Variable Reordering for Shared Binary Decision Diagrams Using Output Probabilities*

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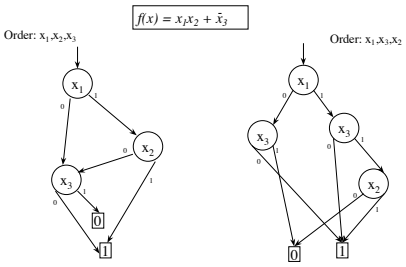
EFFECTS of VARIABLE ORDER

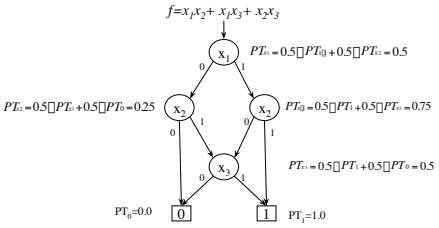


OUTPUT PROBABILITY DEFINITION

AND	$x_1 \bullet x_2$	$X_1 \square X_2$
OR	$x_1 + x_2$	$X_1 + X_2 - (X_1 \square X_2)$
XOR	$x_1 \square x_2$	$X_1 + X_2 - 2(X_1 \square X_2)$
NOT	$(x_1)'$	$1 - X_1$
idempotence	$x_1 \bullet x_1$	X_1

- Likelihood that a function will yield a *logical-1* value given the probability distributions of each of its inputs
- If all inputs have an equal probability of being a "1" or a "0", then it is the percentage of ones in the truth table.

EXAMPLE CALCULATION



Cir. Name	Inputs	Ouputs	SBDD size	depth	breadth
c432.isc	36	7	31172	0.221	0.948
c880.isc	60	26	10348	0.064	0.548
duke2.pla	22	29	973	0.007	0.095
misex2.pla	25	18	136	0.001	0.004
misex3.pla	14	14	1301	0.014	0.152
apex2.pla	39	3	7096	0.068	0.209

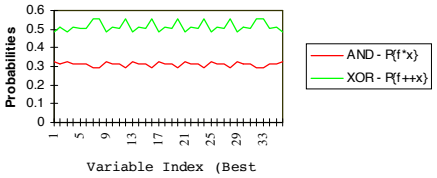
OBSERVATIONS

- Output Probabilities provide Global Information Regarding PIs and Function Behavior
- Good Orderings *Often* Exploit Symmetry
- Necessary Condition for Symmetry:

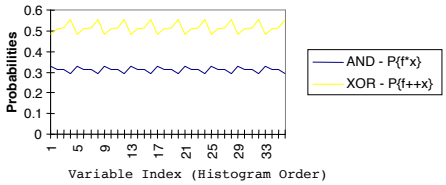
THEOREM: $P(f \square x_i) = P(f \square x_k)$ if $x_i, x_k \square S$, where S is the set of all independent variables that support f and $x_i \square x_k$ (that is, x_i and x_k are symmetric in f).

$$P(f \square x_i) = P(f \square x_j)$$

Probability Values for c432, output370gat

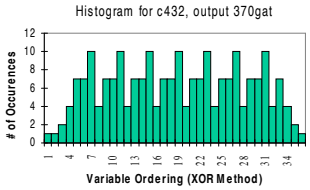


Probability Values for c432, output 370gat



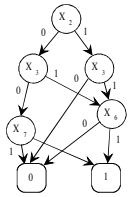
Histogram for c432, output 370gat

Histogram for c432, output 370gat



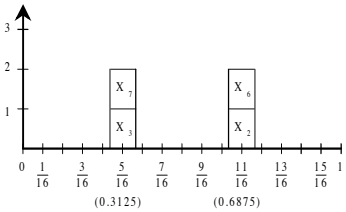
EXAMPLE (c17 output 23gat)

ORIGINAL BDD

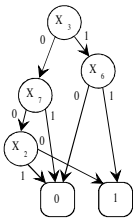


Probability Metrics

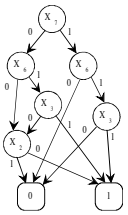
$\square\{f \square X_2\} = 0.6875$
$\square\{f \square X_3\} = 0.3125$
$\square\{f \square X_6\} = 0.6875$
$\square\{f \square X_7\} = 0.3125$



BIN-SIFTED BDD



HISTOGRAM DRIVEN BDD



Original Order: X_2, X_3, X_6, X_7

Histogram Order: X_7, X_6, X_3, X_2

Bin-Sift Order: X_3, X_6, X_7, X_2

BIN-SIFTING ALGORITHM

- No Criteria for Selecting PI from Histogram Bin
- Apply *SIFTING* for all PIs in Common Bin
- Series of *SIFTING* Operations Applied
  - Disjoint Subsets of PIs
  - Each PI in Subset has Same Probability Metric
  - Absolute Position (in terms of metric) Preserved
    - Periodicity Behavior Maintained

EXAMPLE - RESULTS SUMMARY

BDD SIZE

- Original Size: 5 Vertices
- Histogram Driven Size: 6 Vertices
- Bin-Sifted Size: 4 Vertices

COMPUTATION

- 4 Probability Computations: $O(n \square N)$
- Histogram Formation: $O(n)$
- Bin-Sifting: $O(N^2)$

Circuit Name	Original Size	Sifting Size	Sifting Time	Prob. Meth. Size	Prob. Meth. Time
5xp1	74	51	<1	42	<1
alu4	1197	800	<1	639	1
bw	108	103	<1	99	<1
duke2	973	394	<1	339	1
misex1	41	37	<1	37	<1
misex2	136	89	<1	84	<1
misex3	1301	704	<1	683	4
sao2	155	94	<1	87	<1
vg2	1044	335	<1	93	1
misex3c	828	504	<1	472	1
clip	226	87	<1	87	<1
e64	1441	231	1	231	2
apex1	28336	1356	3	1333	58
apex2	7096	708	1	573	76
apex4	928	895	<1	889	1
apex5	2679	1130	<1	1130	4

Circuit	Sift-Best	SA	EA	Prb. Meth.
c432	1210	1087	1064	1119
c499	25866	25866	25866	34432
c880	4083	4053	4053	4603
c1355	25866	25866	25866	34432
c1908	5708	5526	5526	6170
c2670	2008	-	1774	-
c3540	23828	23828	23828	38096
c5315	2104	-	1719	3447
c6822	-	-	-	-
c7552	3730	-	2212	-

Circuit Name	Original Size	Sifting Size	Sifting Time	Bin-sift Size	Bin-sift Time	Prob. Meth. Size	Prob. Meth. Time
c432	31172	20891	28	20891	216	1119	13
c499	53866	38196	84	36234	156	32338	277
c880	10348	5089	9	4603	20	4574	288
c1355	53866	38186	88	36234	159	32338	321
c1908	13934	8175	11	7762	23	6170	48
c3540	72858	41918	73	37644	1421	38096	257
c5315	747662	4078	225	2334	506	2314	1555

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