FIRST LEVEL

$$\Delta w_{ji} = -\eta \frac{\partial E}{\partial w_{ji}} = -\eta \frac{\partial E}{\partial y_i} \frac{\partial y_i}{\partial S_i} \frac{\partial S_i}{\partial w_{ji}}$$

$$\frac{\partial E}{\partial y_i} = \frac{\partial \left[\frac{1}{2} \sum_m (d_m - y_m)^2 \right]}{\partial y_i} = -(d_i - y_i)$$

Note: the partial of all other terms will be 0.

$$\frac{\partial y_i}{\partial S_i} = \frac{\partial \frac{1}{(1+e^{-S_i})}}{\partial S_i}$$

$$= -(1+e^{-S_i})^{-2}(-e^{-S_i})$$

$$= \left(\frac{e^{-S_i}}{1+e^{-S_i}}\right)\left(\frac{1}{1+e^{-S_i}}\right)$$

$$= (1-y_i)\, y_i$$

Note: $1 - y_i = 1 - \frac{1}{1+e^{-S_i}} = \frac{1+e^{-S_i}-1}{1+e^{-S_i}}$

$$\frac{\partial S_i}{\partial w_{ji}} = \frac{\partial \left[\sum_h w_{hi} x_{hi} \right]}{\partial w_{ji}} = x_{ji} = y_j$$

$$\therefore \Delta w_{ji} = (-\eta)(-d_i + y_i)(1-y_i)(y_i)(y_j)$$

SECOND LEVEL

$$\Delta w_{kj} = -\eta \frac{\partial E}{\partial w_{kj}}$$

$$= -\eta \sum_m \frac{\partial E}{\partial y_m} \frac{\partial y_m}{\partial s_m} \frac{\partial s_m}{\partial y_j} \frac{\partial y_j}{\partial s_j} \frac{\partial s_j}{\partial w_{kj}}$$

We already know

$$\frac{\partial E}{\partial y_m} = -(d_m - y_m)$$

$$\frac{\partial y_m}{\partial s_m} = (1 - y_m)\, y_m$$

$$\frac{\partial s_m}{\partial y_j} = \frac{\partial \left(\sum_h w_{hm}\, y_h \right)}{\partial y_j} = w_{jm}$$

$$\frac{\partial y_j}{\partial s_j} = \frac{\partial \left(\frac{1-e^{-s_j}}{1+e^{-s_j}} \right)}{\partial s_j}$$

$$= \frac{(1+e^{-s_j}) \frac{\partial (1-e^{-s_j})}{\partial s_j} - (1-e^{-s_j}) \frac{\partial (1+e^{-s_j})}{\partial s_j}}{(1+e^{-s_j})^2}$$

$$= \frac{(1+e^{-s_j})(e^{-s_j}) - (1-e^{-s_j})(-e^{-s_j})}{(1+e^{-s_j})^2}$$

$$= \frac{e^{-s_j}\left[1 + e^{-s_j} + 1 - e^{-s_j}\right]}{(1 + e^{-s_j})^2}$$

$$= \frac{2\, e^{-s_j}}{(1 + e^{-s_j})^2}$$

$$= \frac{(1 - y_j^2)}{2}$$

<u>Note:</u> $$\frac{(1 - y_j^2)}{2} = \frac{1 - \left(\frac{1 - e^{-s_j}}{1 + e^{-s_j}}\right)^2}{2}$$

$$= \frac{1 - \frac{(1 - e^{-s_j})^2}{(1 + e^{-s_j})^2}}{2} = \frac{(1 + e^{-s_j})^2 - (1 - e^{-s_j})^2}{2\,(1 + e^{-s_j})^2}$$

$$= \frac{(1 + 2e^{-s_j} + e^{-2s_j}) - (1 - 2e^{-s_j} + e^{-2s_j})}{2\,(1 + e^{-s_j})^2}$$

$$= \frac{4\, e^{-s_j}}{2(1 + e^{-s_j})^2} = \frac{2\, e^{-s_j}}{(1 + e^{-s_j})^2}$$

$$\frac{\partial s_j}{\partial w_{kj}} = \frac{\partial \left(\sum_h w_{hj}\, y_h\right)}{\partial w_{kj}} = y_k = x_{kj}$$

$$\therefore \; \Delta w_{kj} = \left(\eta \sum_m (d_m - y_m)(1 - y_m)\, y_m\, w_{jm} \right) \frac{(1 - y_2{}^2)}{2} \; y_k$$

Reference Figure: