Optimal Diversity Multiplexing Tradeoff Region in Asymmetric Multiple Access Channels

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***Abstract*— We consider a multiple access channel (MAC) with multiple antennas, where each user has different diversity and multiplexing gain requirement. For this configuration, we characterize the fundamental tradeoff region for each user. Specifically, we compute the maximum achievable diversity gain for a user in a MAC, given an achievable point in the multiplexing gain region.**

I. Introduction

Communication innovations aspire for increased reliability and speed. Multiple antennas at the transmitter and receiver operating over a wireless medium have been shown to improve both. The improvements in reliability and speed quantified using diversity and multiplexing gains occur with a fundamental tradeoff. This tradeoff has been characterized for an i.i.d. quasi-static Rayleigh fading multiple-input, multiple-output (MIMO) channel [1]. Specifically, the maximum achievable diversity gain, $d^*_{m,n}(r)$, at a given multiplexing gain, r, for a MIMO system with m transmit and n receive antennas, is given by,

$$d^*_{m,n}(r) = (m-r)(n-r), \tag{1}$$

for all coherence times, $t_c \geq m+n-1$. Recently, the diversity multiplexing (D-MG) tradeoff was extended to non i.i.d. Rayleigh and other fading distributions in [2]. In a multiuser scenario like the MIMO multiple access channels (MAC), in addition to diversity and multiplexing gains, multiple access gain is observed. In an i.i.d Rayleigh channel, when all users have the same number of transmit antenna, the combined effect from all users at the receiver is no different from a point-to-point MIMO link. Thus user interference is exploited as multiple access gain which is defined as the ability of the receiver's antennas to spatially separate signals from different users. For a MAC with symmetric diversity and rate requirements, [3] characterizes the tradeoff between the three types of gains. In this paper, we focus on a similar tradeoff for a MIMO MAC with asymmetric requirements.

In this paper, we consider a MIMO MAC where each user has different number of transmit antennas. This asymmetry translates to a situation where all users have different maximum achievable diversity and multiplexing gains. In this paper, we derive the optimal D-MG tradeoff region for each user in an asymmetric MAC. This tradeoff is achieved using i.i.d Gaussian codebooks(uniform superposition coding) for all users and a typical set decoder. In particular,

- The tradeoff region establishes conclusively, that each user in an asymmetric MAC can achieve its maximum diversity simultaneously with all other users and is not limited by the user with the least number of transmit antennas. This result is an improvement over [3], where authors assume a common diversity requirement for all users in the MAC.
- In an asymmetric MAC, the received signals from all users pass through independent but not identical Rayleigh channels. But, in light of a recent result in [2] for non i.i.d Rayleigh channels, we show that multiple access gain is guaranteed for all users in an asymmetric MAC.

Additionally, we find the maximum achievable rate region for an asymmetric MAC, given a set of diversity requirements for all users. A achievable rate-tuple for a set of diversity gains is fully defined by the user with the minimum diversity requirement. A bound on the achievable diversity region for a given set of multiplexing gains was found for a two user asymmetric MAC in [4]. It was shown, using a joint ML receiver and superposition coding, the users' diversity gains are not independent of each other and thus multiple access gains are not exploited.

The rest of the paper is structured as follows. Section II formally introduces the system model and the problem being studied. Our main result which characterizes the maximum diversity gain as a function of achievable multiplexing gain-tuples is in Section III. The derived tradeoff is illustrated in Section IV followed by conclusions in Section V.

Notation: Uppercase boldface letters represent matrices. $\mathbf{H}^\dagger$ represents the hermitian transpose of matrix $\mathbf{H}$. $\mathbb{C}\mathcal{N}(0,1)$ denotes a complex Gaussian random variable with zero mean and unit variance. The notation, $\triangleq$ is read, "is defined as" and the notation, $\doteq$, in $f(\beta) \doteq \beta^a$ is used to denote exponential equality [1]. $\dot{\leq}, \dot{\geq}$ are similarly defined.

