.42
US470	7607	9643	186474	0:00:51	0:03:11	17.37	4.0%	16.67
US480	7788	9253	183882	0:01:47	0:04:07	17.15	3.7%	16.51
NA510	4444	3796	59472	0:21:54	0:22:54	6.03	6.5%	5.64
NA520	5051	4527	69968	0:10:39	0:11:39	7.07	6.6%	6.60
NA530	4457	4299	56482	0:40:59	0:42:00	5.86	7.0%	5.45
NA560	6962	6305	97852	0:10:32	0:12:01	9.87	5.6%	9.32
NA570	6665	6406	91960	1:00:00[*]	1:01:32	9.36	7.4%	8.67
NA580	6957	7125	94204	0:54:17	0:55:44	9.69	6.2%	9.09
NA610	9043	9578	124260	1:00:00[*]	1:02:02	12.79	6.7%	11.93
NA620	8409	7797	120476	0:15:14	0:17:12	12.13	5.9%	11.41
NA630	8851	8131	126662	0:59:11	1:01:09	12.75	7.8%	11.75
NA660	10907	11226	154096	0:43:00	0:45:30	15.72	6.1%	14.76
NA670	11064	12312	153774	0:20:44	0:23:14	15.87	5.2%	15.04
NA680	10866	10736	155694	0:06:34	0:09:04	15.76	5.1%	14.95

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[*] Terminated due to time limit of one hour

Table 7. Heuristic Model (Time Limit = 1 hour, MIP Gap = 5%)

Problem	As	Rs	TEs	CPLEX Time	Total Time	Cost (000,000)	Gap	Lower Bound
EU110	1268	1425	18788	0:00:02	0:00:27	1.91	6.8%	1.78
EU120	1369	1463	19676	0:00:02	0:00:30	2.00	4.5%	1.91
EU130	1349	1442	19122	0:00:02	0:00:33	1.95	5.1%	1.85
EU160	2081	2168	29366	0:00:06	0:00:37	3.00	5.3%	2.84
EU170	2280	2396	32068	0:00:07	0:00:39	3.27	7.6%	3.02
EU180	2194	2233	30736	0:00:10	0:00:41	3.14	5.7%	2.96
EU210	2827	3115	41222	0:00:03	0:00:44	4.20	6.2%	3.94
EU220	3112	3516	45826	0:00:07	0:00:48	4.66	6.0%	4.38
EU230	2780	3221	41026	0:00:05	0:00:47	4.19	5.7%	3.95
EU260	3470	4108	50816	0:00:05	0:00:56	5.20	4.8%	4.95
EU270	3711	4077	52212	0:00:06	0:00:58	5.36	4.9%	5.31
EU280	3631	4637	53400	0:00:05	0:00:57	5.51	7.1%	5.12
US310	3180	3491	76546	0:00:02	0:00:42	7.08	4.0%	6.80
US320	3222	3329	76820	0:00:02	0:00:42	7.08	4.7%	6.75
US330	3169	3511	76078	0:00:02	0:00:42	7.04	5.4%	6.66
US360	4561	5491	110692	0:00:03	0:01:22	10.29	5.2%	9.75
US370	4736	5529	114148	0:00:01	0:01:21	10.60	5.5%	10.02
US380	4789	5646	113976	0:00:03	0:01:22	10.61	6.2%	9.95
US410	6110	7584	147730	0:00:03	0:01:52	13.78	5.3%	13.05
US420	6162	7592	150516	0:00:02	0:01:51	14.00	5.4%	13.24
US430	6147	7540	150650	0:00:03	0:01:52	14.00	5.5%	13.23
US460	7829	9487	185446	0:00:02	0:02:23	17.32	5.2%	16.42
US470	7781	9435	186862	0:00:02	0:02:23	17.41	4.3%	16.67
US480	7680	9227	187002	0:00:02	0:02:23	17.37	5.0%	16.51
NA510	4250	4023	61028	0:02:02	0:03:09	6.15	8.3%	5.64
NA520	4881	4668	10622	0:01:09	0:02:11	7.11	7.2%	6.60
NA530	4157	4403	57920	0:02:47	0:03:49	5.93	8.1%	5.45
NA560	6828	7270	98688	0:00:43	0:02:12	10.04	7.2%	9.32
NA570	6435	6236	93974	0:00:46	0:02:16	9.44	8.2%	8.67
NA580	6725	7015	96576	0:01:24	0:02:54	9.81	7.3%	9.09
NA610	8703	9382	126728	0:00:55	0:02:56	12.89	7.4%	11.93
NA620	8260	8349	123166	0:00:36	0:02:37	12.38	7.8%	11.41
NA630	8784	9075	129636	0:01:09	0:03:10	13.08	10.2%	11.75
NA660	10663	12734	157264	0:00:28	0:03:01	16.11	8.4%	14.76
NA670	10930	11638	159038	0:00:24	0:02:59	16.16	6.9%	15.04
NA680	10851	11705	159568	0:00:30	0:03:04	16.19	7.7%	14.95

Table 8. Selected Test Problem Runs using NA Test Network
(12 Hours Time Limit, MIP Gap = 5%)

Problem	Global Model		Relaxed Model		Path Reduction Heuristic	
	Total Time	Cost	Total Time	Cost	Total Time	Cost
NA610	9:49:35	12.75	1:08:50	12.79	0:02:56	12.89
NA620	4:42:54	12.20	0:17:12	12.13	0:02:37	12.38
NA630	12:02:45[*]	12.88	1:01:09	12.75	0:03:10	13.08
NA660	3:12:42	15.73	0:45:30	15.72	0:03:01	16.11
NA670	2:24:30	15.86	0:23:14	15.87	0:02:59	16.16
NA680	2:46:38	15.86	0:09:04	15.76	0:03:04	16.19

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[*] Terminated due to time limit of twelve hours

Table 9. Multiple Path Model (Time Limit = 1 hour, MIP Gap = 5%)

