Space-Variant Optical Super-Resolution using Sinusoidal Illumination

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Abstract: The present work extends the scope of Optical Super-Resolution to imaging systems with spatially-varying blur, by using sinusoidal illumination. It also establishes that knowledge of the space-variant blur is not a pre-requisite for super-resolution.

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1. Introduction
Despite technical advances in imaging, the issue of overcoming the band-limited nature of an imaging system continues to remain a challenge. The term “optical super-resolution” (OSR) has been used in literature to describe techniques for overcoming the band-limited nature of an imaging system. While bulk of the innovation in OSR has focused on space-invariant imaging systems, we attempt to realize OSR in space-variant imaging systems, by building on developments in structured illumination optical super-resolution (abbreviated SI-OSR). The following paragraph reviews key developments in the area, and highlights differences between the present work and prior art.

Luksos & Marchand [1] were the first to recognize that modulation of spatial detail by a sinusoidal pattern produces replicas of the Fourier spectrum of the spatial detail about the frequency of the sinusoidal pattern, and in the process shifts spatial frequencies that lie outside the optical passband into the passband. This insight has been employed successfully in microscopy [2] to resolve features smaller than the wavelength of light. We established in previous work [3,4] that SI-OSR techniques developed in microscopy are not applicable to perspective imaging systems such as consumer cameras. In order to address the problem, we leveraged concepts from projective geometry. A unique aspect of our prior work is the ability to recover topographic information (scene geometry), in addition to super-resolving spatial detail in the scene. However, the scope of the work is limited to imaging systems whose point spread function (PSF) is not only space-invariant, but also unchanged over the illuminated volume.

The present work generalizes our previous finding, and extends the scope of our work to perspective imaging systems with spatially varying blur. Typical causes for the presence of space-variant blur include the finite nature of an imaging system’s depth-of-field, optical aberrations and/or other design limitations.

2. Principle Behind the Proposed Approach
For the sake of simplicity, we restrict our attention to incoherent imaging systems with spatially varying blur and monochromatic illumination. Following the notation of Lohmann & Paris [5], we identify the following linear relationship between the scene radiance and image intensity, namely

\[ i(x, y) = \int u \int v \{ s(u, v) r(u, v) \} h(x - u, y - v ; u, v) \, du \, dv. \tag{1} \]

where \( i(x, y) \) is the image intensity under sinusoidal illumination, \( s(u, v) \) is the sinusoidal illumination as perceived by the camera, \( r(u, v) \) is the scene reflectance or spatial detail and \( h(x - u, y - v ; u, v) \) is the spatially varying blur or PSF. As a first step towards realizing OSR, we illuminate the scene with a complex sinusoidal pattern of the form \( e^{-j2\pi(\xi x + \eta y)} \). This step shifts the Fourier spectrum of \( r(x, y) \) by \((\xi_0, \eta_0)\) prior to filtering. A portion of the shifted spatial frequencies survive optical blurring and afford us the opportunity to capture previously unresolved spatial detail. But, in order to truly realize OSR, it is necessary to restore the shifted spatial frequencies to their rightful position. It is not obvious that the current practice of demodulating the captured image \( i(x, y) \) with the conjugate pattern \( s^*(x, y) = e^{+j2\pi(\xi_0 x + \eta_0 y)} \) realizes this objective. The following paragraph takes a closer look at the problem.

In the special case that the scene is illuminated with the complex sinusoidal pattern \( s(x, y) = e^{-j2\pi(\xi_0 x + \eta_0 y)} \), the expression for the demodulated image \( s^*(x, y) i(x, y) \) reduces to the special form

\[ s^*(x, y) i(x, y) = \int u \int v \{ e^{j2\pi(\xi_0 (x-u) + \eta_0 (y-v))} h(x - u, y - v ; u, v) \} \, du \, dv, \tag{2} \]

where the term within the curly parentheses represents the PSF modulated by the complex sinusoid. Following the definition of the optical transfer function (OTF) of a space-variant imaging system [6], we find that the effect of modulating the spatially varying blur \( h(x, y ; u, v) \) by a complex sinusoid is that of shifting the passband of the spatially varying OTF \( H(\xi_0, \eta_0, u, v) \) by the carrier frequency \((\xi_0, \eta_0)\). Consequently, the demodulated image
$s^*(x,y)$ $i(x,y)$ contains spatial frequencies outside the optical passband. More importantly, the process of demodulation using the conjugate pattern, restores the modulated spatial frequencies to their rightful position. This behavior is evident in the first two rows of Figure 3. It may be readily observed that the imaging system is unable to resolve the sinusoidal target in certain areas of the image plane, owing to the space-variant blur. However, we are able to resolve the sinusoidal target in all areas of the super-resolved image, following complex sinusoidal modulation and demodulation.

