NAME SOLUTIONS

CSE 2353 Spring 2008

Test 3

May 5, 2008

Open Books. Open Notes. Show all Work.

1. (20 pts) If R is a partial order, is R^{-1}? Justify your answer. (Note: You can find the answer in the book, but that is not a justification.)

Reflexive: Since xRx then $xR^{-1}x$

i.e. $(x,x) \in R \Rightarrow (x,x) \in R^{-1}$

Transitive: $\ni xR^{-1}y$ and $yR^{-1}z$

$\therefore yRx$ and zRy

Since R is transitive zRx

$\therefore xR^{-1}z$

Antisymmetric: $\ni xR^{-1}y$ and $yR^{-1}x$

$\therefore yRx$ and xRy

Since R is antisymmetric

$y = x$

2. (20 pts) What is wrong with the following proof?

Thm: For all positive integers n, n=1.

Proof by Induction on n.

Basis: n=1

Hypothesis: Suppose k=1

Induction: Show k+1 = 1 if k=1.

Suppose k+1=1.

iff (k+1)(k-1) = (k-1)

iff k^2-1 = k-1

iff k^2 = 1

Since k=1 we know $1^2 = 1$

1) Hypothesis must be for $k > 1$

2) starts off assuming what you want to prove

3. (20 pts) Let A={2,3,4,5,6,7,8,9}. Define $f, g : A \to A$ by *f(x)*= smallest prime number that divides x and $g(x) = \frac{(x^2 + x)}{(x+1)}$.

   a. List elements in *f*.
   b. List elements in g.
   c. List elements in $g \circ f$.
   d. Is f a bijection? Justify your answer.

a) $f = \{(2,2), (3,3), (4,2), (5,5), (6,2), (7,7), (8,2), (9,3)\}$

b) $g = \{(2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9)\}$

c) $g \circ f = \{(2,2), (3,3), (4,2), (5,5), (6,2), (7,7), (8,2), (9,3)\} = f$

d) no - not onto; also not 1-1.

4. (20 pts) (Exercise 15 on p 63 in your text) Test the validity of the logical consequence. You must symbolize the statements using quantifiers, predicates, and any needed logical connectives. Justify your answer:

All men are mortal.

Randy is a man.

Therefore, Randy is mortal.

$M(x)$ x is a man

$O(x)$ x is mortal

Argument: $\forall x(M(x) \rightarrow O(x))$

$M(Randy)$

$\therefore O(Randy)$

Yes it is valid

1) $\forall x(M(x) \rightarrow O(x))$ Given

2) $M(Randy) \rightarrow O(Randy)$ US, 1

3) $M(Randy)$ Given

4) $O(Randy$ MP, 2, 3

5. (20 pts) Prove that $\sum_{i=1}^{n} i < \frac{(2n+3)^2}{7}$

Proof by Induction on n:

Basis $n=1$ $\quad \sum_{i=1}^{1} i = 1$

$$\frac{(2i+3)^2}{7} = \frac{(5)^2}{7} = \frac{25}{7}$$

Hypo: $\quad \sum_{i=1}^{k} i < \frac{(2k+3)^2}{7} \quad k > 1$

Induction: $\quad \sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + k+1$

$$< \frac{(2k+3)^2}{7} + (k+1)$$

$$= \frac{4k^2 + 12k + 9 + 7(k+1)}{7}$$

$$= \frac{4k^2 + 19k + 16}{7}$$

$$< \frac{4k^2 + 20k + 25}{7}$$

$$= \frac{4(k+1)^2 + 9 + 12(k+1)}{7}$$

$$(2(k+1)+3)^2$$