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| **CSE 7350/5350**  **D.W. Matula** | **Homework Set # 3** | **Due date:**  **01 November 2011** |
|  | **Total Points: 60** |  |
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|  | **Greedy Paradigm Applications** |  |
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| **Grading Policy:**  ***Unless otherwise stated all problems are 10 points each. Partial credit is available on all problems. Homework submitted after the due date may incur a 10 point penalty on that homework set. (Distance students add one day to deadlines.)*** | | |

**1. [FIFO Scheduling](15 points)** Consider the greedy first-come first-serve coloring of interval graphs, where the n intervals all start and stop at integral times 0, 1, 2, …, t for t ≤ n. Intervals that overlap only at end points are *not* to be considered to yield edges in the interval graph. Describe an O (n) procedure for first building the *event list data structure* from the list of intervals assumed given by start and stop times. The event list is a temporally ordered sequence with each interval occurring twice. The first occurrence of each interval is determined by its start time and the second occurrence by its stop time. Then describe and analyze an implementation of the coloring procedure employing the event list data structure that operates in O(n) time and space that colors with a minimum number of colors. Provide a walk-through of your coloring algorithm for the graph with the event list **ZWXTZVYYXQWUVSUTRSRQ**.

**[7350 only]** Provide an algorithm determining the degrees of all vertices in O(n) time from the event list for the interval graph. Walkthrough your algorithm for the above graph (provide a drawing of the graph).

**Reference: Text Problem 16.1-4, p. 422**

**2. [Matching Substrings]** A matching substring pair of length k in a binary bit string b1, b2...bn is a pair bibi+1... bi+k-1 = bjbj+1...bj+k-1, with i not equal to j, of identical k-bit substrings.

(a) Determine a maximum length binary string with no matching substrings of length 4.   
(b) Describe the suffix tree algorithm for determining a longest matching substring pair and give its time and space complexity. Apply the algorithm to find the longest binary string match for the first 62 bits of

π = 3.11037 55242 10264 302158

**Reference: See notes on web page.**

**3. [Graph Degree Structures] (15 points)** Describe data and record structures for vertex ordering and vertex or edge coloring (or labeling) and a suitably greedy graph search algorithm to solve each of the following problems in the time bound indicated. Illustrate each algorithm with vertex or edge coloring (or labeling) on a graph or tree designed to teach your algorithm. The graph (or tree) should have at least 18 vertices and a maximum degree of at least 4. The graph should be connected with a minimum degree 3.

i) Find a maximum independent set in a tree in time O (|V|).

ii) Find a smallest last vertex ordering and a subgraph maximizing the minimum degree for your graph.

iii) Find some “k-connected” pair of vertices u, v in the graph having k = min {degree (u), degree (v)} edge disjoint distinctly colored (labeled) paths between u and v in time O (|V|+ |E|) using maximum adjacency search. Identify all the sets of pairwise k-edge-connected vertices using the "k-1 cycle" observation discussed in class.

Reference: For (ii) see web page Project Description for Exam Scheduling.   
                    For (iii) see web page Project Description for Maximum Adjacency  
                    Search and the example walkthrough.

**4. [GCD]** Find the greatest common divisor GCD (40902, 24140) by each of the following algorithms, showing the intermediate steps. Convert the input arguments to binary for parts (b) and (c).

1. Euclid’s Algorithm employing division in decimal and illustrating the successive division invariants x = q ∙ y + r.
2. Binary normalization shift-and-subtract.
3. Binary parity “right-normalize” and subtract.
4. “Ternary parity right-normalize” and subtract using decimal.

**5. [Division]** Divide the 16 digit value N by the six digit integer D obtaining the quotient Q and remainder (or sign of the remainder) R by the following division algorithms. Discuss your steps and method for obtaining the remainder (or remainder sign in (c)).

(a) Long division (show only six digits of the answer with partial remainder);

(b) Prescaled division (using a 4 digit reciprocal);

(c) Convergence division (using a 4 digit reciprocal).

N = 3579097531086357

D = 427583

Reference: See chapter 5 of Kornerup and Matula reference.