

CSE2353 - GRAPH THEORY OUTLINE

- What is a Graph?
- Special Types of Graphs
- Trees
- Fun Graph Problems
- Graphs in Computer Science

WHAT IS A GRAPH?

- An (Undirected) Graph G consists of:
 - A finite non-empty set V of Vertices
 - A finite set E of Edges
 - A function $\delta: E \rightarrow P(V)$ such that, for every edge e , $\sigma(e)$ is a one- or two-element subset of V .

GRAPH TERMS

- Adjacent - Vertices which are adjacent if they have an edge in common. Edges are adjacent if they have at least one vertex in common.
- Incident - Two vertices are incident to an edge if they are connected via that edge. In this case the edge is incident to the two vertices.
- Degree - The degree of a vertex is the number of edges which are incident to it.
- Subgraph - H is a subgraph of G if its vertices and edges are subsets of those in G .
- Edge Sequence - Sequence of edges such that any two consecutive ones are adjacent.
- Path - Edge sequence in which all edges are distinct
- Circuit - Closed (starting and ending vertex the same) simple (all vertices distinct) path with at least one edge.

GRAPH REPRESENTATIONS

- Suppose $G = \{V, E\}$ where $|V| = n$ and $|E| = m$
- Adjacency Matrix $n \times n$ matrix A where $A_{i,j}$ is the number of edges joining v_i and v_j . (Note if only one edge exists, the value may be a weight associated to that edge.)
- Incidence Matrix $m \times n$ binary matrix B where $B_{i,j}$ is 1 iff e_i is incident with v_j .

SPECIAL GRAPHS

- Simple - No loops or multiple edges
- Null - No edges.
- Complete K_n - Simple graph where every pair of distinct vertices is joined by an edge.
- Bipartite - Vertices can be partitioned such that every edge joins a vertex from one partition to that in another partition.
- Connected - Any pair of distinct vertices are connected via a path.

TREES

- Tree - Connected graph with no circuits
- Rooted Tree
 - Root
 - Leaves
 - Parent
 - Child
- Binary Tree - Every vertex has cardinality 2 or 3 (0 or 1 children)

DIRECTED GRAPHS

- Graph where edges have a direction
- Edges are sometimes called arcs

FUN GRAPH PROBLEMS

- Königsberg Bridge Problem
- Hamiltonian Circuit
- Stable Marriage Problem
- Four Color Conjecture

GRAPHS IN COMPUTER SCIENCE

- Graph Representation
 - Array
 - Linked List
 - Tree - One dimensional array
- Algorithms
- Applications
 - Indexing Data Structures (B^+ Tree)
 - Spelling Checkers (Trie)
 - Game Trees
 - Efficient Routing
 - Parsing
 - Sorting
 - Scheduling programs - Heap
 - Hierarchical and Network Databases
- NOTE - Graph proofs often use Induction