II. System Model and Problem formulation

Consider a multiple access MIMO channel with K users as shown in Fig. 1. The wireless link between user i and the base station is described by a single flat fading path between any of its m_i transmit and n receive antennas. The channel gains between all pairs of transmit and receive antennas for the i^{th} user are lumped in a $n \times m_i$ -dimensional channel matrix, $\mathbf{H}_i$, whose elements are assumed to be i.i.d. $\mathbb{C}\mathcal{N}(0,1)$. Further, the channel is assumed to be block fading in which the entries in the channel matrix may change independently after coherence

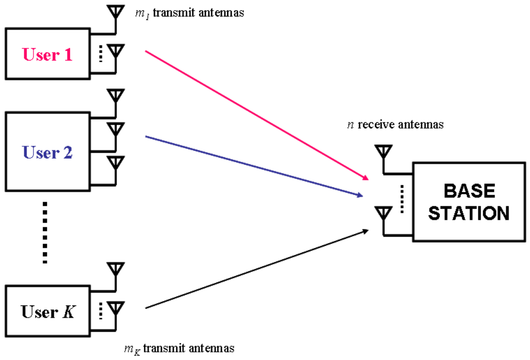


Fig. 1. A MAC with K users with each user having different number of transmit antennas

time, t_c symbols. Let $\mathbf{X}_i$ be the $m_i \times t_c$ -dimensional complex transmit matrix for the i^{th} user. Then, the $n \times t_c$ -dimensional received matrix, $\mathbf{Y}$, at the base station is given by,

$$\mathbf{Y} = \sum_{i=1}^{K} \sqrt{\frac{\mathsf{SNR}}{m_i}} \mathbf{H}_i \mathbf{X}_i + \mathbf{Z} \tag{2}$$

where $\mathbf{Z}$ is the $n \times t_c$ -dimensional additive white Gaussian noise - $\mathbb{C}\mathcal{N}(0,1)$. SNR is the average signal to noise ratio (SNR) at the each receive antenna in the base station. Since the noise is assumed to have unit variance, SNR is indicative of the total transmit power. Since, each user in the MAC can have different number of transmit antennas, we will refer to this channel as the asymmetric MAC in the sequel.

Let the users in the MAC communicate over a fixed block of t_c symbols. A SNR-indexed codebook for user i, denoted by $\mathcal{C}_i\{\mathsf{SNR}\}$ is made up of $2^{t_c R_i}$ codewords, where R_i denotes its rate of communication. The $m \times t_c$ dimensional codewords are denoted as $\{\mathbf{X}_j^{(i)}, j = 1, \ldots 2^{t_c R_i}\}$. We wish to characterize the maximum achievable diversity d_k^* for any user, $k = 1, \ldots, K$, i.e.,

$$P_e^{(k)}(\mathsf{SNR}) \doteq \mathsf{SNR}^{-d_k^*} \tag{3}$$

for a given set of the K-tuple spatial multiplexing gain $(r_1, \ldots, r_K)$, in the achievable rate region of the MAC, where,

$$r_i = \lim_{\mathsf{SNR} \to \infty} \frac{R_i(\mathsf{SNR})}{\log \mathsf{SNR}}. \tag{4}$$

For each rate-tuple, define $d_k^*(r_1, \ldots, r_K)$ as the supremum of the diversity gains for the k^{th} user over all coding schemes.

III. Optimal Diversity Multiplexing Tradeoff of Asymmetric MAC

Theorem: Consider a MAC with K users, where the i^{th} user has m_i transmit antennas and the base station has n receive antennas. If the block length $t_c \geq (n-1) + \sum_{i=1}^{K} m_i$, the optimal diversity-multiplexing tradeoff for any user k in the MAC, given a achievable rate vector, $(r_1, \ldots, r_K)$, is described by

$$d_k^*(r_1, \ldots, r_K) = \min_{S_k} d^*_{\sum_{i \in S_k} m_i, n} \left(\sum_{i \in S_k} r_i \right) \tag{5}$$

where $S_k = \left\{ \{k\} \cup \tilde{S}_k, \ \forall \tilde{S}_k \subseteq \{1, \ldots, k-1, k+1, \ldots, K\} \right\}$ is the subset of users containing user k and $d^*_{m,n}(r)$ is the maximum diversity for a point-to-point channel with m transmit, n receive antennas and a multiplexing gain r defined in (1).

