

Capacity Bounds of Half-Duplex Gaussian Cooperative Interference Channel

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Abstract—In this paper, we study the capacity region of a two user Gaussian interference channel with half duplex node constraints. We develop an achievable region and outer bound for the case when the system allows either transmitter or receiver cooperation. We show that by using our transmitter cooperation scheme, there is significant capacity improvement compared to the previous results [1], [2], especially when the cooperation link is strong. Further, if the cooperation channel gain is infinity, both our transmitter and receiver cooperation rates achieve their respective outer bound. It is also shown that transmitter cooperation provides larger achievable region than receiver cooperation under the same channel and power conditions.

I. INTRODUCTION

In wireless ad hoc networks, spatially dispersed radio terminals can exploit cooperative diversity [3], [4] by relaying signals for each other. With cooperation, different clusters of terminals can act like transmit/receive antenna arrays and achieve increased spatial diversity and throughput by joint encoding and/or decoding.

The capacity of the two-user Gaussian interference channel (IC) is an open problem for many years and is completely known only in some special cases (*e.g.*, in the strong interference case [5]). The capacity region has been studied under various cooperative strategies. Most of these schemes assume that nodes operate in full-duplex mode. A coding scheme for transmitter cooperation using decode-and-forward (DF) for relaying and dirty paper coding (DPC) for codeword generation is proposed in [6]. Compress-and-forward (CF) and DF relaying strategies for receiver cooperation are proposed in [7] and generalized to both transmitter and receiver cooperation in [8]. A comparison of different coding schemes for transmitter cooperation in terms of the relative geometry of transmit and receive clusters is given in [9]. The sum rate capacity with transmitter only, receiver only and both transmitter and receiver cooperation is studied in [10]. By using DF and DPC at the cooperative transmitters and Wyner-Ziv CF at the receivers and assuming equal power gain for all channels, the proposed scheme in [10] is shown to have significant capacity gain over strong IC [5]. While full-duplex cooperative IC has been significantly studied, only limited results are known in the half-duplex scenario. Cooperative diversity with transmitter cooperation for fading channels is considered in [4]. A 2-phase transmitter cooperation scheme using DF and the so called

recycling DPC (RDPC) is introduced in [1]: Similar schemes are also proposed in [2], where the transmitters have additional flexibility in choosing the order of DPC.

In this paper, we compute bounds on the capacity of two user Gaussian IC in two different scenarios: i) transmitter cooperation (TC) and ii) receiver cooperation (RC). Specifically, we allow all nodes to operate in half-duplex mode only, which requires simpler and cheaper hardware.

In TC, the two transmit nodes serve as relays to each other. We assume that the channel gain between the two transmitters is much higher than the others. In this case, it is well known that DF strategy is superior [11], [12]. Thus, in this paper we derive the achievable region with TC using only the DF strategy. We show that the achievable region of the proposed TC strategy is strictly larger than the results in [1], [2], especially when the cooperation link is strong. In case when the cooperation channel gain is infinity, the proposed achievable region achieves the capacity upper bound. In contrast, for the schemes in [1], [2], there is a significant performance gap between the lower and upper bounds.

In RC, the two receive nodes serve as relays to each other. In this case, we assume that the relay to destination channel is strong, and CF [11] is preferable at the relays. Thus, to derive the achievable region with RC, we only consider the CF strategy. The proposed scheme achieves the corresponding MIMO multiple access channel (MAC) capacity [13] when the cooperation channel gain is infinity. To the best of our knowledge, the achievable rate with RC has not been studied under the half-duplex assumption. We also show that under identical channel conditions and equal transmit power constraints on all nodes, TC achieves larger rates than RC.