Problem	As	Rs	TEs	CPLEX Time	Total Time	Cost (000,000)	Gap	Lower Bound
EU110	1140	1502	15910	0:00:01	0:00:21	1.67	1.2%	1.65
EU120	1217	1481	16976	0:00:01	0:00:21	1.77	1.1%	1.75
EU130	1275	1383	16744	0:00:02	0:00:22	1.74	0.6%	1.73
EU160	1905	2177	25334	0:00:02	0:00:33	2.65	0.4%	2.64
EU170	2092	2455	26934	0:00:01	0:00:32	2.84	0.7%	2.82
EU180	2020	2445	26406	0:00:01	0:00:32	2.79	0.7%	2.77
EU210	2641	3269	36068	0:00:01	0:00:42	3.78	0.3%	3.77
EU220	2889	3628	39924	0:00:01	0:00:43	4.17	0.2%	4.16
EU230	2633	3337	36326	0:00:01	0:00:42	3.81	0.5%	3.79
EU260	3252	4213	44743	0:00:01	0:00:53	4.70	0.4%	4.68
EU270	3436	4314	46316	0:00:02	0:00:53	4.88	0.4%	4.86
EU280	3372	4427	46518	0:00:01	0:00:53	4.90	0.4%	4.88
US310	2898	3454	69470	0:00:01	0:00:41	6.47	0.2%	6.46
US320	2897	3511	69578	0:00:01	0:00:40	6.48	0.2%	6.47
US330	2835	3405	67854	0:00:01	0:00:40	6.31	0.2%	6.30
US360	4091	5492	99006	0:00:02	0:01:20	9.29	0.2%	9.27
US370	4263	5517	102390	0:00:01	0:01:20	9.59	0.2%	9.57
US380	4180	5459	99628	0:00:01	0:01:20	9.35	0.2%	9.33
US410	5488	7554	131622	0:00:01	0:01:49	12.39	0.2%	12.37
US420	5530	7520	13368	0:00:01	0:01:50	12.53	0.2%	12.51
US430	5472	7603	132824	0:00:02	0:01:50	12.48	0.1%	12.47
US460	6912	9270	166476	0:00:01	0:02:21	15.62	0.1%	15.60
US470	6958	9439	167192	0:00:01	0:02:21	15.71	0.1%	15.70
US480	6845	9327	165204	0:00:01	0:02:22	15.52	0.1%	15.51
NA510	3573	3804	50958	0:00:05	0:01:05	5.20	0.6%	5.17
NA520	4245	4588	60930	0:00:05	0:01:05	6.22	0.6%	6.18
NA530	3763	4560	50274	0:00:10	0:01:10	5.27	0.6%	5.24
NA560	6178	7049	85020	0:00:05	0:01:32	8.81	0.6%	8.76
NA570	5593	6561	80178	0:00:06	0:01:33	8.25	0.6%	8.20
NA580	5903	7185	83750	0:00:10	0:01:47	8.68	0.3%	8.65
NA610	7895	9562	108178	0:00:03	0:02:00	11.30	0.4%	11.25
NA620	7285	8393	104510	0:00:01	0:01:59	10.74	0.5%	10.69
NA630	7661	8819	109492	0:00:05	0:02:04	11.25	0.3%	11.22
NA660	9653	11779	136328	0:00:03	0:02:32	14.15	0.4%	14.10
NA670	9683	12276	137166	0:00:06	0:02:37	14.29	0.2%	14.26
NA680	9757	11672	136878	0:00:03	0:02:32	14.20	0.4%	14.14

Table 10. Path-Based Shared Protection Model with a Maximum of 6 Cycles per (o,d) Demand Pair
(Time Limit = 1 hour, MIP Gap = 5%)

Problem	As	Rs	TEs	CPLEX Time	Total Time	Cost (000,000)	Gap	Lower Bound
EU110	1339	1756	18632	0:00:07	0:00:38	1.96	2.6%	1.91
EU120	1417	1882	19018	0:00:03	0:00:34	2.02	4.0%	1.94
EU130	1554	1746	21830	0:00:02	0:00:32	2.24	4.5%	2.14
EU160	1971	2274	25930	0:00:10	0:00:54	2.72	2.6%	2.65
EU170	2044	2632	28428	0:00:02	0:00:46	2.98	4.4%	2.85
EU180	1839	2620	25298	0:00:10	0:00:54	2.69	4.5%	2.57
EU210	2729	3487	38208	0:00:17	0:01:19	4.00	1.5%	3.94
EU220	2609	3243	36272	0:00:18	0:01:19	3.78	2.1%	3.70
EU230	2787	3540	38294	0:00:14	0:01:15	4.02	2.2%	3.93
EU260	3147	3717	43924	0:00:47	0:02:04	4.55	1.8%	4.47
EU270	3060	3714	43198	0:00:24	0:01:51	4.48	1.3%	4.42
EU280	3022	3784	42642	0:00:30	0:01:57	4.44	1.6%	4.37
US310	2231	3333	52960	0:00:48	0:01:58	5.03	1.6%	4.95
US320	2273	3472	53938	0:00:32	0:01:43	5.13	1.8%	5.04
US330	2372	3860	56692	0:00:26	0:01:37	5.41	0.6%	5.38
US360	3585	5007	87274	0:01:45	0:03:42	8.20	1.0%	8.12
US370	3733	5950	90170	0:01:07	0:03:14	8.59	0.5%	8.55
US380	3457	5160	84136	0:01:09	0:03:15	7.95	0.8%	7.89
US410	4716	6242	118314	0:01:20	0:04:22	11.02	0.2%	11.00
US420	4976	7792	120240	0:02:34	0:05:35	11.43	0.3%	11.39
US430	5034	7165	125224	0:02:45	0:05:49	11.75	0.8%	11.66
US460	6078	8984	148874	0:04:02	0:08:09	14.05	0.2%	14.02
US470	6248	8818	154308	0:04:16	0:08:21	14.48	0.4%	14.42
US480	5983	8854	148440	0:07:40	0:11:44	13.97	0.2%	13.94
NA510	4040	4557	53112	0:00:18	0:01:41	5.55	2.2%	5.43
NA520	3975	4293	53950	0:00:40	0:02:09	5.58	4.1%	5.35
NA530	3959	4900	50850	0:00:40	0:02:10	5.41	1.8%	5.31
NA560	5918	6839	77620	0:00:57	0:03:12	8.15	2.1%	7.98
NA570	5733	6574	77560	0:01:01	0:03:17	8.07	3.0%	7.83
NA580	5557	8199	74464	0:01:02	0:03:18	8.02	4.9%	7.63
NA610	7641	9576	102776	0:01:10	0:04:10	10.84	1.0%	10.73
NA620	7618	10977	101900	0:02:04	0:05:03	10.94	3.7%	10.54
NA630	7591	9762	101932	0:02:24	0:05:24	10.79	1.4%	10.64
NA660	9396	11380	127252	0:03:15	0:07:11	13.33	1.2%	13.17
NA670	9370	11276	126858	0:02:58	0:06:54	13.29	0.5%	13.22
NA680	9063	10588	123586	0:04:31	0:08:28	12.88	1.1%	12.74