Proof: The proof involves the computation of the probability of error that is asymptotic in SNR. The dominant error event for block length, $t_c \geq (n-1) + \sum_{i=1}^{K} m_i$ occurs when the channel is in outage i.e., the target rate-tuple does not lie in the multiple access region defined by the realized channel matrices $\{\mathbf{H}_i\}_i$. Following similar arguments as in [1], the error probability for the k^{th} user can be bounded as,

$$P^{(k)}(\mathcal{O}) \leq P_e^{(k)}(\mathsf{SNR}) \leq P^{(k)}(\mathcal{O}) + P_e^{(k)}(\text{no outage}), \tag{6}$$

where $P^{(k)}(\mathcal{O})$ is the outage probability and $P_e^{(k)}(\text{no outage})$ is the error probability under no outage for user k. We first compute the outage probability and show using a random coding argument that conditioned on no-outage, the SNR exponent of the error probability is no worse than the probability of outage for all block lengths, $t_c \geq (n-1) + \sum_{i=1}^{K} m_i$.

A. Outage Formulation

Definition (Outage Event [3]): For an asymmetric MAC with K users, the outage event is

$$\mathcal{O} \triangleq \bigcup_{S} \mathcal{O}_S, \tag{7}$$

where the union is taken over all subsets $S \subseteq \{1, \ldots, K\}$. The outage event $\mathcal{O}_S$ represents the event that outage occurs for a subset of S users and is defined as,

$$\mathcal{O}_S \triangleq \left\{ H \in \mathbb{C}^{n \times M} : I\left(\mathbf{X}_S; \mathbf{Y} | \mathbf{X}_{S^c}, \mathbf{H} = H\right) < \sum_{i \in S} R_i \right\} \tag{8}$$

where $\mathbf{H} = [\mathbf{H}_1 \ \mathbf{H}_2 \ \ldots \ \mathbf{H}_K]$ is the concatenated channel matrix and $\mathbf{X}_s = \left[\mathbf{X}_1^\dagger \ \ldots \ \mathbf{X}_{|S|}^\dagger \right]^\dagger$ contains the input signals from the users in S and $M = \sum_{i=1}^{K} m_i$.

To compute $\mathcal{O}_S$, we assume that the receiver has the knowledge of the correct data symbols $\mathbf{X}_{S^c}$ transmitted by users in S^c and cancels the contribution of $\mathbf{X}_{S^c}$ from the received signal. Then, for the remaining $|S|$ users, outage occurs when the mutual information is smaller than the desired sum rate. The behavior of the outage probability for the asymmetric MAC is summarized in the following Lemma.

Lemma: For an asymmetric MAC with K users, where the i^{th} user is equipped with m_i transmit antennas, and the base station with n receive antennas, let the data rate of user i be $R_i = r_i \log \mathsf{SNR}$, for $i \in \{1, \ldots, K\}$. The probability of error for user k with any coding scheme is lower bounded as,

$$P_e^{(k)}(\mathsf{SNR}) \dot{\geq} P^{(k)}(\mathcal{O}) \doteq \mathsf{SNR}^{-d_k^*(r_1, \ldots, r_K)}, \tag{9}$$

where $d_k^*(r_1, \ldots, r_K)$ is defined in (5).

Proof: Since the receiver cancels the contribution of the data $\mathbf{X}_{S_k^c}$ from the received signals given in (2), the channel can now be rewritten as,

$$\mathbf{Y}_{S_k} = \sqrt{\mathsf{SNR}} \sum_{i \in S_k} \sqrt{\frac{1}{m_i}} \mathbf{H}_i \mathbf{X}_i + \mathbf{Z}, \quad (10)$$

$$= \sqrt{\mathsf{SNR}} \mathbf{H}_{S_k} \mathbf{X}_{S_k} + \mathbf{Z}, \quad (11)$$

where $\mathbf{H}_{S_k} \in \mathbb{C}^{n \times \sum_{i \in S_k} m_i}$ is the concatenated channel matrix of users in S_k with entries $\left\{ \sqrt{\frac{1}{m_i}} \mathbf{H}_i \right\}_{i \in S_k}$. Thus, the problem is now reduced to a point-to-point problem with $M_{S_k} = \sum_{i \in S_k} m_i$ transmit antennas and n receive antennas, and a channel matrix $\mathbf{H}_{S_k}$ whose entries belong to an independent but no longer identical complex Gaussian distribution. Let the target data rate of user i be $R_i = r_i \log \mathsf{SNR}$ for $i \in \{1, \ldots, K\}$. Then, the SNR exponent of the outage probability for such non-i.i.d. Gaussian channel is shown in [2] to be,