II. SYSTEM MODEL

Consider a two-transmitter two-receiver network shown in Fig. 1, where node 3 is the intended receiver of node 1 and node 4 is the intended receiver of node 2. The independent messages transmitted by node i , $i \in \{1, 2, 3, 4\}$ are encoded into N complex symbols $x_i[1], x_i[2], \dots, x_i[N]$, under the power constraint $\frac{1}{N} \sum_{n=1}^N |x_i[n]|^2 \leq P_i$. If the messages transmitted by nodes 1 and 2 have a total alphabet of M_1 and M_2 respectively, their respective rates are then $R_1 = \log M_1/N$ and $R_2 = \log M_2/N$ bits/transmission. The channel gain from node i to node k and $k > i$, is represented by a complex constant $h_{ik} = c_{ik}e^{j\theta_{ik}}$. It is assumed that all nodes have perfect knowledge of the channel gain and all the phase offsets can be perfectly synchronized. Let \mathbf{z}_i denote the *i.i.d.* complex circularly symmetric Gaussian noise process at node

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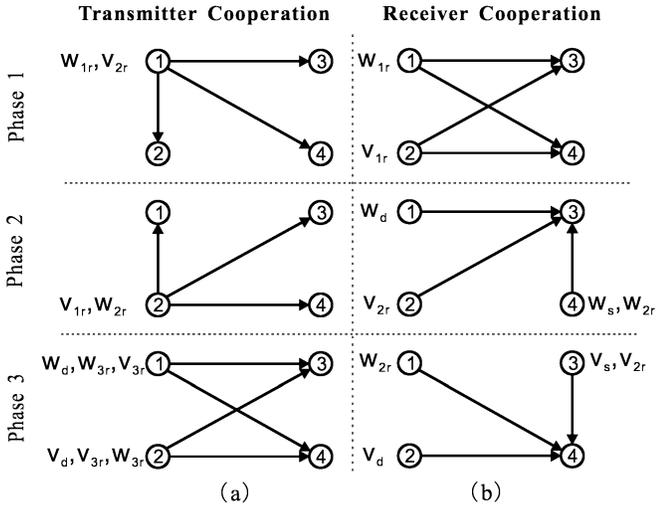


Fig. 1. System models of half-duplex Gaussian cooperative interference channel with (a) transmitter cooperation and (b) receiver cooperation.

i , with the n^{th} element $z_i[n] \sim \mathcal{CN}(0, 1)$. We assume that the communication is in a half-duplex fashion, *i.e.*, each of the nodes can be either in the transmit mode or the receive mode. For TC, only the two transmit nodes (node 1 and 2) can cooperate with each other while for RC, only the two receive nodes (node 3 and 4) can cooperate with each other. It is also assumed that the cooperation nodes are close together, *i.e.*, c_{12} and c_{34} are large compared to the other c_{ik} 's. Further, we define the following non-negative parameters satisfying $\alpha_1 + \alpha_2 = 1$, $\beta_1 + \beta_2 = 1$, $\kappa_1 + \kappa_2 = 1$, $\gamma_1 + \gamma_2 = 1$, $\mu_1 + \mu_2 + \mu_3 = 1$, $\eta_1 + \eta_2 + \eta_3 = 1$ and $\lambda_1 + \lambda_2 + \lambda_3 = 1$. Also define $\mathbf{g}_1 = [c_{13} \ c_{23}]$, $\mathbf{g}_2 = [c_{14} \ c_{24}]$, $\mathbf{h}_1 = [c_{13} \ c_{14}]$ and $\mathbf{h}_2 = [c_{23} \ c_{24}]$. Let $C(x) = \log(1 + x)$.

III. TRANSMITTER COOPERATION

A. Achievable Rates

Theorem 1: For the half-duplex Gaussian interference channel where the transmitters can cooperate with each other, all rate pairs (R_1^{TX}, R_2^{TX}) satisfying

$$R_1^{TX} \leq \min \left\{ R_{1,d}^{TX} + R_{1,r_1}^{TX}, R_{1,d}^{TX} + R_{1,r_2}^{TX} \right\} \quad (1)$$

$$R_2^{TX} \leq \min \left\{ R_{2,d}^{TX} + R_{2,r_1}^{TX}, R_{2,d}^{TX} + R_{2,r_2}^{TX} \right\} \quad (2)$$

are achievable, where $R_{i,d}^{TX}$ is given by (5) and (7), R_{i,r_1}^{TX} is given by (3) and (4), and R_{i,r_2}^{TX} is given by (6) and (8).