Table 11. Path-Based Shared Protection Model with a Maximum of 12 Cycles per (o,d) Demand Pair
(Time Limit = 1 hour, MIP Gap = 5%)

Problem	As	Rs	TEs	CPLEX Time	Total Time	Cost (000,000)	Gap	Lower Bound
EU110	1288	1741	18114	0:00:06	0:00:54	1.90	3.7%	1.83
EU120	1426	1768	18492	0:00:12	0:01:13	1.96	4.1%	1.88
EU130	1486	1951	20664	0:00:10	0:01:11	2.17	4.6%	2.07
EU160	1919	2232	25180	0:00:38	0:02:15	2.65	3.0%	2.57
EU170	1996	2501	27620	0:00:24	0:02:02	2.89	3.8%	2.78
EU180	1842	2186	24696	0:00:34	0:02:10	2.59	3.1%	2.51
EU210	2707	3616	37212	0:01:04	0:03:31	3.93	2.5%	3.83
EU220	2593	4199	34612	0:00:50	0:03:18	3.78	4.8%	3.60
EU230	2707	3701	37410	0:00:54	0:03:23	3.96	3.0%	3.84
EU260	3055	3915	42566	0:03:12	0:06:36	4.46	2.5%	4.35
EU270	3010	4082	41954	0:02:18	0:05:45	4.42	2.9%	4.29
EU280	2956	3851	41286	0:02:35	0:06:01	4.33	2.1%	4.24
US310	2209	3061	51992	0:00:12	0:04:01	4.91	2.2%	4.80
US320	2230	3321	52066	0:03:42	0:06:38	4.96	1.0%	4.91
US330	2345	3737	55886	0:03:45	0:06:42	5.33	1.7%	5.24
US360	3543	4773	84950	0:12:29	0:17:49	7.98	0.4%	7.95
US370	3721	5444	88592	0:04:49	0:10:04	8.39	0.7%	8.33
US380	3433	4830	83164	0:13:59	0:19:16	7.83	1.5%	7.71
US410	4707	5917	115892	0:10:34	0:18:22	10.78	0.6%	10.71
US420	5021	7454	119550	0:31:18	0:39:08	11.34	2.1%	11.10
US430	4967	6882	122422	0:12:43	0:20:38	11.48	0.9%	11.38
US460	6057	8540	145956	0:27:25	0:38:22	13.75	0.4%	13.70
US470	6166	8392	150394	0:45:56	0:56:56	14.10	0.2%	14.07
US480	5941	8308	144836	0:49:42	0:38:46	13.61	0.6%	13.53
NA510	3855	4203	50030	0:03:07	0:06:16	5.23	3.1%	5.07
NA520	3773	4177	50318	0:04:34	0:07:45	5.23	4.4%	5.00
NA530	3859	4601	49718	0:03:26	0:06:36	5.26	4.4%	5.03
NA560	5680	6825	74922	0:14:30	0:19:35	7.90	4.1%	7.58
NA570	5426	5896	71930	0:13:36	0:18:40	7.48	2.4%	7.30
NA580	5271	6000	71730	0:11:00	0:16:05	7.46	3.5%	7.20
NA610	7339	8909	97052	0:30:38	0:38:06	10.23	2.0%	10.03
NA620	7311	8227	97640	0:20:33	0:27:59	10.18	2.3%	9.95
NA630	7200	8934	96420	0:34:08	0:41:35	10.17	2.0%	9.97
NA660	8948	10322	120960	0:54:27	1:04:42	12.61	1.5%	12.42
NA670[*]								
NA680	8572	9866	116940	0:55:31	1:05:42	12.17	2.4%	11.88

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[*] Terminated due to time limit without a feasible integer solution

Table 12. Path-Based Shared Protection Multiple Path Model with a Maximum of 12 Cycles per (o,d) Demand Pair
(Time Limit = 1 hour, MIP Gap = 5%)