$$P(\mathcal{O}_{S_k}) = Pr\left[I\left(\mathbf{X}_{S_k}; \mathbf{Y} | \mathbf{X}_{S_k^c}, \mathbf{H} = H\right) < \sum_{i \in S_k} R_i \right]$$

$$\overset{(a)}{\doteq} Pr\left[\det(I + \mathsf{SNR} \mathbf{H}_{S_k} \mathbf{H}_{S_k}^{\dagger}) < \mathsf{SNR}^{\sum_{i \in S_k} r_i} \right]$$

$$\doteq \mathsf{SNR}^{-d^*_{M_{S_k}, n}\left(\sum_{i \in S_k} r_i\right)}, \quad (12)$$

where (a) is derived by choosing the input $\{\mathbf{X}_i\}_i$ of all users to be i.i.d Gaussian distribution - $\mathbb{C}\mathcal{N}(0,1)$ which is shown to minimize the $P(\mathcal{O}_{S_k})$ for all S_k simultaneously. Thus, even when the entries of the point-to-point link are no longer identically Gaussian, the achievable diversity for a given multiplexing gain is no different than when the entries are i.i.d. Gaussian in the high SNR regime and the multiple access gain is guaranteed for all users.

Hence, the total outage probability [3] can be shown to be,

$$P^{(k)}(\mathcal{O}) = P\left(\bigcup_{S_k} \mathcal{O}_{S_k} \right) \leq \sum_{S_k} P(\mathcal{O}_{S_k}) \doteq P(O_{S_k^*}), \quad (13)$$

where $S_k^* = \arg\min_{S_k} d^*_{M_{S_k}, n}\left(\sum_{i \in S_k} r_i \right)$ is the subset with the slowest decay rate of $P(\mathcal{O}_{S_k})$. Combining (13) with the fact that $P^{(k)}(\mathcal{O}) \geq P(\mathcal{O}_{S_k^*})$, we have

$$P^{(k)}(\mathcal{O}) \doteq P(\mathcal{O}_{S_k^*}) \doteq \mathsf{SNR}^{-\min_{S_k} d^*_{M_{S_k}, n}(\sum_{i \in S_k} r_i)}. \quad (14)$$

Combining (14) with (6), we see that the outage probability lower bounds the error probability and the result follows. ■

B. Achievability

To complete the proof of the theorem, we need to show that the lower bound in (9) is tight in the limit $\mathsf{SNR} \to \infty$, for block length $t_c \geq (n-1) + \sum_{i=1}^{K} m_i$. We now show that for any rate K-tuple, $(r_1, \ldots, r_K)$, there exists a coding scheme that achieves the diversity $d_k^*(r_1, \ldots, r_K)$ for user k.

Consider the ensemble of i.i.d. complex Gaussian random codes where the i^{th} user generates a SNR indexed codebook $\mathcal{C}_i\{\mathsf{SNR}\}$ containing $2^{t_c R_i}$ independent codewords, denoted as $\mathbf{X}_1^{(i)}, \ldots, \mathbf{X}_{2^{t_c R_i}}^{(i)}$. Each codeword is a $m \times t_c$ matrix with i.i.d $\mathbb{C}\mathcal{N}(0,1)$ entries. These codewords are revealed to the senders and the receiver. In each coherence time, t_c, the transmitted signal of user i is chosen equiprobably from the codebook $\mathcal{C}_i\{\mathsf{SNR}\}$.

Let $A_\epsilon^{t_c}$ denote the set of typical $(\mathbf{X}^{(1)}, \ldots, \mathbf{X}^{(K)}, \mathbf{Y})$ sequences. The decoder is a typical set decoder [5], where the decoding function chooses a K-tuple, $\left(a_1, \ldots, a_K : a_i \in \{1, 2^{t_c R_i}\}\right)$, such that

$$\left(\mathbf{X}_{a_1}^{(1)}, \mathbf{X}_{a_2}^{(2)}, \ldots, \mathbf{X}_{a_K}^{(K)}, \mathbf{Y} \right) \in A_\epsilon^{t_c}. \quad (15)$$

Due to inherent symmetry in the random codeword construction, the probability of error does not depend on which K-tuple was sent. Therefore, without loss of generality, we can assume that all users transmit their first codeword i.e., $(a_1, \ldots, a_K) = (1, \ldots, 1)$.