Proof: We construct a 3-phase transmission strategy as shown in Fig. 1-(a), to show the achievability. Let w_i 's and v_i 's be the messages intended for node 3 and 4 respectively. The specific message sent in each phase is detailed in Fig. 1-(a). In phase 1 and 2, the two source nodes transmit messages w_{1r} and v_{1r} to each other, and w_{2r} and v_{2r} to the receive nodes by broadcasting their signals using DPC. In phase 3, after the sources have exchanged their information, the system is equivalent to a two user 2-transmit-1-receive antenna MIMO BC. The source nodes can then jointly broadcast w_{3r} and v_{3r}

to the receivers using DPC [14]. Further, the two source nodes can also send w_d and v_d in phase 3, respectively. Due to the limited space, we only outline the results for each phase.

Transmission Scheme: The transmission is divided into 3 phases as shown in Fig. 1-(a), with time portion λ_1 , λ_2 and λ_3 . In *Phase 1*, node 1 is in transmit mode and all other nodes are in receive mode. The received signal at node i is $y_i[n] = h_{1i}x_1[n] + z_i[n]$, $n \in \{1, 2, \dots, \lfloor \lambda_1 N \rfloor\}$, $i = 2, 3$, and 4. In *Phase 2*, node 2 is in the transmit mode and all the other nodes are in receive mode. In *Phase 3*, nodes 1 and 2 are in transmit mode and nodes 3 and 4 are in the receive mode. The received signal in phases 2 and 3 can be easily expressed similar to phase 1.

Outline of Achievability:

1) *Phase 1:* If $c_{13} > c_{14}$, generate codeword $\mathbf{X}_1(v_{2r})$ with length $\lambda_1 N$, $N \rightarrow \infty$ and power $\alpha_2 P_1^{(1)}$, $P_1^{(1)} = \kappa_1 P_1 / \lambda_1$. Given $\mathbf{X}_1(v_{2r})$, use DPC to generate $\mathbf{X}_1(w_{1r})$ with length $\lambda_1 N$ and power $\alpha_1 P_1^{(1)}$. If $c_{13} \leq c_{14}$, do DPC with the reverse order. Since v_{2r} is known to node 2, it can subtract $\mathbf{X}_1(v_{2r})$ and decode w_{1r} if the rate of w_{1r} satisfies [1]

$$R_{1,r_1}^{TX} \leq \lambda_1 C \left(c_{12}^2 \alpha_1 P_1^{(1)} \right). \quad (3)$$

Node 3 can decode v_{2r} if the rate of v_{2r} satisfies

$$R_{2,1}^{TX} \leq \begin{cases} \lambda_1 C \left(c_{14}^2 \alpha_2 P_1^{(1)} / (1 + c_{14}^2 \alpha_1 P_1^{(1)}) \right), & \text{if } c_{13} > c_{14} \\ \lambda_1 C \left(c_{14}^2 \alpha_2 P_1^{(1)} \right), & \text{otherwise} \end{cases}$$

2) *Phase 2:* If $c_{24} > c_{23}$, generate codeword $\mathbf{X}_1(w_{2r})$ with length $\lambda_2 N$ and power $\beta_2 P_2^{(1)}$, $P_2^{(1)} = \gamma_1 P_2 / \lambda_2$. Given $\mathbf{X}_1(w_{2r})$, use DPC to generate $\mathbf{X}_2(v_{1r})$ with length $\lambda_2 N$ and power $\beta_1 P_2^{(1)}$. If $c_{24} \leq c_{23}$, do DPC in the reverse order. Node 1 can decode v_{1r} if the rate of v_{1r} satisfies [1]

$$R_{2,r_1}^{TX} \leq \lambda_2 C \left(c_{12}^2 \beta_1 P_2^{(1)} \right) \quad (4)$$

and node 3 can decode w_{2r} if the rate of w_{2r} satisfies

$$R_{1,2}^{TX} \leq \begin{cases} \lambda_2 C \left(c_{23}^2 \beta_2 P_2^{(1)} / (1 + c_{23}^2 \beta_1 P_2^{(1)}) \right), & \text{if } c_{24} > c_{23} \\ \lambda_2 C \left(c_{23}^2 \beta_2 P_2^{(1)} \right), & \text{otherwise} \end{cases}$$

3) *Phase 3:* After phases 1 and 2, v_{1r} and w_{1r} have been exchanged between the sources. Nodes 1 and 2 can then send messages jointly using the coding scheme of a two user 2-transmit-1-receive antenna MIMO BC [14]. The problem now is to find the optimal covariance matrices of the two transmit signals intended to both nodes 3 and 4. A simple method of generating MIMO BC covariance matrices is proposed in [15] by transforming the covariance matrices from its dual, MIMO MAC. We use this method to find the covariance matrices Σ_i and Σ'_i for our coding scheme.

If $c_{13} + c_{23} > c_{14} + c_{24}$, generate codeword $\mathbf{X}_2(v_d)$ with length $\lambda_3 N$ and power $\eta_1 P_2^{(2)}$, $P_2^{(2)} = \gamma_2 P_2 / \lambda_3$ at node 2. Generate codeword $\mathbf{X}_1(v_{3r})$ and $\mathbf{X}_2(v_{3r})$ with length $\lambda_3 N$ at nodes 1 and 2 respectively with covariance matrix Σ_2 , where Σ_2 can be found by using the results given in [15]. Let $\mathbf{B}_1 = \mathbf{I} + \mathbf{g}_2^T \mathbf{g}_2 (\mu_3 P_1^{(2)} + \eta_2 P_2^{(2)})$, $P_1^{(2)} = \kappa_2 P_1 / \lambda_3$, then $\Sigma_1 =$

$\mathbf{B}_1^{-1}(\mu_2 P_1^{(2)} + \eta_3 P_2^{(2)})$. Let $A_2 = 1 + \mathbf{g}_2 \Sigma_1 \mathbf{g}_2^T$, then $\Sigma_2 = A_2(\mu_3 P_1^{(2)} + \eta_2 P_2^{(2)})\mathbf{I}$. Given $\mathbf{X}_2(v_d)$ and $\mathbf{X}_1(v_{3r})$, use DPC to generate codeword $\mathbf{X}_1(w_d)$ with length $\lambda_3 N$ and power $\mu_1 P_1^{(2)}$ at node 1. Generate codeword $\mathbf{X}_1(w_{3r})$ and $\mathbf{X}_2(w_{3r})$ with length $\lambda_3 N$ at node 1 and 2 respectively with covariance matrix Σ_1 . If $c_{13} + c_{23} \leq c_{14} + c_{24}$, do DPC with the reverse order. Note that in this case, the covariance matrix becomes $\Sigma'_1 = \mathbf{B}'_1^{-1}(\mu_3 P_1^{(2)} + \eta_2 P_2^{(2)})$, where $\mathbf{B}'_1 = \mathbf{I} + \mathbf{g}_1^T \mathbf{g}_1 (\mu_2 P_1^{(2)} + \eta_3 P_2^{(2)})$ and $\Sigma'_2 = A'_2(\mu_2 P_1^{(2)} + \eta_3 P_2^{(2)})\mathbf{I}$, where $A'_2 = 1 + \mathbf{g}_1 \Sigma_1 \mathbf{g}_1^T$. Node 3 first decodes w_{3r} , it can do so if the rate of w_{3r} satisfies

$$R_{1,3}^{Tx} \leq \begin{cases} \lambda_3 C \left(\frac{\mathbf{g}_1 \Sigma_1 \mathbf{g}_1^T}{c_{13}^2 \mu_1 P_1^{(2)}} \right), & \text{if } c_{13} + c_{23} > c_{14} + c_{24} \\ \lambda_3 C \left(\frac{\mathbf{g}_1 \Sigma'_2 \mathbf{g}_1^T}{1 + \mathbf{g}_1 \Sigma'_1 \mathbf{g}_1^T + c_{13}^2 \mu_1 P_1^{(2)} + c_{23}^2 \eta_1 P_2^{(2)}} \right), & \text{otherwise} \end{cases}$$

Node 3 then decodes w_d , if the rate of w_d satisfies

$$R_{1,d}^{Tx} \leq \begin{cases} \lambda_3 C \left(c_{13}^2 \mu_1 P_1^{(2)} \right), & \text{if } c_{13} + c_{23} > c_{14} + c_{24} \\ \lambda_3 C \left(\frac{c_{13}^2 \mu_1 P_1^{(2)}}{1 + \mathbf{g}_1 \Sigma'_1 \mathbf{g}_1^T + c_{23}^2 \eta_1 P_2^{(2)}} \right), & \text{otherwise} \end{cases} \quad (5)$$

After decoding w_{2r} and w_{3r} , node 3 can finally decode w_{1r} if the rate of w_{1r} satisfies

$$R_{1,r_2}^{Tx} \leq R_{1,1}^{Tx} + R_{1,2}^{Tx} + R_{1,3}^{Tx} \quad (6)$$

where

$$R_{1,1}^{Tx} \leq \begin{cases} \lambda_1 C \left(c_{13}^2 \alpha_1 P_1^{(1)} \right), & \text{if } c_{13} > c_{14} \\ \lambda_1 C \left(c_{13}^2 \alpha_1 P_1^{(1)} / (1 + c_{13}^2 \alpha_2 P_1^{(1)}) \right), & \text{otherwise} \end{cases}$$

Similarly, node 4 first decodes v_{3r} , if the rate of v_{3r} satisfies

$$R_{2,3}^{Tx} \leq \begin{cases} \lambda_3 C \left(\frac{\mathbf{g}_2 \Sigma_2 \mathbf{g}_2^T}{1 + \mathbf{g}_2 \Sigma_1 \mathbf{g}_2^T + c_{14}^2 \mu_1 P_1^{(2)} + c_{24}^2 \eta_1 P_2^{(2)}} \right), & \text{if } c_{13} + c_{23} > c_{14} + c_{24} \\ \lambda_3 C \left(\frac{\mathbf{g}_2 \Sigma'_1 \mathbf{g}_2^T}{c_{24}^2 \eta_1 P_2^{(2)}} \right), & \text{otherwise} \end{cases}$$

Node 4 can then decode v_d if the rate of v_d satisfies

$$R_{2,d}^{Tx} \leq \begin{cases} \lambda_3 C \left(\frac{c_{24}^2 \eta_1 P_2^{(2)}}{1 + \mathbf{g}_2 \Sigma_1 \mathbf{g}_2^T + c_{14}^2 \mu_1 P_1^{(2)}} \right), & \text{if } c_{13} + c_{23} > c_{14} + c_{24} \\ \lambda_3 C \left(c_{24}^2 \eta_1 P_2^{(2)} \right), & \text{otherwise} \end{cases} \quad (7)$$

¹It is easy to show that if we let $\Sigma_1 = \Sigma'_2 = \text{diag} \{ \mu_2 P_1^{(2)}, \eta_3 P_2^{(2)} \}$ and $\Sigma'_1 = \Sigma_2 = \text{diag} \{ \mu_3 P_1^{(2)}, \eta_2 P_2^{(2)} \}$, the achievable rates of our scheme reduces to the rates given by parallel coding DPC in [2] (or RDPC in [1], if we further restrict the condition to $c_{13} \leq c_{14}$ and $c_{24} \leq c_{23}$). Note that using the above covariance matrices pairs is equivalent to assuming random phase shifts for different channels, *i.e.*, the received signal from different transmitters can not be synchronized.

After decoding v_{2r} and v_{3r} , node 4 can decode v_{1r} if the rate of v_{1r} satisfies²

$$R_{2,r_2}^{Tx} \leq R_{2,1}^{Tx} + R_{2,2}^{Tx} + R_{2,3}^{Tx} \quad (8)$$

where

$$R_{2,2}^{Tx} \leq \begin{cases} \lambda_2 C \left(c_{24}^2 \beta_1 P_2^{(1)} \right), & \text{if } c_{24} > c_{23} \\ \lambda_2 C \left(c_{24}^2 \beta_1 P_2^{(1)} / (1 + c_{24}^2 \beta_2 P_2^{(1)}) \right), & \text{otherwise} \end{cases}$$

B. Outer Bound

For TC, when $c_{12} \rightarrow \infty$, the system becomes a two user 2-transmit-1-receive antenna MIMO BC. The capacity region of this MIMO BC [14] is an outer bound on achievable rate. Further, when one user is silent, the achievable rate for the active user is bounded by the single user half-duplex relay channel max-flow-min-cut bound [12]. Hence, with TC, the set of achievable rate pairs (R_1^+, R_2^+) satisfies

$$R_i^+ \leq \max_{0 \leq \rho_i \leq 1} \min \{ R_{i,1}^+(\rho_i), R_{i,2}^+(\rho_i) \}, \quad i = 1, 2 \quad (9)$$

$$R_1^+ + R_2^+ \leq \bigcup_{\forall P_1 + P_2 < P} \mathbf{C}(\mathbf{g}_1^T P_1 \mathbf{g}_1 + \mathbf{g}_2^T P_2 \mathbf{g}_2). \quad (10)$$

where $\mathbf{C}(\mathbf{x}) = \log |\mathbf{I} + \mathbf{x}|$ and \bigcup is the union of all rates with any power allocations P_1 and P_2 that satisfies the total power constraint P , and

$$\begin{aligned} R_{1,1}^+(\rho_1) &= \alpha_1 C(c_{12}^2 + c_{13}^2 P_1) + \alpha_2 C((1 - \rho_1)c_{13}^2 P_1) \\ R_{1,2}^+(\rho_1) &= \alpha_1 C(c_{13}^2 P_1) + \alpha_2 C(c_{13}^2 P_1 + c_{23}^2 P_2 + 2\varphi_1) \\ R_{2,1}^+(\rho_2) &= \alpha_1 C(c_{12}^2 + c_{24}^2 P_2) + \alpha_2 C((1 - \rho_2)c_{24}^2 P_2) \\ R_{2,2}^+(\rho_2) &= \alpha_1 C(c_{24}^2 P_2) + \alpha_2 C(c_{14}^2 P_1 + c_{24}^2 P_2 + 2\varphi_2) \end{aligned}$$

where $\varphi_1 = \sqrt{\rho_1 c_{13}^2 c_{23}^2 P_1 P_2}$ and $\varphi_2 = \sqrt{\rho_2 c_{14}^2 c_{24}^2 P_1 P_2}$,

IV. RECEIVER COOPERATION

A. Achievable Rates

Theorem 2: For the half-duplex Gaussian interference channel where the receivers can cooperate with each other, all rate pairs (R_1^{Rx}, R_2^{Rx}) satisfying

$$R_1^{Rx} \leq R_{1,d}^{Rx} + R_{1,r_1}^{Rx} + R_{1,r_2}^{Rx} \quad (11)$$

$$R_2^{Rx} \leq R_{2,d}^{Rx} + R_{2,r_1}^{Rx} + R_{2,r_2}^{Rx} \quad (12)$$

are achievable, where $R_{i,d}^{Rx}$ is given by (16) and (22), R_{i,r_1}^{Rx} is given by the inequalities from (25) to (31), and R_{i,r_2}^{Rx} is given by (18) and (21).

Proof: The 3-phase RC scheme is shown in Fig. 1-(b). In phase 1, the signals from node 1 and 2 are received at node 3 and 4. Rather than decoding the signals, the two receive nodes exchange information in phase 2 and 3 by sending each other a compressed version of signals that they received. The receive nodes then perform decoding by using the aggregation of the compressed signal and the signal directly received in phase 1. Let w_i 's and v_i 's be the messages intended for nodes

²Note that for the transmission order given in Fig. 1-(a), v_{1r} is encoded and transmitted in phase 2, the receiver can decode it only after v_{2r} and v_{3r} been decoded at phase 1 and 3 of the next transmission block.

