Achievable Rate of Gaussian Cognitive Z-Interference Channel with Partial Side Information

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***Abstract*— In this paper, we compute an achievable rate of a Gaussian Z-interference channel as shown in Fig. 1, when transmit node C has causal, imperfect cognitive knowledge of the signal sent by transmit node A. This achievable rate is derived using a two-phase transmission scheme in which node C uses a combination of a linear minimum mean square error (LMMSE) estimator and dirty paper code and node D employs a combination of LMMSE estimator and partial interference canceler. Numerical results indicate that the achievable rate of the Gaussian Z-interference channel increases significantly with cognition under certain channel conditions. We also derive an upper bound on the capacity of this channel with cognition and quantify the channel conditions under which the proposed achievable scheme equals the upper bound.**

I. Introduction

Consider a Z-channel as shown in Fig. 1, in which the primary transmitter (node A) communicates with its intended receiver (node B). There is also a secondary transmitter (node C) who wishes to communicate with its receiver (node D) on the same frequency as the primary nodes. Further, node C can cognitively monitor the interfering signal from node A, hence reducing its effect at node D. We call nodes C and D as the cognitive transmitter and receiver, respectively. We focus on the case when nodes C and D are relatively closer to node A than node B. Such a scenario might occur for instance when node A is a cellular base station and nodes C and D are two nearby nodes, while node B is at the cell-edge. We assume that node B is much farther away from the other nodes and hence do not explicitly consider the interference that node C causes at node B. Since node D receives a combination of both the intended signal from node C and interfering signal from node A, and node B is interference free, this system model is sometimes also referred to as a Z-interference channel (ZIC), or one-sided interference channel. Since we also assume that node A's signal is cognitively known at node C, we call the network in Fig. 1 as a Gaussian cognitive ZIC. The achievable rate and sum capacity of ZIC without cognition is studied in [1], [2], where the two receive nodes can cooperate through a rate-limited relay link. The capacity of a discrete memoryless cognitive ZIC in which the cognitive transmitter has *perfect*, *non-causal* side information of the interference is evaluated in [3].

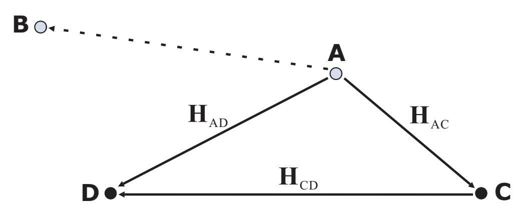

Fig. 1. Channel model.

In this paper, we compute an achievable rate of the Gaussian cognitive ZIC, when only *partial*, *causal* side information of the interference is available at the cognitive transmitter. Let node A communicates with its receiver, node B, at rate R using transmit power P_A. Let the transmit power of node C equals P_C. A simple lower bound, R_{CD-lb} on the rate that nodes C and D can communicate is

$$R_{CD-lb} = C\left(\frac{|h_{CD}|^2 P_C}{N_D + |h_{AD}|^2 P_A}\right) \quad (1)$$

which is achieved by treating the signal from node A as noise at node D. In (1), $C(x) = \log(1+x)$ and note that all the rates are in nats throughout the paper. Similarly, a trivial upper bound on this rate is obtained (if either nodes C or D has perfect, non-causal knowledge of node A's signal) as

$$R_{CD-ub} = C\left(\frac{|h_{CD}|^2 P_C}{N_D}\right). \quad (2)$$

We first compute the capacity of this channel when the signal from node A is non-causally observed at both nodes C and D, with certain Gaussian observation noise. We show that the interference at node D can be written as the linear combination of three components: 1) A linear minimum mean square error (LMMSE) estimate of node A's signal at node C; 2) a LMMSE estimate of node A's signal at node D; and 3) a residual noise. We prove that by using dirty paper coding

[1]This work has been supported in part by the National Science Foundation through grant CCF 0546519.

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

(DPC) [4], [5] at node C and interference cancelation at node D, in conjunction with these LMMSE estimators, the interference in parts 1) and 2) can be completely eliminated. Further, this scheme achieves the channel capacity extended from [6] for the Gaussian partial side-information case. For simplicity, we refer to this scheme as noisy DPC.

