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Edge Reduction for EVMDDs to Speed Up Analysis of Multi-State Systems

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***Abstract*—This paper introduces a reduction rule for edge-valued multi-valued decision diagrams (EVMDDs), which improves the speed of analysis of multi-state systems (MSSs). Most reduction rules for decision diagrams remove redundant nodes, while the introduced rule removes redundant edges in EVMDDs. Since the time to do an analysis in an MSS depends on the number of edges in the EVMDD, the introduced rule makes analysis of MSSs faster, especially when used with edge minimization algorithms based on variable grouping. Experimental results show that the introduced rule reduces the number of edges by up to 30%, and this results in an analysis time that is reduced by up to 30%.**

***Keywords*-Reduction rules for decision diagrams; EVMDDs; edge reduction of EVMDDs; multi-state systems; system analysis based on decision diagrams.**

I. Introduction

A multi-state system (MSS) [8], [9] is a model in which performance, reliability, safety, efficiency, power consumption, etc. are represented by states. It is widely used to model various fault tolerant systems including computer server systems, telecommunication systems, water, gas, electrical power distribution systems, flight control systems, and nuclear power plant monitoring systems [3]–[5], [17], [21], [23], [25], [26]. To design dependable fault tolerant systems, intensive analysis of MSSs in terms of various assessment measures, such as reliability and availability is needed [17]. An approach to solving this complex problem is to compute the probability of each state of an MSS [23], [25]. This is because many measures can be easily computed from the state probabilities [1], [8], [9]. However, assessing the state probabilities is time-consuming.

For fast assessment of the state probabilities, many methods based on binary decision diagrams (BDDs) [2]–[4], [6], [25] and multi-valued decision diagrams (MDDs) [10], [14], [22]–[24] have been proposed, and they have attracted much attention in recent years. Especially, MDD based methods hold promise, as [18] showed that an MDD based method is more efficient than a BDD based one.

Most of these methods assume only stationary probability distributions for analysis, and thus, they do not accommodate probability distributions that depend on time. This is because in many practical systems (especially in systems working for a long time), probabilities converge to constant values after enough time goes by, and people are interested only in probabilities on systems working stably [8], [9]. However, in safety-critical systems such as flight control systems and nuclear power plant monitoring systems, not only static analysis but also dynamic analysis should be done.

Although methods based on the Markov model [5], [8], [9], [21] can deal with probability distributions that depend on time, they are impractical for a large MSS. This is because their time complexity is $O(m^{3n})$, where m is the number of states, and n is the number of components in an MSS [3], [4]. For faster dynamic analysis of large MSSs, an MDD based method has been proposed [20]. This method can make a dynamic analysis in the same way as a static analysis by assigning dynamic probability distributions to edges in an MDD. However, even this method is slow, since for a dynamic analysis, a system has to be analyzed many times while advancing time by a small amount. Thus, a faster analysis method is desired.

In this paper, we introduce a reduction rule [19] for edge-valued MDDs (EVMDDs) [13], [14] to speed up the analysis of MSSs. Most of reduction rules of decision diagrams are intended to reduce redundant nodes, but the introduced rule is intended to reduce redundant edges in EVMDDs. Since analysis time of MSSs depends on *the number of edges* in an EVMDD, the introduced edge reduction rule can also reduce analysis time. The proposed technique can be applied to a wide range of analyses including static analysis, dynamic analysis, and analysis of systems having dependent components [15]. This is because the proposed technique is a fundamental one.

This paper is organized as follows: Section II defines MSSs, EVMDDs, and variable grouping. Section III introduces the analysis method of MSSs using EVMDDs, and in Section IV, we introduce an edge reduction of EVMDDs, and propose an analysis method using EVMDDs with reduced edges. Experimental results are shown in Section V.

II. Preliminaries

This section defines MSSs, structure functions, EVMDDs to represent structure functions, and variable grouping.

A. Multi-State Systems and Structure Functions

Definition 1: A **multi-state system (MSS)** is a model of a system that represents, as states, a capability, such as performance, capacity, or reliability. There are usually more

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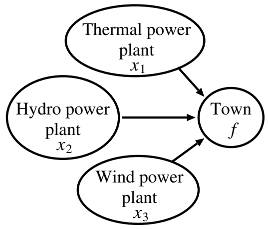


(a) MSS.