There are two error events that determine the probability of error when the channel is not in outage: i) when the correct codewords are not typical with the received sequence and ii) when incorrect codewords are jointly typical with the received sequence i.e., an *undetected* error. Define the events,

$$\mathcal{E}_{(a_1, \ldots, a_K)} = \left\{ \left(\mathbf{X}_{a_1}^{(1)}, \mathbf{X}_{a_2}^{(2)}, \ldots, \mathbf{X}_{a_K}^{(K)}, \mathbf{Y} \right) \in A_\epsilon^{t_c} \right\}. \quad (16)$$

Then, by the union bound, the probability that the k^{th} user is in error is given by,

$$P_e^{(k)}(\text{no outage}) \leq P\left(\mathcal{E}^c_{(1, \ldots, 1)} \right) + P\left(\bigcup_{S_k} \mathcal{U}_{S_k} \right) \quad (17)$$

where $\mathcal{U}_{S_k}$, the undetected error event involving user k is defined as,

$$\mathcal{U}_{S_k} = \left\{ \mathcal{E}_{(a_1, \ldots, a_K)} : a_i \neq 1 \forall i \in S_k \right\}. \quad (18)$$

From the asymptotic equipartition property, $P(\mathcal{E}^c_{1, \ldots, 1}) \to 0$. Further, from [5], we know that the probability of undetected error can be simplified as,

$$P\left(\bigcup_{S_k} \mathcal{U}_{S_k} \right) \leq \sum_{S_k} P(\mathcal{U}_{S_k})$$

$$= \sum_{S_k} 2^{t_c \sum_{i \in S_k} R_i} 2^{-t_c (I(\mathbf{X}_{S_k}; \mathbf{Y} | \mathbf{X}_{S_k^c}, \mathbf{H}_{S_k}) - \epsilon)} \quad (19)$$

The term $2^{t_c \epsilon}$ in (19) does not contribute to the SNR exponent and is ignored. The mutual information in (19) is expressed as,

$$I(\mathbf{X}_{S_k}; \mathbf{Y} | \mathbf{X}_{S_k^c}, \mathbf{H}_{S_k}) = \log\det\left(I + \mathsf{SNR} \mathbf{H}_{S_k} \mathbf{H}_{S_k}^{\dagger} \right). \quad (20)$$

Substituting (20) in (19) and recalling that $\sum_{i \in S_k} R_i = \sum_{i \in S_k} r_i \log \mathsf{SNR}$, the probability of undetected error in (19) simplifies to,

$$P\left(\bigcup_{S_k} \mathcal{U}_{S_k} \right) \leq \sum_{S_k} \mathsf{SNR}^{t_c \sum_{i \in S_k} r_i} \det\left(I + \mathsf{SNR} \mathbf{H}_{S_k} \mathbf{H}_{S_k}^{\dagger} \right)^{-t_c}. \quad (21)$$

Since the codewords of all users are i.i.d. $\mathbb{C}\mathcal{N}(0,1)$, for each subset, S_k, the term inside the summation in (21) is equivalent to computing the probability of error of a point-to-point link with $M_{S_k} = \sum_{i \in S_k} m_i$ transmit and n receive antennas and a overall data rate of $\sum_{i \in S_k} r_i \log \mathsf{SNR}$. Also, notice that each term inside the summation in (21) is the same as error expression for a point-to-point channel derived for i.i.d. Rayleigh channel (equation (19) in [1]). The result in [1] was based on the conditional pairwise error probability averaged over the ensemble of Gaussian codewords. However, we arrive at the same expression using the standard error analysis of a typical set decoder, the key difference in the current problem being the nature of the concatenated channel matrix $\mathbf{H}_{S_k}$, which is made up of independent but no longer identical Gaussian entries because of the asymmetry in the MAC. However, [2] shows that even for non-identical point-to-point complex Rayleigh channels, the SNR exponent that in the high SNR regime, is the same as that for a i.i.d. Rayleigh channel. Therefore, for $t_c \geq (n-1) + \sum_{i=1}^{K} m$, the probability of error for the k^{th} user in a MAC averaged over the Gaussian code ensemble and the no outage region of the channel statistics, evaluates to