We then apply noisy DPC derived in Section II to the Gaussian cognitive ZIC, where the interference from node A can only be *causally* observed at nodes C and D. Specifically, we propose a two-phase transmission strategy over n symbols. In the first phase that lasts m symbols ($m < n$), nodes C does not transmit; both nodes C and D attempt to decode node A's signal based on the portion of codeword they observe. The probability of decoding error is bounded using Gallager's random coding exponent [7]. These decoding errors result in a noisy estimate of the interference at nodes C and D. By approximating the estimation error as Gaussian, we can apply the results from the non-causal side information case, to compute a lower bound on achievable rate R_{CD}. We quantify the increase in achievable rate using the proposed causal transmission scheme in Section III.

II. Gaussian Channel with Non-Causal Partial Side Information

A. System Model

Consider a channel as depicted in Fig. 2, in which the received signal, $\mathbf{Y}$, is corrupted by two independent additive white Gaussian noise (AWGN) sequences, $\mathbf{S} \sim \mathbb{CN}(0, Q\mathbf{I}_n)$ and $\mathbf{Z}_0 \sim \mathbb{CN}(0, N_0\mathbf{I}_n)$, where $\mathbf{I}_n$ is the identity matrix of size n. The received signal is of the form,

$$\mathbf{Y} = \mathbf{X} + \mathbf{S} + \mathbf{Z}_0 \tag{3}$$

where $\mathbf{X}$ is the complex transmitted sequence. The transmitter

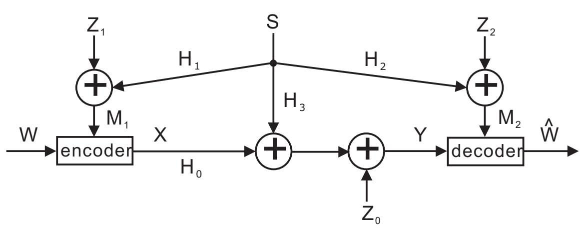

Fig. 2. Gaussian channel with partial side information of the interference at both encoder and decoder.

sends an index, $W \in \{1, 2, \ldots, K\}$, to the receiver in n uses of the channel at rate $R = \frac{1}{n}\log K$ nats per transmission. Side information $\mathbf{M}_1 = \mathbf{S} + \mathbf{Z}_1$, which is noisy observations of the interference is available at the transmitter. Similarly, noisy side information $\mathbf{M}_2 = \mathbf{S} + \mathbf{Z}_2$, is available at the receiver. The noise vectors are distributed as $\mathbf{Z}_1 \sim \mathbb{CN}(0, N_1\mathbf{I}_n)$ and $\mathbf{Z}_2 \sim \mathbb{CN}(0, N_2\mathbf{I}_n)$.

Based on index W and $\mathbf{M_1}$, the encoder transmits one complex codeword, $\mathbf{X}$, from a $(2^{nR}, n)$ code book, which satisfies average power constraint, $\frac{1}{n}\|\mathbf{X}\|^2 \leq P$. Let $\hat{W}$ be the estimate of W at the receiver; an error occurs if $\hat{W} \neq W$.

B. Channel Capacity and Achievability

Theorem 1: Consider a channel of the form (3) with an average transmit power constraint P. Let independent noisy observations $\mathbf{M}_1 = \mathbf{S} + \mathbf{Z}_1$ and $\mathbf{M}_2 = \mathbf{S} + \mathbf{Z}_2$ of the interference $\mathbf{S}$ be available, respectively, at the transmitter and receiver. The noise vectors have the following distributions: $\mathbf{Z}_i \sim \mathbb{CN}(0, N_i\mathbf{I}_n)$, $i = 0, 1, 2$ and $\mathbf{S} \sim \mathbb{CN}(0, Q\mathbf{I}_n)$. The capacity of this channel equals $C\left(\frac{P}{\mu Q + N_0}\right)$, where $0 \leq \mu = \frac{1}{1+\frac{Q}{N_1}+\frac{Q}{N_2}} \leq 1$.

Remark 1: Clearly, $\mu = 0$ when either $N_1 = 0$ or $N_2 = 0$ and the capacity is $C(P/N_0)$, which is consistent with [4]. Further, $\mu = 1$ when $N_1 \to \infty$ and $N_2 \to \infty$, and the capacity is $C(P/(Q+N_0))$, which is the capacity of a Gaussian channel with noise $Q + N_0$. Thus, one can interpret μ as the residual fractional power of the interference that cannot be canceled by the noisy observations at the transmitter and receiver.