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	0	2	1
0	1	0	1
0	1	1	2
	⋮		⋮
2	2	2	5

(b) Structure function.

Figure 1. MSS for an electrical power distribution system and its structure function [16].

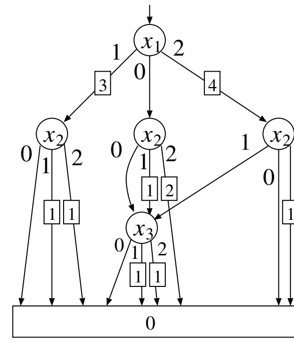


Figure 2. EVMDD for the structure function [16].

than two states, and so a multiple-valued analysis is required. When components in a system are modeled as well, it is called an **MSS with multi-state components**. In this paper, it is simply called an MSS.

Definition 2: A state of an MSS depends only on states of components in the system. A system with n components can be considered as a multi-valued function $f(x_1,x_2,\ldots,x_n) : R_1 \times R_2 \times \ldots \times R_n \to M$, where each x_i represents a component with r_i states, $R_i = \{0,1,\ldots,r_i-1\}$ is a set of the states, and $M = \{0,1,\ldots,m-1\}$ is a set of the m system states. This multi-valued function is called a **structure function** of the MSS.

Example 1: Fig. 1(a) shows an MSS for an electrical power distribution system. In this figure, the thermal power plant x_1, the hydro power plant x_2, and the wind power plant x_3 have three states which correspond to supply levels: 0 (breakdown), 1 (partially supply), and 2 (full supply). And, the system has six states which correspond to the percentage of area of a town that is blacked out: 0 (complete blackout), 1 (90% blackout), 2 (60% blackout), 3 (30% blackout), 4 (10% blackout), and 5 (0% blackout).

In this way, by assigning a value to each state, we obtain the 6-valued structure function f shown in Fig. 1(b). Note that Fig. 1(b) shows a part of the $3^3 = 27$ entry table since it is too large to be included in its entirety. (End of Example)

B. Edge-Valued Multi-Valued Decision Diagrams

Definition 3: An **edge-valued multi-valued decision diagram (EVMDD)** [13] is an extension of the MDD [7], [11], and represents a multi-valued function. It consists of one terminal node representing 0 and non-terminal nodes with edges having integer weights; 0-edges always have zero weights. In the EVMDD, the following two reduction rules are applied:

1) Share equivalent sub-graphs.
2) Delete a non-terminal node satisfying the two conditions: 1) its outgoing edges all point to the same node v, and 2) all of these outgoing edges have 0 weights. And, redirect edges, which point to the deleted node, to v

In the EVMDD, the function value is represented as a sum of weights of edges traversed from the root node to the terminal node. That is, the weights of edges traversed are summed, using integer addition, and the resulting value is the function value.

Example 2: Fig. 2 shows an EVMDD for the structure function of Example 1. (End of Example)

C. Variable Grouping

Definition 4: Let $X = (x_1,x_2,\ldots,x_n)$ be an ordered set of n multi-valued variables. Partition X into u ordered subsets as follows:

$$\begin{aligned} X_1 &= (x_1,x_2,\ldots,x_{k_1}), \\ X_2 &= (x_{k_1+1},x_{k_1+2},\ldots,x_{k_1+k_2}), \\ &\vdots \\ X_u &= (x_{k_1+k_2+\ldots+1},x_{k_1+k_2+\ldots+2},\ldots,x_n). \end{aligned}$$

Then, $(X_1,X_2,\ldots,X_u)$ is a **grouping** of X. Each ordered set $X_i = (x_{j+1},x_{j+2},\ldots,x_{j+k_i})$ forms a **composite variable** whose domain is $\{0,1,\ldots,r_{j+1} \times r_{j+2} \times \ldots \times r_{j+k_i} - 1\}$, where $|X_i| = k_i$ and $k_1 + k_2 + \ldots + k_u = n$. Note that the order of the original multi-valued variables is preserved in a grouping.

By considering each composite variable X_i as a larger-valued variable, the original multi-valued function $f(x_1,x_2,\ldots,x_n) : R_1 \times R_2 \times \ldots \times R_n \to M$ can be converted into another multi-valued input function $g(X_1,X_2,\ldots,X_u) : P_1 \times P_2 \times \ldots \times P_u \to M$, where $P_i = \{0,1,\ldots,r_{j+1} \times r_{j+2} \times \ldots \times r_{j+k_i} - 1\}$.

