Stern-Gerlach Experiments

The Concept of Spin

Overview

- Experiment in 1922 by Otto Stern and Walther Gerlach
- Discovered Intrinsic Property of SPIN of Atoms and Particles
- Spin is a prototypical way to implement a Qubit
- Theorized to exist by Paul Dirac, experimentally shown to exist by Stern and Gerlach

Magnetic Dipole

- Classical Electrodynamics embodied in Maxwell's Equations
- Ampere's Law Describes Relationship between Electric Current in a Loop and Magnetic Field
- Magnetic Moment is a Vector Quantity given by:

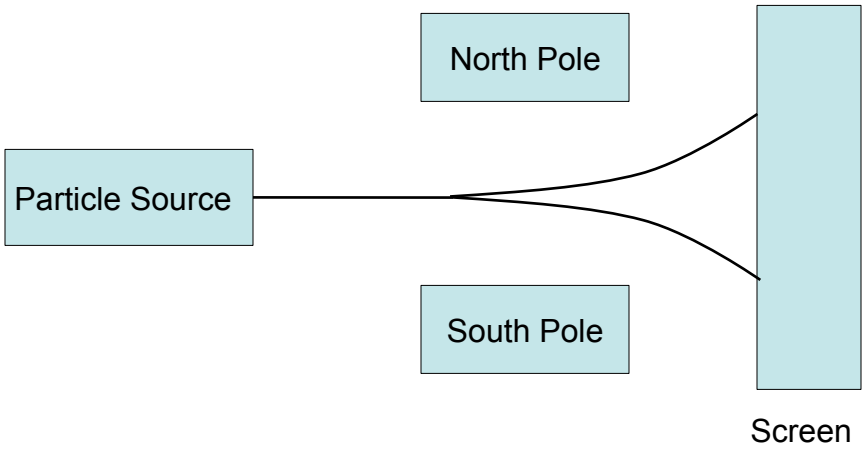
$$\overline{\mu} = I \int da$$

- Where I is a constant Current and da is the area of the loop
- Thought is that electron orbiting an atom nucleus would have a magnetic moment

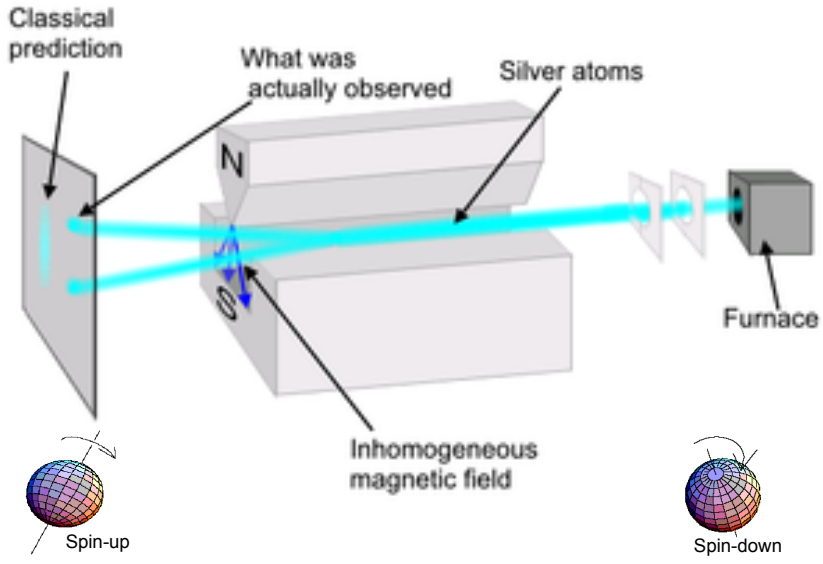
Magnetic Dipole

- Orbiting electron would be equivalent to a tiny Magnet Dipole (bar magnet)
- If orbiting electron passes through a Fixed Magnetic Field, should be Deflected in Direction based on its Orientation
- Hypothesis is that a Stream of Atoms would have Randomly Distributed Orientations of there Magnetic Dipoles
- After Passing Through Magnetic Field, Should have Random Exiting Angles

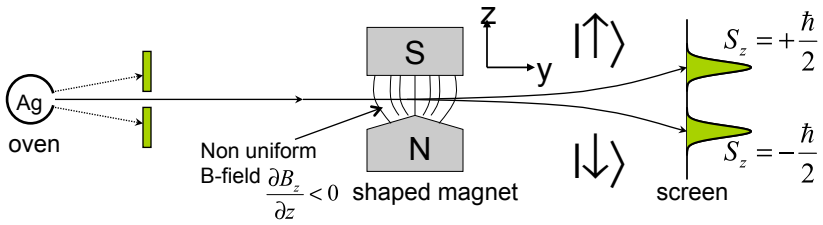
Simplified Experimental Setup



Experimental Setup



Experimental Setup



$$\vec{F} = -\nabla U = -\nabla(-\vec{\mu}\cdot\vec{B}) \quad \Rightarrow \quad F_z = \mu_z \frac{\partial B_z}{\partial z} = -\frac{e}{m_e c} S_z \frac{\partial B_z}{\partial z}$$