Capacity Upper Bound: The capacity of the channel can be calculated by extending the results given in [6] to the Gaussian case. It is clear that the channel capacity can not exceed $\max_{p(x|m_1,m_2)} I(X; Y|M_1, M_2)$, which is the capacity when both M_1 and M_2 are known at the transmitter and receiver. Thus, a capacity bound is given by

$$\begin{aligned}
&I(X; Y|M_1, M_2) = I(X; Y, M_1, M_2) - I(X; M_1, M_2) \\
&\leq I(X; Y, M_1, M_2) \qquad (4) \\
&= H(X) + H(Y, M_1, M_2) - H(X, Y, M_1, M_2)
\end{aligned}$$

$$= \log(2\pi e)^4 P \begin{vmatrix} P+Q+N_0 & Q & Q \\ Q & Q+N_1 & Q \\ Q & Q & Q+N_2 \end{vmatrix} - \log(2\pi e)^4 \begin{vmatrix} P & P & 0 & 0 \\ P & P+Q+N_0 & Q & Q \\ 0 & Q & Q+N_1 & Q \\ 0 & Q & Q & Q+N_2 \end{vmatrix}$$

$$= C\left(\frac{P}{\mu Q + N_0}\right) \tag{5}$$

where $\mu = \frac{1}{1+\frac{Q}{N_1}+\frac{Q}{N_2}}$. Note that the inequality in (4) is actually a strict equality since $I(X; M_1, M_2) = 0$.

Proof of Achievability: The proof follows closely from standard DPC with LMMSE estimation of $\mathbf{S}$ (which we named the noisy DPC). Thus, we only outline the main steps. Given the noisy observations, $\mathbf{M}_1 = \mathbf{S} + \mathbf{Z}_1$ and $\mathbf{M}_2 = \mathbf{S} + \mathbf{Z}_2$, and assumed that the statistics of $\mathbf{S}$, $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are known at both transmitter and receiver, the LMMSE estimation of the interference, $\hat{\mathbf{S}}$, can be computed as

$$\hat{\mathbf{S}} = \alpha\mathbf{M}_1 + \beta\mathbf{M}_2 \tag{6}$$

where $\alpha = \frac{N_2 Q}{N_1N_2+N_1Q+N_2Q}$ and $\beta = \frac{N_1 Q}{N_1N_2+N_1Q+N_2Q}$. At the transmitter, the encoder apply DPC by treating $\alpha\mathbf{M}_1$ as the non-causal side information. Thus, part of the total interference, $\alpha\mathbf{M}_1$, can be removed due to DPC. Since $\mathbf{M}_2$ is non-causally known at the receiver, the receiver can directly subtract $\beta\mathbf{M}_2$ from the received signal. Thus, the residual noise is, $\hat{\mathbf{Z}} = \mathbf{S} - \hat{\mathbf{S}}$. Since $\mathbf{S}$, $\mathbf{M}_1$ and $\mathbf{M}_2$ are all Gaussian sequences, the residual noise vector $\hat{\mathbf{Z}}$ is also Gaussian, and is orthogonal to $\mathbf{M}_1$ and $\mathbf{M}_2$. After simple algebra, the variance of the residual noise is found to be $\hat{N} = \mu Q$. Since $\hat{\mathbf{Z}}$ is independent of the channel noise $\mathbf{Z}_0$, the variance of the total noise is then the summation of $\hat{N}$ and N_0. Thus, the achievable rate using noisy DPC is given by, $C\left(P/(\mu Q + N_0)\right)$, which is the channel capacity derived in (5).

The converse of the capacity theorem follows naturally from [4], [8], we omit the proof for simplicity.

Remark 2: When the observation of $\mathbf{S}$ is only available at the transmitter or receiver, the channel is equivalent to the model of Fig. 2, when $N_2 \to \infty$ and $N_1 \to \infty$, respectively. Their capacity are, respectively

$$I(X;Y|M_1) = C(P/(Q[N_1/(Q+N_1)] + N_0)) \tag{7}$$

$$I(X;Y|M_2) = C(P/(Q[N_2/(Q+N_2)] + N_0)). \tag{8}$$

Note that when $N_1 = 0$, the channel model further reduces to Costa's DPC channel model [4]. Indeed, by setting $N_1 = N_2$ in (7) and (8), we can see that the observation of $\mathbf{S}$ made at the transmitter and the receiver are equivalent in achievable rate, as long as the corrupting Gaussian noises have the same statistics.