In this paper, for convenience, an EVMDD representing the function g obtained by grouping variables is called a **GEVMDD** (grouped EVMDD).

Example 3: Let the multi-valued variables x_1,x_2,x_3 in Example 1 be grouped into two composite variables, as follows:

$$X_1 = (x_1,x_2) \text{ and } X_2 = (x_3)$$

Note that since x_1 and x_2 are 3-valued variables, the composite variable X_1 consisting of x_1 and x_2 is a 9-valued variable. The GEVMDD representing the obtained function $g(X_1,X_2)$ is shown in Fig. 3. (End of Example)

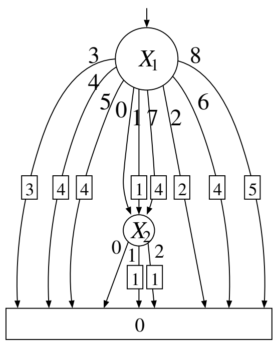
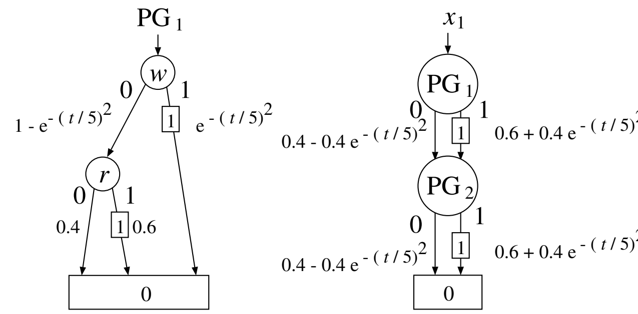


Figure 3. GEVMDD for the function $g(X_1, X_2)$ [16].



(a) Power generator. (b) Thermal power plant.

Figure 5. EVMDDs for a power generator and the thermal power plant.

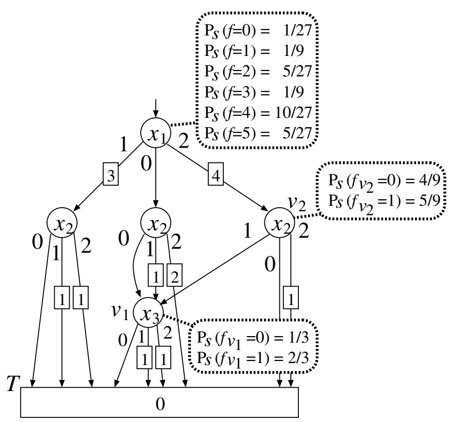


Figure 4. Analysis of the MSS using EVMDD [16].

III. Analysis Method Using EVMDDs

A. Static Analysis of Multi-State Systems

Definition 5: The probability that a structure function f has the value s is denoted by $P_s(f = s)$, where $s \in \{0, 1, \ldots, m-1\}$. The probability that a component x_i has the value c is denoted by $P_c(x_i = c)$, where $c \in \{0, 1, \ldots, r_i - 1\}$.

An analysis of MSSs solves the following:

Problem 1: Given a structure function f of an MSS and the probability of each state of each component $P_c(x_i = c)$, compute the probability of each state of the MSS $P_s(f = s)$. For simplicity, we assume that the probabilities of all component states are independent of each other.

To solve this problem efficiently, a method using EVMDDs has been proposed [14]. The method represents given structure functions using EVMDDs, and computes probabilities for a structure function by merging probabilities for sub-functions represented by nodes in a bottom-up manner.

Example 4: Let us compute the probability of each state of the MSS using the EVMDD in Fig. 4. In this example, we assume that all states of each component occur with the same probability, 1/3.