The key insight provided by the above derivation is that the standard practice of demodulating the image captured under complex sinusoidal illumination by the conjugate illumination pattern, is also applicable to space-variant imaging systems. Consequently, the space-invariant SI-OSR strategy proposed in [3,4] and illustrated in Figure 1 can be used to realize OSR in space-variant imaging systems. A potential benefit of the strategy is that it permits us to realize SI-OSR without prior knowledge of the imaging system’s space-variant blur.

Due to the fact that the imaging chain involves incoherent illumination, we begin the process of super-resolution with the acquisition of four images $i_0(x,y)$ obtained under phase-shifted sinusoidal illumination

$$s_0(x,y) = A + A\sin(2\pi(\xi_0 x + \eta_0 y) + \theta),$$  

where $\theta$ sequentially takes on the four values of 0, $\pi/2$, $\pi$ and $3\pi/2$. The corresponding camera images $i_\theta(x,y)$ are then digitally recombined to yield images of the scene under complex sinusoidal illumination. The modulated spatial frequencies are restored to their rightful position, by multiplying the complex images $m^i(x, y)$ with the conjugate patterns $e^{-j2\pi(\xi_0 x + \eta_0 y)}$ and $e^{+j2\pi(\xi_0 x + \eta_0 y)}$ respectively. The final super-resolved image is then obtained by combining the demodulated images with the image captured by the camera under ambient illumination.

3. Proof-of-Concept Simulation

In an effort to validate the findings reported in Section 1, we simulate OSR in a space-variant imaging system afflicted with spherical aberration. Figure 2 visually illustrates the severity of the space variance in the image plane, and also provides the MTF’s at specific image locations. The on-axis/off-axis optical cutoff frequency was observed to be 200 lp/mm and 80 lp/mm respectively. For simulation purposes, we attempt to super-resolve images of a 130 lp/mm sinusoidal target, and an ISO 3334 resolution chart that contains fine spatial detail.

The process of OSR begins with the modulation of the scene (ground-truth image) by phase-shifted sinusoidal patterns at 65 lp/mm, strictly in the horizontal direction. The modulated images are subject to space-variant blurring prior to demodulation using the strategy outlined in Figure 1. The result of OSR is provided in Figure 3. The thumbnails underneath the bigger images provide a closer look at specific areas in the sinusoidal target and the ISO 3334 resolution chart. For the purpose of comparison, we also provide the result of digital sharpening in the second column of Figure 3. It is obvious that digital sharpening cannot recover the fine spatial detail lost to blurring. In contrast, SI-OSR is able to recover spatial detail lost to blurring, but only in the horizontal direction. The lack of resolution enhancement in the vertical direction is attributed to the absence of modulation in the vertical direction.

Careful examination of the super-resolved images reveals the presence of a vignetting like artifact, attributed to the absence of OTF compensation in the demodulation strategy of Figure 1. It is expected that OTF compensation will improve the appearance of the super-resolved image. Further examination of the super-resolved images reveals that the absolute gain in resolution is non-uniform, owing to the space-variant nature of the imaging system.
4. Summary

Current approaches to realizing OSR implicitly assume that the optical blur is space-invariant and possibly diffraction-limited. The present work has extended the scope of OSR to imaging systems with spatially-varying blur. It established that the SI-OSR strategy originally proposed in [3,4], is applicable to space-variant imaging systems. It also established that explicit knowledge of the space-variant blur is not a prerequisite for realizing SI-OSR.

It is expected that odd-order aberrations such as coma will induce spatially varying phase distortions in the captured illumination pattern, and require an improved demodulation strategy.

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5. References