Remark 3: Let there be n_1 independent observations $\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_{n_1}$ of $\mathbf{S}$ at the transmitter and n_2 independent observations $\mathbf{M}_{n_1+1}, \mathbf{M}_{n_1+2}, \ldots, \mathbf{M}_{n_1+n_2}$ at the receiver. It can be easily shown by applying noisy DPC that, the capacity in this case is given by $C\left(P/(\hat{\mu}Q + N_0)\right)$, where $\hat{\mu} = \frac{1}{1+\frac{Q}{N_1}+\frac{Q}{N_2}+\cdots+\frac{Q}{N_{n_1+n_2}}}$ and $N_1, N_2, \ldots, N_{n_1+n_2}$ are the variances of the Gaussian noise variables, corresponding to the $n_1 + n_2$ observations. To avoid repetitive derivations, the proof is omitted.

In this section, we assumed non-causal knowledge of the interference at the transmit and receive nodes. In the next section, we propose a simple and practical transmission scheme that uses causal knowledge of the interference to increase the achievable rate.

III. Cognitive Z-Interference Channel With Causal Side Information

Theorem 2: Consider the network as shown in Fig. 1. Nodes C can communicate with node D at rate given by $R_{CD} = \max\left(\max_{0<m<n, m\in\mathbb{N}, 0\le\alpha\le 1} R_{CD}(m,\alpha), R_{CD-lb}\right)$, where $R_{CD}(m,\alpha)$ is given in (14), (15), (16) and (17) under different channel conditions, $\mu_r = \frac{1}{1+0.5e^{nE_D(m,r)}}$, $\mu_t = \frac{1}{1+0.5e^{mE_C(m)}}$ and $\mu_{tr} = \frac{1}{1+0.5e^{mE_C(m)}+0.5e^{nE_D(m,r)}}$.

Proof: Construct a two-phase transmission scheme. Phase 1 lasts for $m,\ 0 < m < n$ symbol periods and phase 2 for $n - m$ symbol periods. The average power constraint of the signal transmitted by node C are $\frac{n}{m}\alpha P_C$ and $\frac{n}{n-m}(1-\alpha)P_C$ in phases 1 and 2, respectively, where $0 \le \alpha \le 1$ is the power allocation factor.

The total achievable rate between nodes A and C in phase 1 is given by

$$R_{AC}(m) = mC\left(\frac{|h_{AC}|^2 P_A}{N_C}\right). \tag{9}$$

Let node A transmits to its intended receiver node B in n uses of the channel at rate R nats per second. The codeword node A transmitted is chosen from a (e^{nR}, n) Gaussian code book with each element generated randomly according to Gaussian i.i.d.. For values of m such that $nR \le R_{AC}(m)$, node C can decode node A's signal using the symbols received in phase 1 with an error probability bounded by, $P_{e,C} \le \exp(-mE_C(m))$, where $E_C(m)$ is Gallager's random coding error exponent [7]. The error exponent, $E_C(m)$, is given by

$$E_C(m) = \max_{0\le\rho\le 1,\ r\ge 1} r(1+\rho)P_A + \log(1 - 2rP_A) + \rho\log\left(1 - |h_{AC}|^2 P_A\left(2r - \frac{1}{(1+\rho)N_D}\right)\right) - \frac{n}{m}R \tag{10}$$

where ρ and r are free parameters to be optimized [7].

Thus, the estimate of node A's codeword at node C, $\hat{\mathbf{M}}_1$, is a combination of the interference, $\mathbf{S}$, and an estimation noise, $\hat{\mathbf{Z}}_1$, *i.e.*, $\hat{\mathbf{M}}_1 = \mathbf{S} + \hat{\mathbf{Z}}_1$. Let $\hat{Z}_1$ be the estimation noise variable at node C of one time instant, its noise variance, $\hat{N}_1$, can be computed as

$$\hat{N}_1 = P_{e,C}\mathbb{E}[(S-\hat{S})^2] = 2P_{e,C}P_A \tag{11}$$

where S is the random Gaussian variable node A transmits at one time instant and $\hat{S}$ is its estimate at node C. Node C can then apply noisy DPC by treating $\hat{\mathbf{M}}_1$ as the noisy side information to reduce the interference for the remaining $n-m$ transmissions.