First, we have $P_s(f_T = 0) = 1$ at the terminal node T. Then, we compute probabilities for a sub-function f_{v_1} at node v_1. Since this node has two edges pointing to T whose values are 1, and the two edges represent $f_{v_1} = 1$, we have

$$\begin{aligned} P_s(f_T = 0) \times P_c(x_3 = 1) &= 1/3, \\ P_s(f_T = 0) \times P_c(x_3 = 2) &= 1/3, \text{ and thus,} \\ P_s(f_{v_1} = 1) &= P_s(f_T = 0) \times P_c(x_3 = 1) \\ &\quad + P_s(f_T = 0) \times P_c(x_3 = 2) \\ &= 2/3. \end{aligned}$$

Thus, $P_s(f_{v_1} = 0) = 1/3$ and $P_s(f_{v_1} = 1) = 2/3$ for v_1. At v_2, the probabilities at the terminal node and v_1 are multiplied by 1/3, and they are merged. Thus, $P_s(f_{v_2} = 0) = 4/9$ and $P_s(f_{v_2} = 1) = 5/9$. Similarly, by performing the same computation at each node in a bottom-up manner, we have the following at the root node: $P_s(f = 0) = 1/27$, $P_s(f = 1) = 1/9$, $P_s(f = 2) = 5/27$, $P_s(f = 3) = 1/9$, $P_s(f = 4) = 10/27$, and $P_s(f = 5) = 5/27$. (End of Example)

B. Dynamic Analysis of Multi-State Systems

In static analysis, it is assumed that the probabilities of component states $P_c(x_i = c)$ are constants, as shown in the previous subsection, since those probabilities converge to constant values after enough time goes by, as will be shown later. On the other hand, dynamic analysis assumes that the probability distributions depend on time, such as Weibull distributions that are widely used for modeling reliability of components considering their degradation due to age. That is, in dynamic analysis, probabilities of component states $P_c(x_i = c)$ are given as functions of time.

However, by assigning function values at a certain time to edges, we can apply the same analysis method to dynamic analysis as well. We can do a dynamic analysis of an MSS by doing many static analyses while advancing time by a small amount [20].

Example 5: Make the following three assumptions: 1) the thermal power plant x_1 in Fig. 1 consists of two identical power generators that work independently each other; 2) the

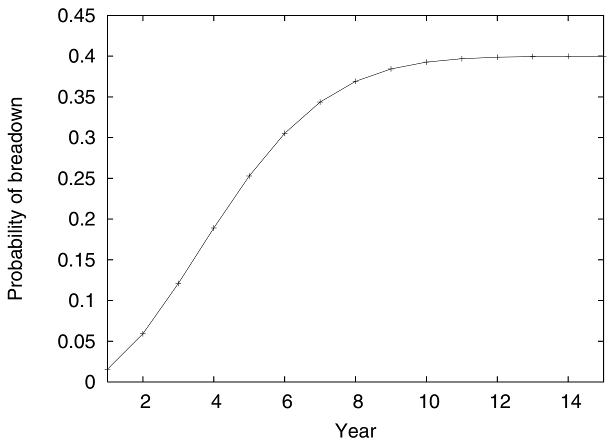


Figure 6. Result of dynamic analysis of a power generator.

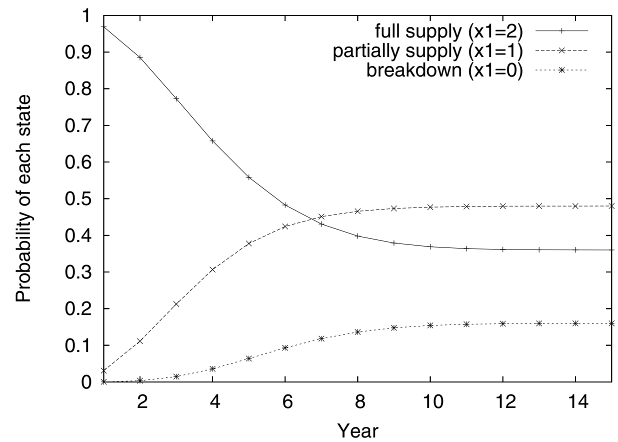


Figure 7. Result of dynamic analysis of the thermal power plant x_1.

probability that a power generator fails is given as a cumulative distribution function (CDF) of a Weibull distribution:

$$1 - e^{-(t/5)^2},$$

where t is time; and, 3) 60% of failures occurring at a power generator are repairable.

Then, the probability of whether a power generator (PG) works (PG=1) or not (PG=0) can be computed using a decision diagram in Fig. 5(a). In this figure, the variable w represents whether the power generator works ($w = 1$) or fails ($w = 0$), and r represents whether a failure is repairable ($r = 1$) or not ($r = 0$). Similarly, the probability of various states of the thermal power plant x_1 can be computed by using a decision diagram in Fig. 5(b) and analysis results of power generators PG_1 and PG_2. In this way, by assigning function values at a certain time to edges, and by analyzing system components hierarchically while advancing time by a small amount, we can perform dynamic analysis of the whole power distribution system in Fig. 1.