$$P_e^{(k)}(\text{no outage}) \leq P\left(\bigcup_{S_k} \mathcal{U}_{S_k}\right) \dot{\leq} \sum_{S_k} \mathsf{SNR}^{-d^*_{M_{S_k},n}(\sum_{i \in S_k} r_i)} \doteq \mathsf{SNR}^{-\min_{S_k} d^*_{M_{S_k},n}(\sum_{i \in S_k} r_i)} \quad (22)$$

Equations (5), (9) and (22) can be combined to derive the error probability for the k^{th} user as,

$$P_e^{(k)}(\mathsf{SNR}) \doteq \mathsf{SNR}^{-\min_{S_k} d^*_{M_{S_k},n}\left(\sum_{i \in S_k} r_i\right)}. \quad (23)$$

This completes the proof of our main result.

A natural extension is to find the multiplexing-diversity region i.e., given the achievable diversity gain $\{d_i\}_i$ for the K users in a MAC, we wish to compute the set of K-tuple of multiplexing gains, $(r_1, \ldots, r_K)$, that can be achieved. This set of multiplexing gains is denoted $\mathcal{R}(d_1, \ldots, d_K)$, and is explicitly characterized in the following Corollary.

Corollary: For an asymmetric MAC, if block length $t_c \geq (n-1) + \sum_{i=1}^{K} m_i$,

$$\mathcal{R}(d_1, \ldots, d_K) = \Big\{(r_1, r_2, \ldots, r_K) : \sum_{s \in S} r_s \leq r^*_{\sum_{i \in S} m_i, n}(\min_{s \in S} d^*_s), \forall S \subseteq \{1, \ldots, K\}\Big\} \quad (24)$$

where $r^*_{m,n}(.)$ is the multiplexing-diversity tradeoff curve for a point-to-point channel with m transmit and n receive antennas.

Proof: From the proof of the Theorem, the SNR exponent of the probability of error for user s is given by (23). By the union bound of error events, the probability of error for a subset of users S is given by,

$$P_e^{(S)}(\mathsf{SNR}) \dot{\leq} \sum_{s \in S} P_e^{(s)}(\mathsf{SNR}) \doteq \mathsf{SNR}^{-\min_{s \in S} d^*_s(r_1, \ldots, r_K)}, \quad (25)$$

where $d^*_s(r_1, \ldots, r_K)$ is defined in (5). Now, using the same point-to-point MIMO analogy as before, the probability that $S \subseteq \{1, \ldots, K\}$ users in error is given by,

$$P_e^{(S)}(\mathsf{SNR}) \doteq \mathsf{SNR}^{-d^*_{\sum_{s \in S} m_s, n}(\sum_{s \in S} r_s)}. \quad (26)$$

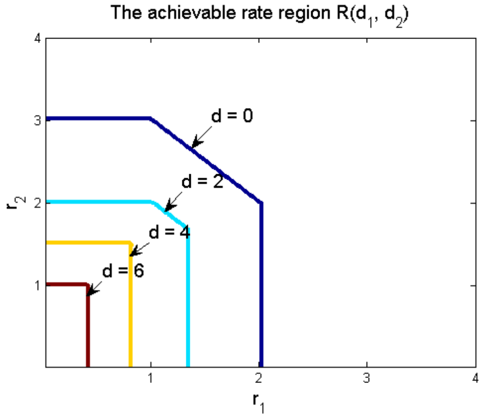


Fig. 3. The achievable rate region, $\mathcal{R}(d_1, d_2)$, defined in (24) for a asymmetric 2 user MAC where users have $m_1 = 2$, $m_2 = 3$ transmit antennas each and $n = 4$ antennas at the reciever. d in the figure is defined as $d = \min\{d_1, d_2\}$.