The total achievable rate, $R_{AD}(m,\alpha)$, between nodes A and D during phases 1 and 2 is given by

$$R_{AD}(m,\alpha) = mC\left(\frac{|h_{AD}|^2 P_A}{\frac{n}{m}\alpha|h_{CD}|^2 P_C}\right) + (n-m)C\left(\frac{|h_{AD}|^2 P_A}{N_D + \frac{n}{n-m}(1-\alpha)|h_{CD}|^2 P_C}\right). \tag{12}$$

For values of m such that $nR \le R_{AD}(m,\alpha)$, node D can decode node A's message with error probability $P_{e,D} \le \exp(-nE_D(m,\alpha))$ at the end of phase 2, where $E_D(m,\alpha)$ is the random coding error exponent for node D to decode node A's signal. Node D experiences time varying interference as a result of node C using different powers in phases 1 and 2. Extending the result from [7], the error exponent, $E_D(m,\alpha)$, is found to be

$$E_D(m,\alpha) = \max_{0\le\rho\le 1,\ r\ge 1} r(1+\rho)P_A + \log(1-2rP_A) + \rho\log\left[1 - \left(\frac{\alpha n}{m} + \frac{(1-\alpha)(n-m)}{m}\right)|h_{AD}|^2 P_A \times\left(2r - \frac{1}{(1+\rho)N_D}\right)\right] - \rho R. \tag{13}$$

Similar to the case of node C, we can express node D's estimation of node A's codeword as $\hat{\mathbf{M}}_2 = \mathbf{S} + \hat{\mathbf{Z}}_2$, where

$\hat{\mathbf{Z}}_2$ is the estimation noise. Let $\hat{Z}_2$ be the estimation noise variance at node D of one time instant. The covariance of $\hat{Z}_2$ is $\hat{N}_2 = 2P_{e,D}P_A$. Node D can thus treat $\hat{\mathbf{M}}_2$ as its noisy side information and apply noisy DPC to partially cancel out the interference.

Depending on nodes C and D's ability to decode node A's signal, the achievable rate, $R_{CD}(m, \alpha)$, is computed for the following four cases.

Case 1 ($nR \leq R_{AC}(m)$ and $nR \leq R_{AD}(m, \alpha)$): In this case, both nodes C and D can decode node A's information. In phase 1, node C transmits codeword generated with random Gaussian coding while receiving node A's signal. At the end of phase 1, node C decodes node A's signal. In phase 2, node C transmits using DPC by treating node A's signal as interference. Node D decodes node A's signal at the end of phase 2. Using Theorem 1, the achievable rate is given by

$$R_{CD}(m, \alpha) = \frac{m}{n} C\left(\frac{\frac{n}{m}\alpha|h_{CD}|^2 P_C}{N_D + \mu_r(m, \alpha)|h_{AD}|P_A}\right) + \frac{n-m}{n} C\left(\frac{\frac{n}{n-m}(1-\alpha)|h_{CD}|^2 P_C}{N_D + \mu_{tr}(m, \alpha)|h_{AD}|^2 P_A}\right) \quad (14)$$

for values such that $nR \leq R_{AC}(m)$ and $nR \leq R_{AD}(m, \alpha)$, where $\frac{1}{\mu_r} = 1 + \frac{P_A}{\hat{N}_2}$ and $\frac{1}{\mu_{tr}} = 1 + \frac{P_A}{\hat{N}_1} + \frac{P_A}{\hat{N}_2}$.

Note that there is no constraint that node C must use codes of length $n-m$ since node A uses codes of length n. Node C can code over multiple codewords of A to achieve its desired probability of error.

Case 2 ($nR \leq R_{AC}(m)$ and $nR > R_{AD}(m, \alpha)$): In this case, only node C can decode node A's signal. Node C decodes node A's signal after phase 1. In phase 2, node C transmits using DPC to cancel out part of node A's interference. Since node D cannot decode node A's signal, it cannot apply partial interference cancelation. Thus, the achievable rate is given by

$$R_{CD}(m, \alpha) = \frac{m}{n} C\left(\frac{\frac{n}{m}\alpha|h_{CD}|^2 P_C}{N_D + |h_{AD}|^2 P_A}\right) + \frac{n-m}{n} C\left(\frac{\frac{n}{n-m}(1-\alpha)|h_{CD}|^2 P_C}{N_D + \mu_t(m)|h_{AD}|^2 P_A}\right) \quad (15)$$

for values such that $nR \leq R_{AC}(m)$ and $nR > R_{AD}(m, \alpha)$, where $\frac{1}{\mu_t} = 1 + \frac{P_A}{\hat{N}_1}$.