Figs. 6 and 7 show results of dynamic analysis of a power generator and the thermal power plant x_1, respectively. From these figures, we can see that probabilities converge to constant values after enough time goes by. In static analysis, these converged values are used. (End of Example)

Since for a dynamic analysis, a system has to be analyzed many times while advancing time by a small amount, a faster analysis method is desired.

IV. Edge Reduction of EVMDDs

Since the computation time of EVMDD based analysis method depends on the number of edges in an EVMDD, minimization of the number of edges by variable grouping is helpful to reduce analysis time [16]. However, variable grouping tends to increase the number of redundant edges that point to the same node and have the same edge value, as shown in Fig. 3. The GEVMDD has redundant edges for

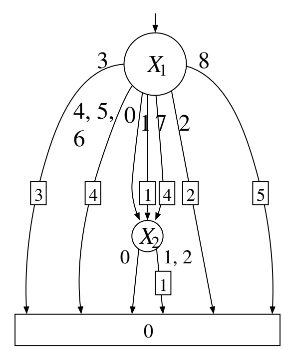


Figure 8. Reduced GEVMDD for the function $g(X_1, X_2)$.

$X_1 = 4,5,6$ and $X_2 = 1,2$. We make the analysis faster by reducing these redundant edges.

To reduce such redundant edges, we introduce the following rule in addition to the reduction rules of EVMDDs.

- Merge equivalent multiple edges, which leave from the same node, point to the same node, and have the same edge values, into a single edge, and share the edge among corresponding values of an input variable.

In this paper, for convenience, an EVMDD and a GEVMDD whose edges are reduced are called an **REVMDD** (reduced EVMDD) and an **RGEVMDD** (reduced GEVMDD), respectively.

Example 6: Fig. 8 shows the RGEVMDD of the GEVMDD in Fig. 3. This RGEVMDD has 9 edges while the original GEVMDD has 12 edges. (End of Example)

To analyze an MSS using an RGEVMDD, we begin by assigning probabilities of component states to edges. Shared edges are assigned sums of probabilities corresponding to merged edges, as shown in Fig. 9. Then, the analysis method shown in Section III is applied to the RGEVMDD.

Since edge reduction is a fundamental technique, it can

Table I
NUMBER OF EDGES IN EVMDDS FOR m-STATE SYSTEMS WITH n 3-STATE COMPONENTS.

n	m	Number of edges							Computation time (sec.)	
		EVMDD	REVMDD	Ratio1	GEVMDD	RGEVMDD	Ratio2	Ratio3	Grouping	Reduction
5	3	27	22	81%	27	19	70%	70%	* < 0.01	* < 0.01
5	10	51	42	82%	48	39	76%	81%	* < 0.01	* < 0.01
10	3	42	34	81%	42	32	76%	76%	* < 0.01	* < 0.01
10	10	168	134	80%	162	120	71%	74%	* < 0.01	* < 0.01
10	100	792	662	84%	750	531	67%	71%	* < 0.01	* < 0.01
10	1,000	2,718	2,548	94%	2,364	2,171	80%	92%	* < 0.01	* < 0.01
15	3	87	66	76%	87	64	74%	74%	* < 0.01	* < 0.01
15	10	312	250	80%	309	246	79%	80%	* < 0.01	* < 0.01
15	100	2,121	1,696	80%	2,076	1,571	74%	76%	* < 0.01	* < 0.01
15	1,000	10,083	8,322	83%	9,597	7,164	71%	75%	* < 0.01	* < 0.01
15	10,000	34,419	31,608	92%	31,212	28,234	82%	90%	0.01	0.01
15	100,000	188,274	182,748	97%	159,768	153,216	81%	96%	0.06	0.02

n: Number of 3-state components. m: Number of states for systems. Ratio1: REVMDD / EVMDD × 100 (%)
Ratio2: RGEVMDD / EVMDD × 100 (%) Ratio3: RGEVMDD / GEVMDD × 100 (%)
* <: It was shorter than 10 msec., but could not be obtained precisely due to precision of the timer.