Comparing the SNR exponents of (25) and (26), we infer that the diversity requirement for each user is met when, $d^*_{\sum_{s \in S} m_s, n}(\sum_{s \in S} r_s) \geq \min_{s \in S} d^*_s$, or equivalently,

$$\sum_{s \in S} r_s \leq r^*_{\sum_{s \in S} m_s, n}\left(\min_{s \in S} d^*_s\right),$$

for all $S \subseteq \{1, \ldots, K\}$. ■

IV. Illustrations

Consider an asymmetric MAC with two users having $m_1 = 2$ and $m_2 = 3$ transmit antennas respectively. Let, the base station have $n = 4$ receive antennas. The achievable rate region [6] is described by the constraints,

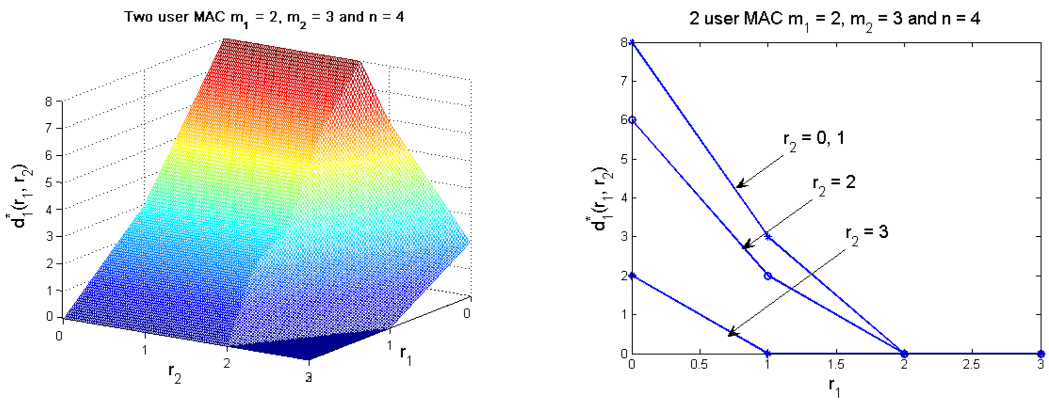
$$\{(r_1, r_2) : r_1 \leq 2, r_2 \leq 3 \text{ and } r_1 + r_2 \leq 4\}, \quad (27)$$

where $r_i = \min\{m_i, n\}$. Thus, given any rate pair belonging to (27), the maximum achievable diversity for user 1, denoted $d^*_1(r_1, r_2)$ is computed using (5) as,

$$d^*_1(r_1, r_2) = \min\left\{d^*_{m_1,n}(r_1), d^*_{m_1+m_2,n}(r_1 + r_2)\right\}. \quad (28)$$

This maximal diversity region is plotted for different rate pairs (r_1, r_2) in Fig. 2(a). Accompanying the diversity region is a two dimensional diversity-multiplexing tradeoff plot in Fig 2(b) which is derived by slicing the diversity region in the d^*_1-r_1 plane at values of $r_2 = \{0, 1, 2, 3\}$. From the figures, we can see that for $r_2 \in (0, 1)$, user 1 attains single user tradeoff i.e., $d^*_1(r_1, r_2) = d^*_{m_1,n}(r_1)$. As r_2 increases beyond one, the maximum achievable diversity-multiplexing tradeoff for user 1 reduces from the single user tradeoff curve, as seen in Fig. 2(a). The maximal diversity region for user 2, $d^*_2(r_1, r_2)$ can be similarly described.

Now, we look at the rate region described in the Corollary. Given each user's diversity requirement, we plot in Fig. 3, the achievable rate region $\mathcal{R}(d_1, d_2)$ given in (24) for the 2 user asymmetric MAC under consideration. Each rate pair (r_1, r_2) in the region has an associated minimum diversity requirement, $d = min\{d_1, d_2\}$. A diversity of $d = 0$ translates to a set



(a) (b)

Fig. 2. (a) The diversity-multiplexing tradeoff region $d_1^*(r_1, r_2)$ for user 1 in a 2 user MAC (b) $d_1^*(r_1, r_2)$ as a function of r_1 for fixed r_2.

of rate pairs that describe the maximum achievable capacity points from (27) and is shown in Fig. 3. Each rate pair in the countour, $d = 0$, describes the user whose diversity has to be zero. For example, to achieve rate pairs, $(r_1 = 2, r_2 \leq 2)$, user 1's diversity $d_1 = 0$ and the associated diversity pairs are described by $(d_1 = 0, d_2 \geq 0)$. Similarly, other diversity pairs can be described for all points in the region.

