CSE 3381 Digital Logic Design

- Instructor Background

- Syllabus:

`http://lyle.smu.edu/~mitch/class/3381`

- Electronic Notes Version MOST Current
- Class Schedule IMPORTANT
- Grading Policy

Class Grades: 40% Lab, 10% HW/Quiz,
15% Exam 1, 15% Exam 2,
20% Final

Teaching Method

- Electronic Notes – Summarize Highlights
  - Suggest you Print-out Before Class to Take Notes on
- Blackboard – Examples, Answer Questions
- Classroom Participation:
  - Interaction and Feedback (Pace of Class)
  - “Quiz” Points: Impromptu In-class Exercises
  - Variable Format
  - Please give me Feedback! I Care!

CSE 3381 Digital Logic Design

What is digital logic?

Generally a name given to electronic circuits that use discrete voltage (or current) levels and modeled by logic expressions. These circuits are the building blocks of computers and other devices.

What is digital logic design?

The study of systematic methods to produce circuits based on the digital logic model.

We will study:

- Number Systems and Logic
- Digital circuit analysis (Logic model)
- Digital circuit design (Logic model)

Number System

- Consists of **TWO** Things:
  - A **BASE** or **RADIX** Value
  - A **SET** of **DIGITS**
    - *Digits* are symbols representing all values ***less than*** the radix value.
- Example is the Common Decimal System:
  - RADIX (BASE) = 10
  - Digit Set = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Decimal Number System

EXAMPLE

- Consider: 5032.21
- Arithmetically, Shorthand for a *RADIX POLYNOMIAL*:

$$5\times(10^3)+0\times(10^2)+3\times(10^1)+2\times(10^0)+2\times(10^{-1})+1\times(10^{-2})$$
$$=5000+0+30+2+0.2+0.01$$

- Other Notation: $(5032.21)_{10}$

Other Number Systems

- Binary
  - Radix = $(2)_{10}=(10)_2$
  - Digit Set = $\{0,1\}$
- Octal
  - Radix = $(8)_{10}=(10)_8$
  - Digit Set = $\{0,1,2,3,4,5,6,7\}$
- Hexadecimal
  - Radix = $(16)_{10}=(10)_{16}$
  - Digit Set = $\{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F\}$

Binary Number System

Example

- Binary
  - Radix = $(2)_{10}$
  - Digit Set = $\{0,1\}$

$$
\begin{aligned}
(1101.01)_2 &= 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 + 0\times 2^{-1} + 1\times 2^{-2} \\
&= 1\times 2^3 + 1\times 2^2 + 1\times 2^0 + 1\times 2^{-2} \\
&= 2^3 + 2^2 + 2^0 + 2^{-2} \\
&= (8)_{10} + (4)_{10} + (1)_{10} + (1/4)_{10} \\
&= (13.25)_{10}
\end{aligned}
$$

Octal Number System

Example

- Octal
  - Radix = $(8)_{10}$
  - Digit Set = $\{0,1,2,3,4,5,6,7\}$

$$
\begin{aligned}
(15.2)_8 &= 1\times 8^1 + 5\times 8^0 + 2\times 8^{-1} \\
&= (8)_{10} + (5)_{10} + (2/8)_{10} \\
&= (13.25)_{10}
\end{aligned}
$$

Hexadecimal Number System

Example

- Hexadecimal
  - – Radix = $(16)_{10}$
  - – Digit Set = $\{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F\}$

$$\begin{aligned}(D.4)_{16} &= D\times 16^{0} + 4\times 16^{-1}\\ &= (13)_{10} + (4/16)_{10}\\ &= (13.25)_{10}\end{aligned}$$

Hexadecimal Digit Set

- Digit Set
  - – Set of *SYMBOLS*
  - – Each Symbol Represents Unique Quantity From:

    Zero Through β-1

    β is the *Base* or *Radix*

    β=16 for Hexadecimal

    β-1=$(15)_{10}$=$(F)_{16}$ for Hexadecimal
  - – Need Additional Symbols for 10 thru 15
  - – Convention is to Use Letters A thru F

Numbers with Different Bases

Decimal (radix 10)	Binary (radix 2)	Octal (radix 8)	Hexadecimal (radix 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	*A*
11	1011	13	*B*
12	1100	14	*C*
13	1101	15	*D*
14	1110	16	*E*
15	1111	17	*F*

General Number Systems

- Base-3 (Ternary)
  - Radix = $(3)_{10}$
  - Digit Set = $\{0,1,2\}$
- Base-4
  - Radix = $(4)_{10}$
  - Digit Set = $\{0,1,2,3\}$
- Base-7
  - Radix = $(7)_{10}$
  - Digit Set = $\{0,1,2,3,4,5,6\}$

Other Number System

Base-7 Example

$$(161.2)_7 = 1\times 7^2 + 6\times 7^1 + 1\times 7^0 + 2\times 7^{-1}$$
$$= 49 + 42 + 1 + 2/7$$
$$= (92.285714285714\ldots)_{10}$$

- Repeated Fraction in Decimal
- Exact Representation in Base-7
- Can Lead to *Round-off Errors* in Digital Circuits
- Example: $(21.3)_{10} = (10101.01\ 0011\ 0011\ \ldots)_2$

Other Number System

Base-3 Example

$$(10102.02)_3 = 1\times 3^4 + 1\times 3^2 + 2\times 3^0 + 2\times 3^{-2}$$
$$= 81 + 9 + 2 + 2/3$$
$$= (92.666666...)_{10}$$

Number System Conversions

- Can Convert the Integral Portion by Performing Repeated Division
  - Accumulate the Remainder Digits
- Can Convert the Fractional Portion by Performing Repeated Multiplication
  - Accumulate Integral Product Digits

$$\underbrace{12012}_{\text{Integral Portion}}.\underbrace{11201}_{\text{Fractional Portion}}$$

Radix Conversion Example

$X_I = 346_{10}$ $\beta_s=10$ $\beta_d=3$

Decimal to Ternary Integer Conversion

$$\begin{array}{r|l}
3 & \underline{346} \rightarrow 1 \\
3 & \underline{115} \rightarrow 1 \\
3 & \underline{38} \rightarrow 2 \\
3 & \underline{12} \rightarrow 0 \\
3 & \underline{4} \rightarrow 1 \\
 & 1 \rightarrow 1
\end{array}$$

remainders

$X_I = 110211_3$

Check by evaluating the radix polynomial

$$1\times 3^5 + 1\times 3^4 + 0\times 3^3 + 2\times 3^2 + 1\times 3^1 + 1\times 3^0$$

$$= [243+81+18+3+1]_{10} = 346_{10}$$

Radix Conversion Example

$X_I = 0.291_{10}$ β_s=10 β_d=5

Fixed-point Decimal to Pentary Fractional Conversion

$$\begin{aligned}
0.291 \times 5 = 1.455 &\rightarrow 1\\
0.455 \times 5 = 2.275 &\rightarrow 2\\
0.275 \times 5 = 1.375 &\rightarrow 1\\
0.375 \times 5 = 1.875 &\rightarrow 1\\
0.875 \times 5 = 4.375 &\rightarrow 4\\
0.375 \times 5 = 1.875 &\rightarrow 1\\
0.875 \times 5 = 4.375 &\rightarrow 4
\end{aligned}$$

$$0.291_{10} = (0.12114141414\ldots)_5$$

0.291_{10} is Finite Fraction for β_s=10, but infinite fraction for β_d=5

Shorthand Number System Conversions

- When Source Radix β_s is a Power of the Destination Radix β_d
  - Conversion is Easier
- Example:
  - Binary → Octal → Hexadecimal
  - $2^1 \rightarrow 2^3 \rightarrow 2^4$
  - Can Group Digits Together