Problem	As	Rs	TEs	CPLEX Time	Total Time	Cost (000,000)	Gap	Lower Bound
EU110	1338	1697	17497	0:00:01	0:00:50	1.86	1.6%	1.83
EU120	1377	1714	17798	0:00:01	0:00:49	1.88	1.1%	1.86
EU130	1506	1853	19708	0:00:01	0:00:48	2.08	1.4%	2.05
EU160	1892	2253	24359	0:00:01	0:01:16	2.58	1.2%	2.55
EU170	1980	2429	26450	0:00:01	0:01:19	2.78	0.4%	2.77
EU180	1848	2168	23884	0:00:01	0:01:15	2.52	1.2%	2.49
EU210	2619	3539	36326	0:00:04	0:02:06	3.83	0.3%	3.82
EU220	2509	3365	34074	0:00:03	0:02:04	3.60	0.3%	3.59
EU230	2691	3669	36092	0:00:01	0:02:02	3.85	0.8%	3.82
EU260	3049	3868	41394	0:00:01	0:02:49	4.36	0.7%	4.33
EU270	2935	4032	40658	0:00:04	0:02:56	4.30	0.2%	4.29
EU280	2928	3759	40422	0:00:04	0:02:53	4.24	0.2%	4.23
US310	2189	3055	50860	0:00:01	0:02:30	4.82	0.4%	4.80
US320	2245	3283	51680	0:00:01	0:02:28	4.92	0.6%	4.89
US330	2289	3644	55112	0:00:04	0:02:32	5.25	0.2%	5.24
US360	3561	4739	84744	0:00:02	0:04:36	7.96	0.3%	7.94
US370	3702	5375	88164	0:00:02	0:04:34	8.35	0.4%	8.32
US380	3384	4818	81950	0:00:02	0:03:00	7.72	0.3%	7.70
US410	4697	5821	115296	0:00:04	0:06:54	10.72	0.2%	10.70
US420	4913	7269	117170	0:00:05	0:06:56	11.11	0.2%	11.09
US430	4960	6696	121738	0:00:09	0:07:04	11.40	0.2%	11.38
US460	6024	8394	145654	0:00:04	0:09:46	13.71	0.0%	13.71
US470	6167	8354	150222	0:00:04	0:09:43	14.08	0.0%	14.08
US480	5914	8169	144362	0:00:06	0:09:45	13.55	0.1%	13.53
NA510	3725	4109	48664	0:00:02	0:02:54	5.09	0.8%	5.05
NA520	3535	3954	48236	0:00:24	0:03:18	5.00	0.4%	4.98
NA530	3648	4337	47984	0:00:14	0:03:08	5.05	0.6%	5.02
NA560	5430	6446	72356	0:00:20	0:05:04	7.59	0.3%	7.57
NA570	5343	5959	69995	0:00:11	0:04:54	7.32	0.5%	7.28
NA580	5275	5901	69186	0:00:03	0:04:49	7.23	0.7%	7.18
NA610	7083	8728	95407	0:00:26	0:07:26	10.04	0.3%	10.01
NA620	7222	8111	95490	0:00:03	0:07:01	9.98	0.6%	9.92
NA630	7172	8859	94404	0:00:04	0:07:03	9.99	0.6%	9.93
NA660	8843	10219	119249	0:00:16	0:09:50	12.44	0.2%	12.41
NA670	8894	10513	119763	0:00:23	0:09:57	12.54	0.3%	12.50
NA680	8462	9871	114014	0:00:34	0:10:07	11.91	0.3%	11.87

Table 13. Failure Rates and Repair Times

Module	Failure Rate (r)	MTTR
Fiber	2.12566E-07	12
Regen	3.35521E-06	2
TE	3.35521E-06	2
MUX/DMUX	4.98153E-07	2
OXC	1.96685E-06	2
AMP	4.22508E-06	2

Table 14. Availability Events for Demand Pair (1,2)

Scenario	**w1**	**w2**	**p1**	**p2**	**p3**	**p4**	**p5**
1	W	▨	▨	▨	▨	▨	▨
2	F	W	W	W	W	▨	▨
3	F	F	W	W	W	F	▨
4	F	F	W	W	W	W	F

Table 15. Availability Events Including Node Failures for Demand Pair (1,2)

Event	**w1**	**w2**	**p1**	**p2**	**p3**	**p4**	**p5**	**Node 1**	**Node 2**	**Node 3**	**Node 4**	**Node 5**	**Node 6**
1	W	▨	▨	▨	▨	▨	▨	W	W	▨	▨	▨	▨
2	F	W	W	W	W	▨	▨	W	W	W	W	▨	▨
3	F	F	W	W	W	F	▨	W	W	W	W	W	W
4	F	F	W	W	W	W	F	W	W	W	W	W	W
5	F	F	W	W	W	▨	▨	W	W	W	W	F	▨
6	F	F	W	W	W	▨	▨	W	W	W	W	W	F

Table 16. Unavailability Calculations for Demand Pair (1,2) in Figure 7

Equipment	**Failure Rate**	**MTTF(yrs)**	**MTTF(hrs)**	**MTTR(hrs)**	**Avail**	**Unavail**
A	4.22508E-06	27.0	236,682	2.0	0.99999155	8.45E-06
Fiber	2.12566E-07	536.7	4,704,420	12.0	0.999997449	2.55E-06
MUX/DMUX	4.98153E-07	229.0	2,007,414	2.0	0.999999004	9.96E-07
OXC	1.96685E-06	58.0	508,428	2.0	0.999996066	3.93E-06
R	3.35521E-06	34.0	298,044	2.0	0.99999329	6.71E-06
TE	3.35521E-06	34.0	298,044	2.0	0.99999329	6.71E-06

Link Availability

	Quantity	**Avail**	**Unavail**
A	11	0.999907053	9.29509E-05
Fiber	1000	0.997452461	0.002550786
MUX/DMUX	2	0.999998007	1.99261E-06
TE	80	0.999463312	0.00053683
Total		0.99681744	0.00318256

$\boxed{A'_{w1}A'_{O_W}}$	0.996809598
$\boxed{(1-A'_{w1})\,A'_{w2}A'_{p1}A'_{p2}A'_{p3}A'_{O_W}\,A'_{O_B}}$	0.003142188
$\boxed{A'_{12}}$	0.999951787
Unavailability	4.82135E-05
hrs/yr	0.422639417
mins/yr	25.358365

Table 17. Availability Events for Demand Pair (7,8)

	Paths										**Nodes**							
Event	**w1**	**w2**	**w3**	**p1**	**p2**	**p3**	**p4**	**p5**	**p6**	**p7**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1			W														W	W
2	W	W	F		W			W	W	W				W	W	W	W	W
3	W	F	F		W		F	W	W	W				W	W	W	W	W
4	W	F	F		W	F	W	W	W	W				W	W	W	W	W
5	W	F	F		W	W	W	W	W	W			F	W	W	W	W	W
6	W	F	F		W	W	W	W	W	W		F	W	W	W	W	W	W
7	F	W	F	F	W			W	W	W				W	W	W	W	W
8	F	W	F	W	W	F		W	W	W				W	W	W	W	W
9	F	W	F	W	W	W		W	W	W	F			W	W	W	W	W
10	F	W	F	W	W	W		W	W	W	W	F		W	W	W	W	W
11	F	F	F	W	W	W	F	W	W	W	F			W	W	W	W	W
12	F	F	F	W	W	W	F	W	W	W	W	F		W	W	W	W	W
13	F	F	F	F	W	W	W	W	W	W			F	W	W	W	W	W
14	F	F	F	F	W	W	W	W	W	W		F	W	W	W	W	W	W
15	F	F	F		W	F		W	W	W				W	W	W	W	W
16	F	F	F	F	W	W	F	W	W	W				W	W	W	W	W
17	F	F	F	W	W	W	W	W	W	W		F		W	W	W	W	W
18	F	F	F	W	W	W	W	W	W	W	F	W	F	W	W	W	W	W

Table 18. Unavailability Calculations Using the Exact and the Approximate Methods

Network	U_{12} Exact	U_{12} Exact (Mins)	U_{12} Approximate	U_{12} Approximate (Mins)	% Error
Figure 7	4.81497E-05	25.3248	4.82135E-05	25.3584	0.13%

Network	U_{78} Exact	U_{78} Exact (Mins)	U_{78} Approximate	U_{78} Approximate (Mins)	
Figure 8	6.80719E-05	35.8031	6.82189E-05	35.8804	0.22%

Table 19. Unavailability of the Demand Pairs in Hours/Year

Problem	Link-Based Shared Protection			Path-Based Shared Protection		
	Min	Ave	Max	Min	Ave	Max
EU210	0.89	9.41	35.59	1.10	11.03	35.13
EU260	1.10	12.64	43.68	1.23	12.56	46.83
US410	0.70	31.74	100.21	2.23	34.68	87.60
US460	0.77	39.94	124.34	3.27	42.96	105.72
NA610	0.33	29.31	92.67	0.43	36.72	104.05
NA660	0.25	35.13	134.24	1.05	41.54	124.15

Table 20. Average Costs[*] for Shared Protection Schemes

	Multi-Path Models		Single-Path Models	
	Link-Based	Path-Based	Link-Based	Path-Based
EU	3.31	3.16	3.66	3.25
US	10.98	9.47	12.16	9.54
NA	9.86	8.68	11.25	8.93

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[*] In million dollars

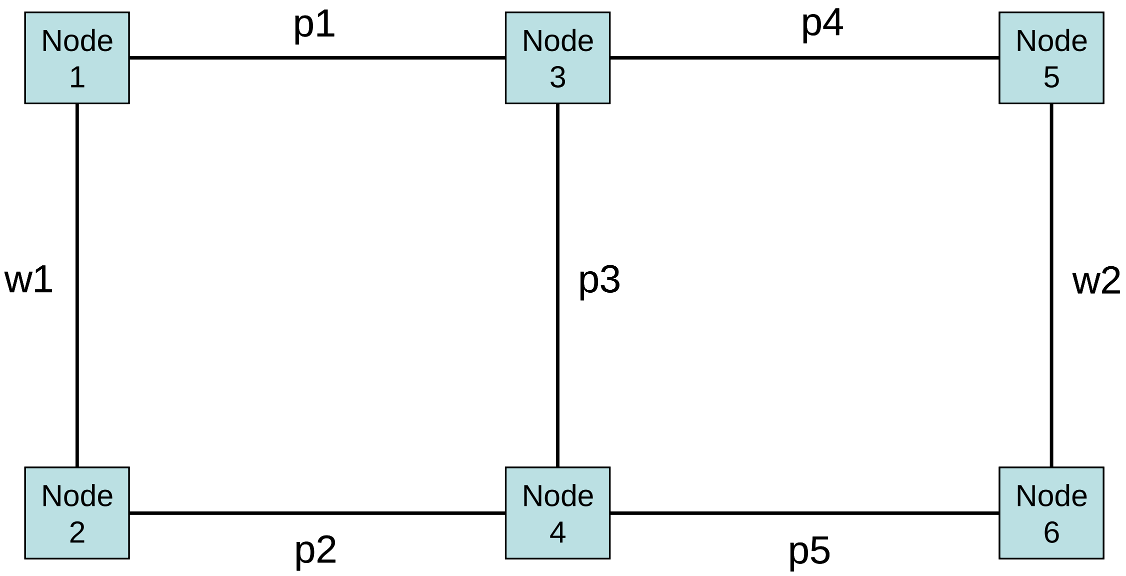


Figure 1. Example network for link-based shared protection

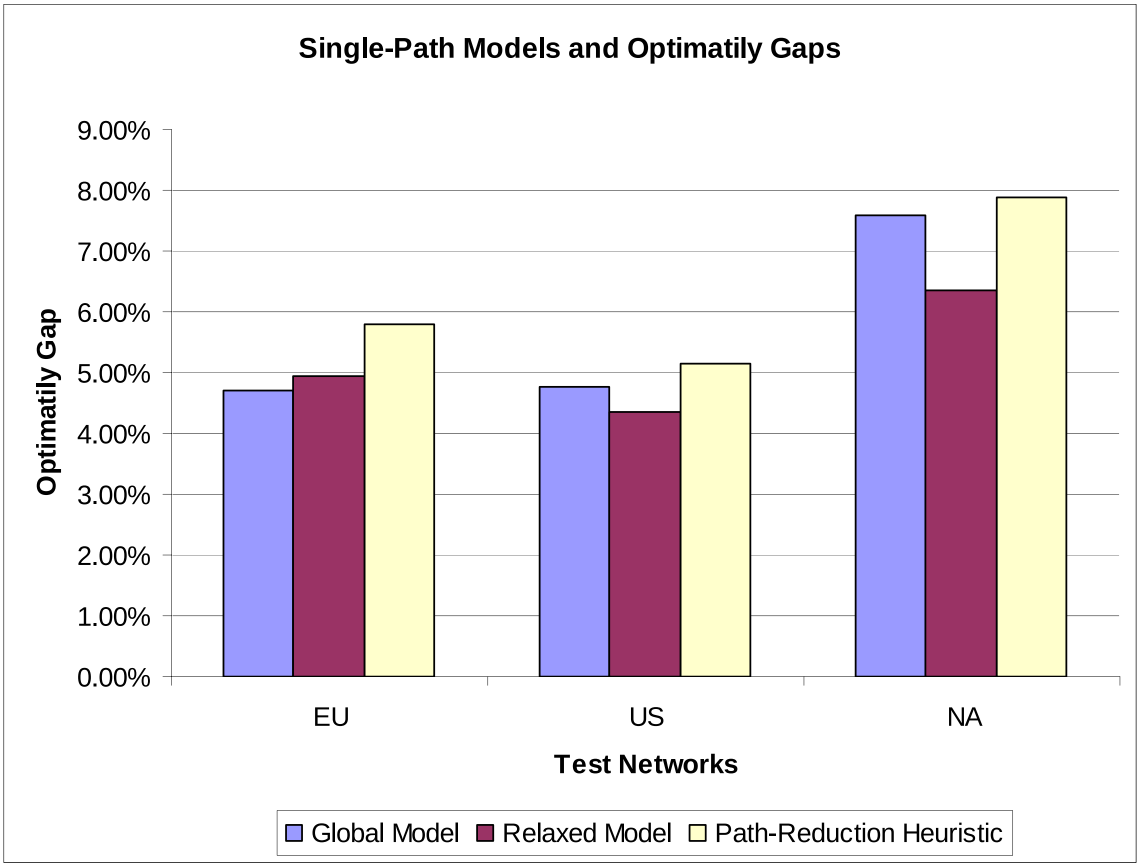


Figure 2. Average optimality gaps for the test cases using single-path link-based shared protection design models

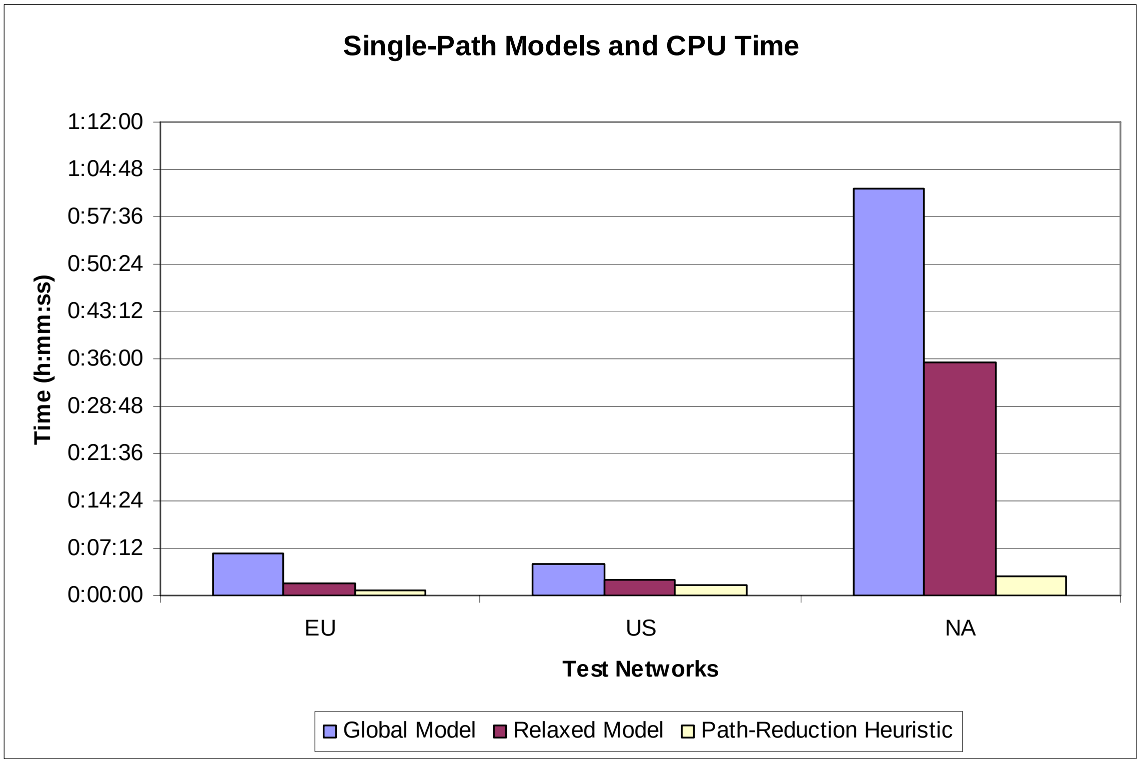
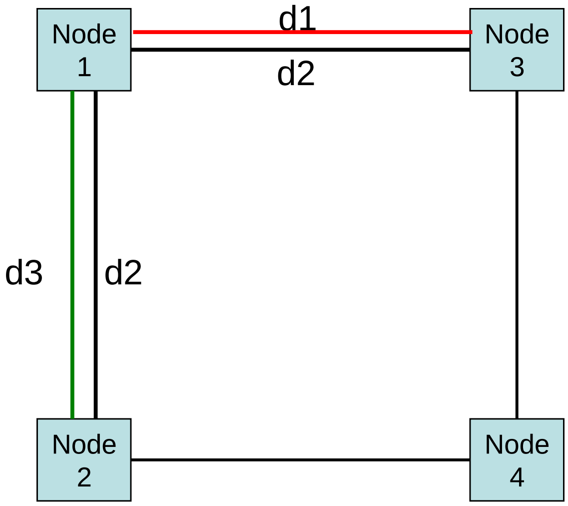


Figure 3. Average processing times for the test cases using single-path link-based shared protection design models



Demands	Links in the Working Path	Links in the Back-up Path
d1	(1,3)	(1,2) (2,4) (4,3)
d2	(2,1) (1,3)	(2,4) (4,3)
d3	(1,2)	(1,3) (3,4) (4,2)