- Eigenvalues of Observable are S_z Values and often referred to as "spin values" of +½ and -½
- Eigenvectors are:

$|\uparrow\rangle$ $|\downarrow\rangle$

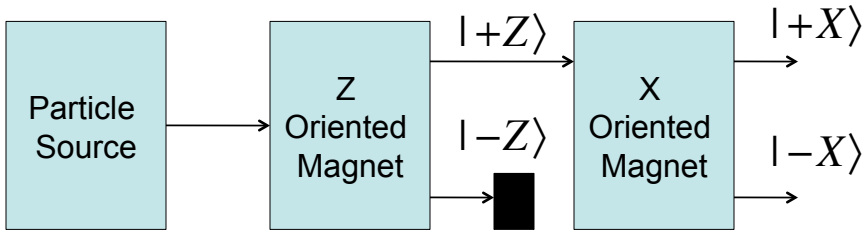
Experimental Results

- Electrons act Like Spinning Tops – but thought of a a point charge
- New Intrinsic Property Called Spin
- Expected Random Orientations Since Particles not "Aligned" before Injection into Magnetic Field
- Instead all Either went Upward or Downward
- Spin can thus be used as a Binary Basis
- Particle with Spin can be a Qubit

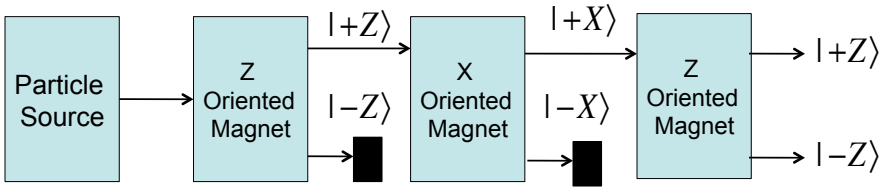
$$|\psi\rangle = \alpha_0 |\uparrow\rangle + \alpha_1 |\downarrow\rangle$$

Further Experiment

- First Experiment had Magnetic Flux Lines Oriented in direction of z-axis
- Next Block off the spin down particles and cascade another external magnet oriented vertically (in x-axis direction)



Even Further Experiment



- How can this happen?
- All Spin Down Particles (in z-direction) "trapped" at Output of First Magnet
- Can be Explained with Quantum Theory

Quantum Explanation of Spin

- Spin-up Particles in z-direction Contain Equal Numbers of Spin-up and Spin-down Particles in x-direction
- Spin-up Particles in x-direction Contain Equal Numbers of Spin-up and Spin-down Particles in z-direction
- Let:

$$|+Z\rangle \mapsto |0\rangle \qquad |-Z\rangle \mapsto |1\rangle$$

- Then Phenomena Explained by:

$$|+X\rangle \mapsto \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} \qquad |-X\rangle \mapsto \frac{(|0\rangle - |1\rangle)}{\sqrt{2}}$$

What is Projection Operator?

$$|+Z\rangle \mapsto |0\rangle \qquad |-Z\rangle \mapsto |1\rangle$$

$$|+X\rangle \mapsto \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} \qquad |-X\rangle \mapsto \frac{(|0\rangle - |1\rangle)}{\sqrt{2}}$$

Projection Operator

$$|\psi\rangle = \alpha_0 |+Z\rangle + \alpha_1 |-Z\rangle = \beta_0 |+X\rangle + \beta_1 |-X\rangle$$

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle = \beta_0 \frac{|0\rangle + |1\rangle}{\sqrt{2}} + \beta_1 \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle = \frac{\beta_0 + \beta_1}{\sqrt{2}} |0\rangle + \frac{\beta_0 - \beta_1}{\sqrt{2}} |1\rangle$$

$$\mathbf{P}|\psi\rangle = |\psi\rangle$$

$$\mathbf{P}\begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} \qquad \mathbf{P} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Projection Operator

$$|\psi\rangle = \alpha_0 |+Z\rangle + \alpha_1 |-Z\rangle = \beta_0 |+X\rangle + \beta_1 |-X\rangle$$

$$\mathbf{P}|\psi\rangle = |\psi\rangle \qquad \mathbf{P} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- This is a Hadamard Operator
- Quantum Logic Hadamard Gate
- Quantum Computing Hadamard Operation
- Application of Hadamard Gate/Operation using Particle Spin as Qubit