Shorthand Number System Conversions - Example

$$
\begin{aligned}
(6C.4)_{16} &= 6\times16^{1}+C\times16^{0}+4\times16^{-1}\\
&= 6\times2^{4}+C\times2^{0}+4\times2^{-4}\\
&= (0110)_2\times2^{4}+(1100)_2\times2^{0}+(0100)_2\times2^{-4}\\
&= (0\times2^{3}+1\times2^{2}+1\times2^{1}+0\times2^{0})\times2^{4}\\
&\quad+(1\times2^{3}+1\times2^{2}+0\times2^{1}+0\times2^{0})\times2^{0}\\
&\quad+(0\times2^{3}+1\times2^{2}+0\times2^{1}+0\times2^{0})\times2^{-4}\\
&= (0\times2^{7}+1\times2^{6}+1\times2^{5}+0\times2^{4})\\
&\quad+(1\times2^{3}+1\times2^{2}+0\times2^{1}+0\times2^{0})\\
&\quad+(0\times2^{-1}+1\times2^{-2}+0\times2^{-3}+0\times2^{-4})\\
&= (01101100.0100)_2
\end{aligned}
$$

More Shorthand Examples

$(10110001101011.111100000110)_2$

For Octal:

Group Into Triplets of Bits
Starting From Radix Point

$(010\ \ 110\ \ 001\ \ 101\ \ 011\ \ .\ 111\ \ 100\ \ 000\ \ 110)_2$

$(2\ \ 6\ \ 1\ \ 5\ \ 3\ \ .\ \ 7\ \ 4\ \ 0\ \ 6)_8$

$(10110001101011.111100000110)_2 = (26153.7406)_8$

More Shorthand Examples

$$(1CF.23)_{16}$$

$$(0001\ 1100\ 1111\ .\ 0010\ 0011)_2$$

$$(000\ 111\ 001\ 111\ .\ 001\ 000\ 110)_2$$

$$(0\ 7\ 1\ 7\ .\ 1\ 0\ 6)_8$$

$(1CF.23)_{16} = (111001111.00100011)_2 = (717.106)_8$

Radix Conversion (integral)

Given a value X represented in source system with radix β_s, represent the same number in a destination system with radix β_d

Consider the integral part of the number, X_I, in the β_d system.

$$X_I = x_{k-1}\beta_d^{\,k-1} + x_{k-2}\beta_d^{\,k-2} + \cdots + x_1\beta_d^{\,1} + x_0\beta_d^{\,0}$$

$$= \{[(x_{k-1}\beta_d + x_{k-2})\beta_d + \cdots + x_2]\beta_d + x_1\}\beta_d + x_0$$

$$0 \le x_i < \beta_d$$

$$\frac{X_I}{\beta_d} = Q + R$$

$$Q = \{[(x_{k-1}\beta_d + x_{k-2})\beta_d + \cdots + x_2]\beta_d + x_1\}$$

$$R = x_0$$

R is the Desired digit (LSD) – Can Repeatedly Divide to Obtain Converted Value

Radix Conversion (fractional)

Consider the fractional part of the value in β_d Number system

$$X_F = x_{-1}\beta_d^{-1} + x_{-2}\beta_d^{-2} + \cdots + x_{-(m-1)}\beta_d^{-(m-1)} + x_{-m}\beta_d^{-m}$$
$$= \beta_d^{-1}\{x_{-1} + \beta_d^{-1}[+x_{-2} + \beta_d^{-1}(x_{-3} + \cdots)]\}$$
$$\beta_d \cdot X_F = P_I + P_F$$
$$P_I = x_{-1}$$
$$P_F = \beta_d^{-1}[+x_{-2} + \beta_d^{-1}(x_{-3} + \cdots)]$$

Thus, P_I is the Desired Digit
We can Repeatedly Multiply by the β_d Value