Table II
COMPUTATION TIMES FOR ANALYSIS OF m-STATE SYSTEMS WITH n 3-STATE COMPONENTS.

n	m	Computation time of bottom-up method [14] (μsec.)					Computation time of hybrid method [15] (μsec.)				
		EVMDD	GEVMDD	RGEVMDD	Ratio1	Ratio2	EVMDD	GEVMDD	RGEVMDD	Ratio1	Ratio2
5	3	0.96	0.98	0.77	80%	79%	1.77	1.12	0.78	44%	70%
5	10	2.24	2.06	1.85	83%	90%	4.76	3.13	2.42	51%	77%
10	3	1.55	1.56	1.37	88%	88%	2.74	1.68	1.24	45%	74%
10	10	6.61	6.39	5.30	80%	83%	12.14	7.53	5.59	46%	74%
10	100	42.32	33.29	26.22	62%	79%	69.60	44.30	32.65	47%	74%
10	1,000	258.74	155.89	151.90	59%	97%	360.14	230.27	199.19	55%	87%
15	3	3.36	3.11	2.74	82%	88%	5.95	4.21	3.11	52%	74%
15	10	12.18	12.48	10.90	89%	87%	21.14	14.61	11.42	54%	78%
15	100	95.90	86.76	78.44	82%	90%	148.64	102.59	79.09	53%	77%
15	1,000	652.68	500.55	442.38	68%	88%	930.28	628.26	480.47	52%	76%
15	10,000	3,253.00	2,397.00	2,328.00	72%	97%	5,408.00	3,753.00	3,169.00	59%	84%
15	100,000	55,097.00	18,257.00	19,465.00	35%	107%	24,238.00	15,845.00	14,940.00	62%	94%

Ratio1: RGEVMDD / EVMDD × 100 (%) Ratio2: RGEVMDD / GEVMDD × 100 (%)
Very short computation times are average times obtained by running the same computation $1,000,000$ times, and dividing its total time by $1,000,000$.

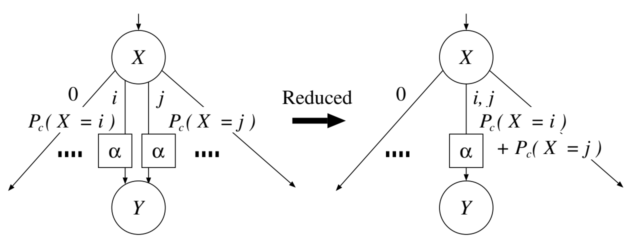


Figure 9. Probability of shared edge.

be used along with the hybrid analysis method [15] that is used for analysis of systems having dependent components.

V. EXPERIMENTAL RESULTS

To show the efficiency of the proposed edge reduction technique, we used the same analysis algorithms as [14], [15], and we randomly generated M1-monotone increasing functions [12] as benchmark structure functions. The methods are implemented on our private EVMDD package, and run on the following computer environment: CPU: Intel Core2 Quad Q6600 2.4GHz, memory: 4GB, OS: CentOS 5.7, and C-compiler: gcc -O3 (version 4.1.2). Table I shows the number of edges in various EVMDDs for randomly generated m-state systems with n 3-state components. In this table, the columns "Grouping" and "Reduction" show computation times of the edge minimization algorithm [16] and edge reduction presented in Section IV, respectively.

From this table, we can see that even ordinary EVMDDs have redundant edges, but GEVMDDs have more redundant edges. Thus, edge reduction works more effectively when it is used along with the edge minimization algorithm by variable grouping. Table I also shows that edge reduction quickly reduces the number of edges.

Table II shows the analysis times when applying the two analysis methods to various EVMDDs. In the analysis using RGEVMDDs, probabilities of shared edges have to be computed before applying the analysis algorithms, and thus, this makes analysis slightly longer when the number of redundant edges is small, like the analysis of a system with $n = 15$ and $m = 100,000$. But, in many cases, edge reduction makes analysis faster. It can shorten analysis time of optimized GEVMDDs by up to 30%.

VI. Conclusion and Comments

This paper introduces a reduction rule for EVMDDs that reduces redundant edges to speed up the analysis of MSSs. Since analysis time depends on the number of edges in an EVMDD, the introduced edge reduction rule can also reduce analysis time. Experimental results show that the introduced rule works more effectively when it is used along with the edge minimization algorithm by variable grouping, and it reduces the number of edges by up to 30%. This results in reducing analysis time of MSSs by up to 30%.

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