Figure 4. Example network for path-based shared protection

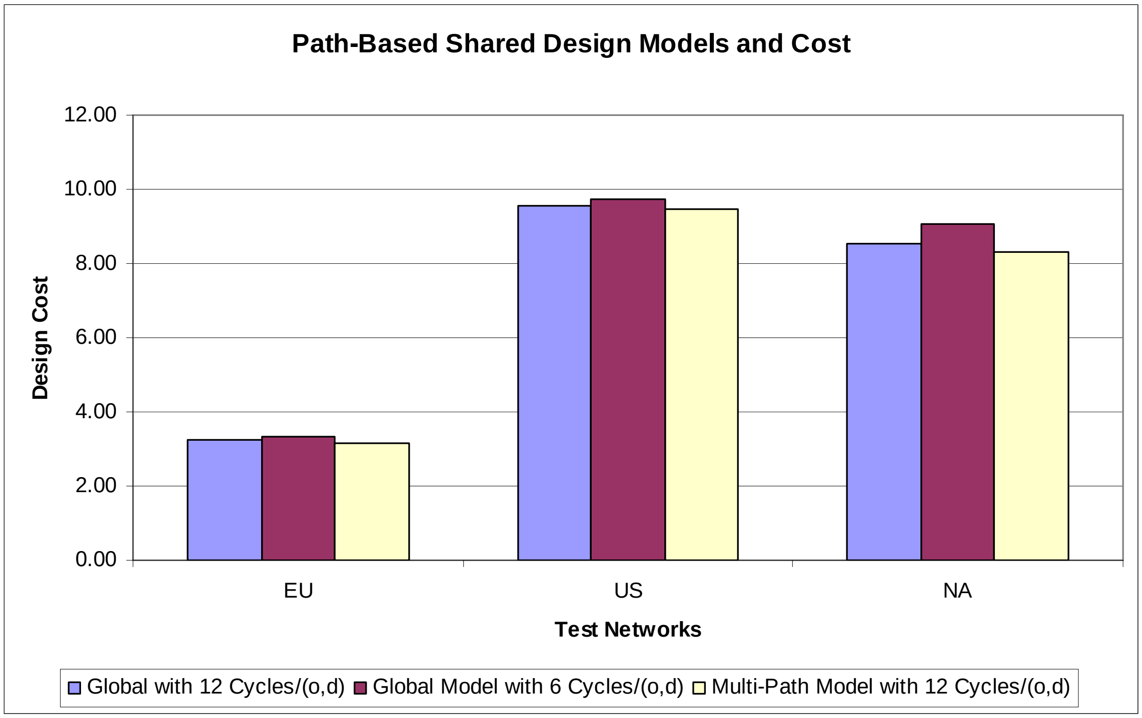


Figure 5. Path-based shared protection design models and average cost

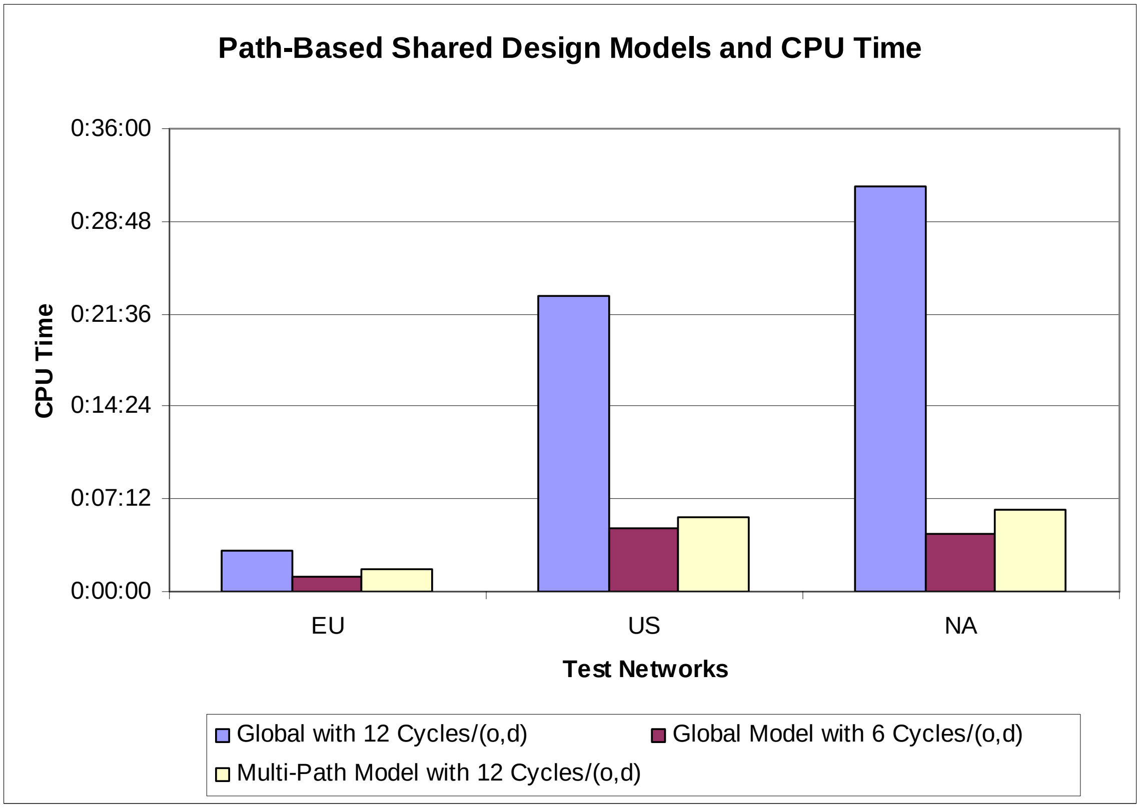


Figure 6. Path-based shared protection design models and average CPU time

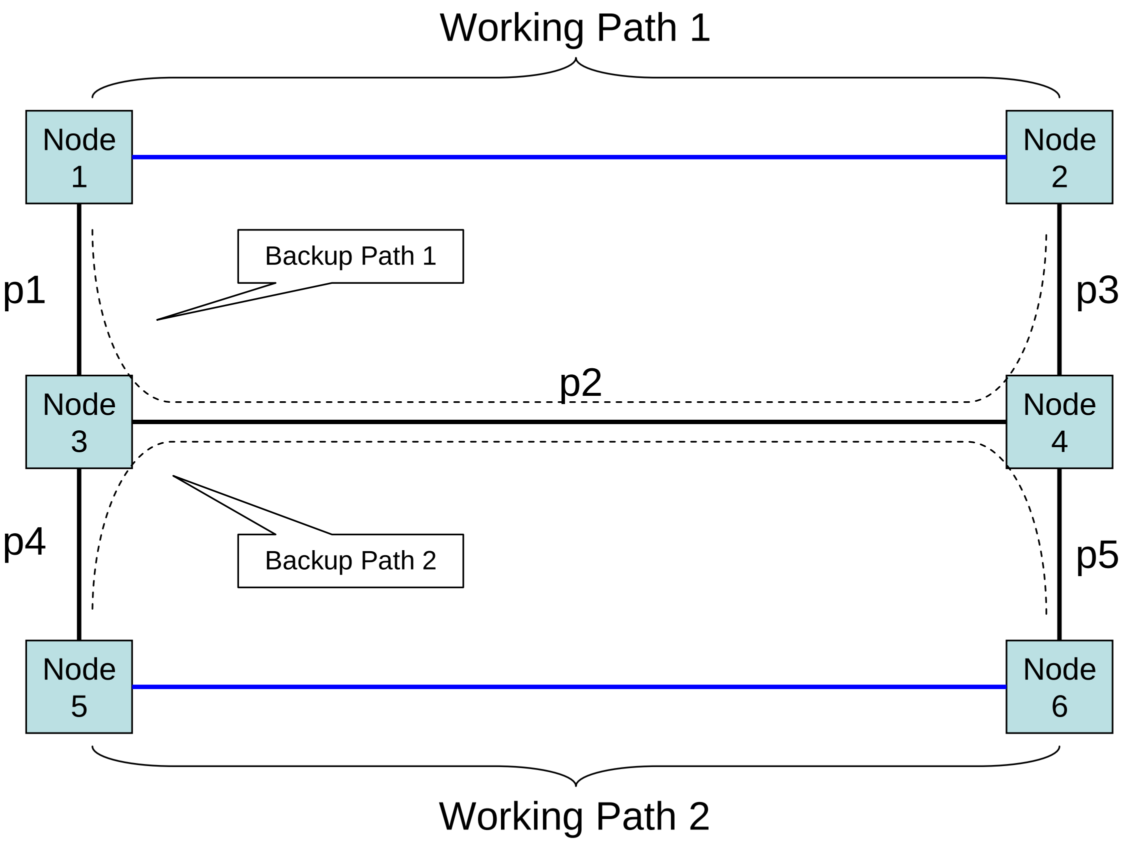
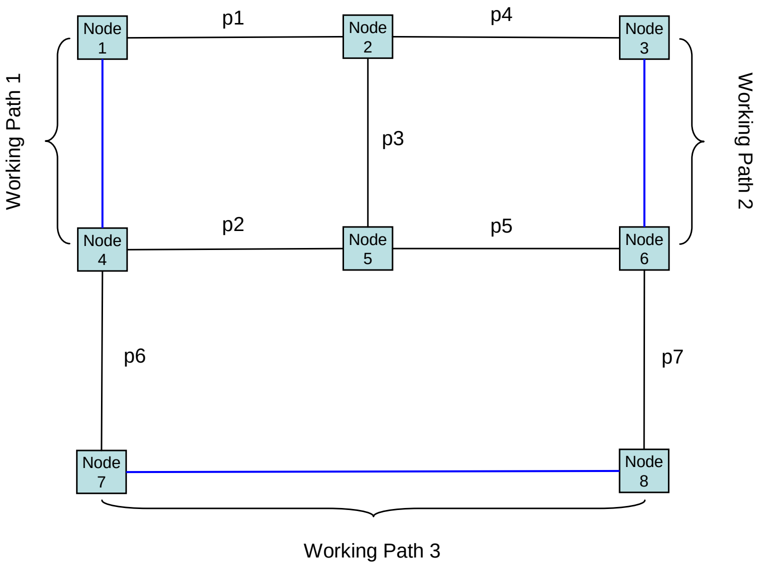


Figure 7. Example with two demand pairs



Demand Pairs	Links in the Back-up Path
(1,4)	(1,2) (2,5) (5,4)
(3,6)	(3,2) (2,5) (5,6)
(7,8)	(7,4) (4,5) (5,6) (6,8)

Figure 8. Example with three demand pairs

```
procedure Link-Based Unavailability (D, $\overline{W}$, r, t, $\overline{S}$, U);
 Inputs: D, $\overline{W}$, r, t, $\overline{S}$
  /* D denotes the set of demand pairs */
  /* $\overline{W}_{od}$ denotes the set of links in the working path for the demand pair (o,d) */
  /* $r_{od}$ denotes the demand in units of wavelengths between nodes o and d */
  /* $t_{ij}$ denotes the working traffic in units of wavelengths on link (i,j) */
  /* $\overline{S}_{ij}$ denotes the set of links sharing backup resources with link (i,j) */
 Output: U
  /* $U_{od}$ denotes the unavailability for the demand between nodes o and d */
 begin
    $\forall (o,d) \in D$ do
       $\bar{A} \leftarrow$ Calculate the availability of the working path using the links in $\overline{W}_{od}$ and