1) Symmetric rates and asymmetric diversity requirements: Consider a case, where the base station has the least number of antennas in the MAC: $n < m_i \ \forall i$. For all subsets, S_k, involving the user k, the multiplexing gain is limited by,

$$\sum_{i \in S_k} r_i < \min \left\{ \sum_{i \in S_k} m_i, n \right\} = n. \tag{29}$$

Let all users have symmetric multiplexing gain requirements, $r_i = r \forall i$. Then, the maximum multiplexing gain achievable by each user that satisfies the rate constraints in (29) is $r^*_{max} = \frac{n}{K}$.

When all users have symmetric rate requirements, then the optimal diversity multiplexing tradeoff for the k^{th} user is calculated from (5) to be, $d_k^*(r) = \min_{S_k} d^*_{\sum_{i \in S_k} m_i, n}(|S_k| r)$. With the same arguments as [3], we can show that this optimal tradeoff simplifies to,

$$d_k^*(r) = \begin{cases} d^*_{m,n}(r) & r \leq r'_k \\ d^*_{\sum_i m_i, n}(Kr) & r \geq r'_k \end{cases} \tag{30}$$

where r'_k is the intersecting point of the curves, $d^*_{m,n}(r)$ and $d^*_{\sum_i m_i, n}(Kr)$ which is different for each user. Therefore, for all rates $r \leq \min_k\{r'_k\}$, the single user tradeoff curve can be achieved by all users and is called the lightly loaded regime. For rates $r \geq \max_k\{r'_k\}$, it is as though the K users are pooled into a single user with $\sum_i m_i$ antennas with multiplexing gain Kr and the MAC is heavily loaded since all users are transmitting at high rates. If all users have the same diversity requirement, we recover the result in [3]. We illustrate the optimal tradeoff for each user in a 3 user MAC with $m_1 = 7, m_2 = 6, m_3 = 5$ and $n = 3$ in Fig. 4. The multiplexing gain, r'_k that differentiates the lightly loaded and heavily loaded regimes in a MAC shown in the figure.

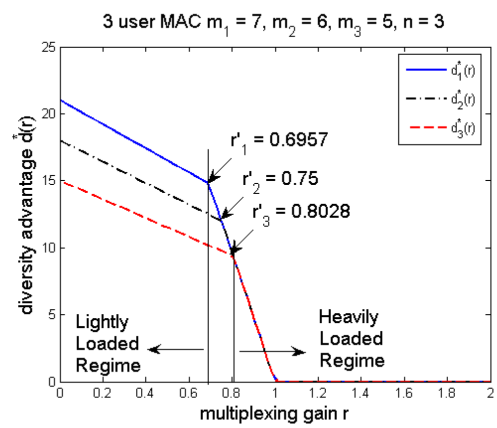


Fig. 4. The diversity multiplexing tradeoff for a asymmetric MAC with symmetric rate requirements

V. Conclusions

In this paper, we characterized the maximum diversity gain achievable by a user in an asymmetric MAC as a function of the achievable multiplexing gain-tuple, when the channel is known only at the base station. When multiplexing gain is zero for all users, we show that each user can achieve the maximum diversity independent of others in the MAC. However, there is a clear deleterious effect on the achievable diversity due to interference from other users transmitting at high rates. An immediate challenge is to characterize the tradeoff when partial channel knowledge is available at the transmitters.

References

[1] L. Zheng and D. Tse, “Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels,” in *IEEE Trans. Inf. Th.*, vol. 49 no. 5, May 2003, pp. 1073 – 1096.

[2] L. Zhao *et al.*, “Diversity and multiplexing tradeoff in general fading channels,” in *Proc. of 40th CISS*, Princeton, NJ, Mar 2006.

[3] D. Tse *et al.*, “Diversity-multiplexing tradeoff in multiple-access channels,” in *IEEE Trans. Inf. Th.*, vol. 50 no.9, Sep 2004, pp. 1859–1874.

[4] L. Weng *et al.*, “Diversity gain regions for MIMO fading multiple-access channels,” in *Proc. Allerton. Conf.*, Sep 2004.

[5] T. Cover and J. Thomas, *Elements of Information Theory*. John Wiley and Sons, 1991.

[6] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.