Case 3: ($nR > R_{AC}(m)$ and $nR \leq R_{AD}(m, \alpha)$): In this case, only node D can decode node A's signal. Node D decodes node A's signal at the end of phase 2 and cancel out part of its interference. The achievable rate is given by

$$R_{CD}(m, \alpha) = \frac{m}{n} C\left(\frac{\frac{n}{m}\alpha|h_{CD}|^2 P_C}{N_D + |h_{AD}|^2 P_A}\right) + \frac{n-m}{n} C\left(\frac{\frac{n}{n-m}(1-\alpha)|h_{CD}|^2 P_C}{N_D + \mu_r(m)|h_{AD}|^2 P_A}\right) \quad (16)$$

for values such that $nR \leq R_{AC}(m)$ and $nR > R_{AD}(m, \alpha)$.

Case 4: ($nR > R_{AC}(m)$ and $nR_{AD}(m, \alpha)$): In this case, neither node C nor D can decode node A's signal. Thus, node D decodes node C's signal by treating node A's signal as noise. The achievable rate is given by

$$R_{CD}(m, \alpha) = R_{CD-lb} \quad (17)$$

for values such that $nR > R_{AC}(m)$ and $nR > R_{AD}(m, \alpha)$.

Combining the above four cases, we can express the achievable rate between nodes C and D by optimizing over all time and power allocation as

$$R_{CD} = \max_{\substack{0<m<n,\ m\in\mathbb{N} \\ 0\leq\alpha\leq 1}} R_{CD}(m, \alpha). \quad (18)$$

Remark 4: The estimation errors, with variances $\hat{N}_1$ and $\hat{N}_2$, are in general not Gaussian. However, it is known that for a given noise variance the capacity is minimum when the noise is Gaussian. Thus for simplicity, we model the residual noise as Gaussian, and hence the achievable rate given in (18) is only a lower bound on capacity.

IV. Numerical Results

In this section, we provide numerical examples to illustrate the increase in the achievable rate using noisy DPC. We set $|h_{AB}| = 0.25$, $|h_{CD}| = 1$,and $N_C = N_D = 1$. We also set the average power constraints $P_A = P_C = 100$ and $n = 50$.

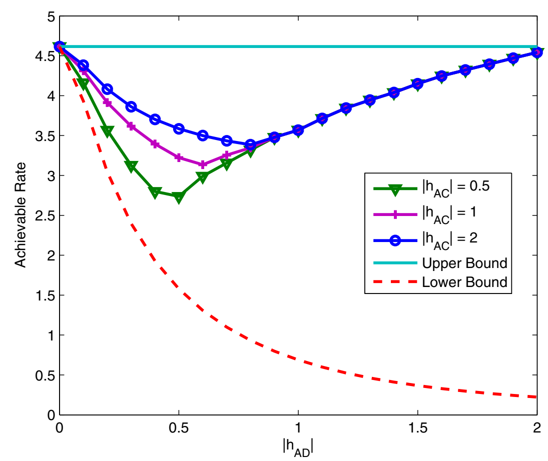

Fig. 3. Variation of achievable rate with $|h_{AD}|$ for different values of $|h_{AC}|$.

Fig. 3 shows the variation of the achievable rate R_{CD} with $|h_{AD}|$ for different values of $|h_{AC}|$. Notice the non-monotonic variation of R_{CD} with $|h_{AD}|$ which can be explained as follows. First consider $|h_{AC}|$ is small. In this case, the transmitter cannot reliably decode node A's signal. If in addition, $|h_{AD}|$ is also small, then node D cannot decode node A's signal either. Thus, as $|h_{AD}|$ increases, the interference of node A at node D increases and the achievable rate R_{CD} decreases. Now, as $|h_{AD}|$ increases beyond a certain value, node D can begin to decode node A's signal and the probability of error is captured by Gallager's error exponents. In this scenario, as $|h_{AD}|$ increases, the error probability decreases and thus node D can cancel out more and more of interference from node A. Consequently, R_{CD} increases. Similar qualitative behavior occurs for other values of $|h_{AC}|$. However, for large $|h_{AC}|$, node C can decode (with some errors) the signal from node A and then use a noisy DPC scheme to achieve higher rates R_{CD}. Notice also that as explained before for large $|h_{AD}|$, the outer bound on the rate is achieved for all values of $|h_{AC}|$.