         equipment requirements for $r_{od}$ wavelengths of traffic;
       $U_{od} \leftarrow (1-\bar{A})$;
       $\forall (i,j) \in \overline{W}_{od}$ do
          $\bar{U} \leftarrow$ Calculate the unavailability of link (i,j) using the equipment
             requirements for $r_{od}$ wavelengths of traffic;
          $\tilde{A} \leftarrow$ Calculate the availability of the backup path for link (i,j) using the
             equipment requirements for $t_{ij}$ wavelengths of traffic;
          $A' \leftarrow \bar{U}\,\tilde{A}$;
          $\forall (k,l) \in \overline{S}_{ij} \setminus \overline{W}_{od}$ do
             $v \leftarrow$ Calculate the availability of link (k,l) using the equipment
             requirements for $t_{kl}$ wavelengths of traffic;
             $A' \leftarrow A'v$;
          end do
          $\forall (k,l) \in \overline{W}_{od} \setminus \{(i,j)\}$ do
             $y \leftarrow$ Calculate the availability of link (k,l) using the equipment
             requirements for $r_{od}$ wavelengths of traffic;
             $A' \leftarrow A'y$;
          end do
          $U_{od} \leftarrow U_{od} + (1-A')$;
       end do
       return $U_{od}$;
    end do
 end
```

Figure 9. Unavailability calculations for the link-based shared protection