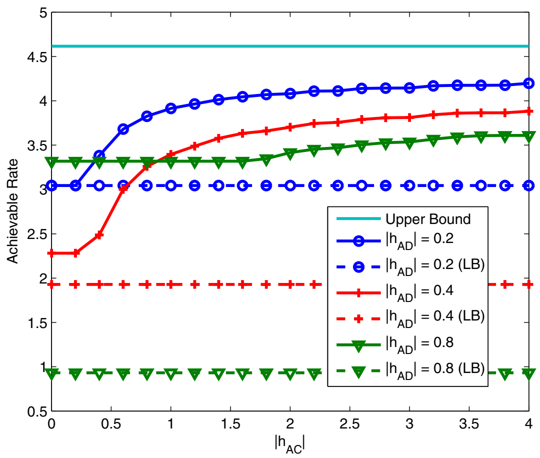

Fig. 4. Variation of achievable rate with $|h_{AC}|$ for different values of $|h_{AD}|$.

The variation of R_{CD} with $|h_{AC}|$ is given in Fig. 4. First consider the case $|h_{AD}| = 0.2$. In this case, node D cannot decode the signal of node A reliably. Now, for small values of $|h_{AC}|$ node C also cannot decode node A's signal. Hence, the achievable rate equals the lower bound, R_{CD-lb}. As $|h_{AC}|$ increases, node C can begin to decode node A's signal and cancel out a part of the interference using the noisy DPC scheme; hence R_{CD} begins to increase. However, when $|h_{AD}| = 0.4$ and $|h_{AD}| = 0.8$, node D can decode node A's signal with some errors and cancel out part of the interference. Hence, in these cases, even for small values of $|h_{AC}|$ the achievable rate R_{CD} is greater than the lower bound. As before, R_{CD} increases with $|h_{AC}|$ since node A can cancel out an increasing portion of the interference using the noisy DPC technique. Note however, that a larger $|h_{AD}|$ causes more interference at node D, which is reflected in the decrease of the lower bound. Thus, for a given $|h_{AC}|$ the achievable rate can be lower or higher depending on the value of $|h_{AD}|$.

Fig. 5 shows the optimal choice of m with respect to the codeword length n for different values of $|h_{AC}|$. As we can see, the optimal m increases almost linearly with n. However, the optimal m decreases with $|h_{AC}|$, since as the channel quality between nodes A and C increases, less time is needed in phase 1 for node C to decode node A's signal.

V. Conclusion

In this paper, we first derive the capacity rate of a Gaussian channel with additive Gaussian interference, when noisy estimates of the interference are known at the transmitter

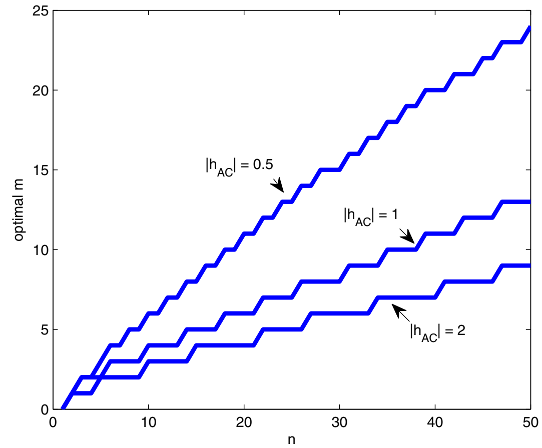

Fig. 5. Variation of achievable rate with m for different values of n.

and receiver. We then construct a causal transmission strategy that uses this noisy dirty paper coding strategy and derive an achievable rate for a cognitive Z-interference channel. Future work should consider optimizing the random coding error exponent to further improve the capacity lower bound. It will also be interesting to apply the current scheme to multiple antenna case and quantify the resultant capacity gain.

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