3 and 4 respectively. The specific message sent in each phase is detailed in Fig. 1-(b). We outline the coding scheme as follows.

Transmission Scheme: In *Phase 1*, nodes 3 and 4 are in receive mode and nodes 1 and 2 are in transmit mode. Again, since the expressions of the received signals can be easily shown, we omit them due to limited space. In *Phase 2*, node 3 is in receive mode and all the other nodes are in transmit mode. In *Phase 3*, node 4 is in receive mode and all the other nodes are in transmit mode.

Outline of Achievability:

Phase 1: At nodes 1 and 2, generate $\lambda_1 N$ length codewords $\mathbf{X}_1(w_{1r})$ and $\mathbf{X}_2(v_{1r})$ with powers $P_1^{(1)} = \mu_1 P_1 \lambda_1$ and $P_2^{(1)} = \eta_1 P_2 \lambda_1$ respectively.

Phase 2: At node 1 and 2, generate $\lambda_2 N$ length codewords $\mathbf{X}_1(w_d)$ and $\mathbf{X}_2(v_{2r})$ with powers $P_1^{(2)} = \mu_2 P_1 / \lambda_2$ and $P_2^{(2)} = \eta_2 P_2 / \lambda_2$ respectively. At node 4, generate $\lambda_2 N$ length codewords $\mathbf{X}_2(w_s)$ and $\mathbf{X}_2(w_{2r})$ with power $P_4^{(1)} = \alpha_1 P_4 / \lambda_2$ and $P_4^{(2)} = \alpha_2 P_4 / \lambda_2$ respectively³. Node 3 first decode w_{2r} , if the rate of w_{2r} satisfies

$$R_{1,2r_2}^{R_x} \leq \lambda_2 C \left(\frac{c_{34}^2 P_4^{(2)}}{1 + c_{13}^2 P_1^{(2)} + c_{23}^2 P_2^{(2)} + c_{34}^2 P_4^{(1)}} \right). \quad (13)$$

Node 3 can then decode w_s , if the rate of w_s satisfies

$$R_{1,s}^{R_x} \leq \lambda_2 C \left(c_{34}^2 P_4^{(1)} / (1 + c_{13}^2 P_1^{(2)} + c_{23}^2 P_2^{(2)}) \right) \quad (14)$$

and decode v_{2r} and w_d , if their respective rates satisfy

$$R_{2,2r_1}^{R_x} \leq \lambda_2 C \left(c_{23}^2 P_2^{(2)} / (1 + c_{13}^2 P_1^{(2)}) \right) \quad (15)$$

$$R_{1,d}^{R_x} \leq \lambda_2 C \left(c_{13}^2 P_1^{(2)} \right). \quad (16)$$

Phase 3: At nodes 1 and 2, generate $\lambda_3 N$ length codewords $\mathbf{X}_1(w_{2r})$ and $\mathbf{X}_2(v_d)$ with powers $P_1^{(3)} = \mu_3 P_1 / \lambda_3$ and $P_2^{(3)} = \eta_3 P_2 / \lambda_3$ respectively. At node 3, generate $\lambda_3 N$ length codewords $\mathbf{X}_1(v_s)$ and $\mathbf{X}_2(v_{2r})$ with powers $P_3^{(1)} = \beta_1 P_3 / \lambda_3$ and $P_3^{(2)} = \beta_2 P_3 / \lambda_3$ respectively. Node 4 can decode v_{2r} if

$$R_{2,2r_2}^{R_x} \leq \lambda_3 C \left(\frac{c_{34}^2 P_3^{(2)}}{1 + c_{14}^2 P_1^{(3)} + c_{24}^2 P_2^{(3)} + c_{34}^2 P_3^{(1)}} \right). \quad (17)$$

Combining (15) and (17), node 4 can decode v_{2r} if

$$R_{2,r_2}^{R_x} \leq \min \left\{ \max \left(R_{2,2r_1}^{R_x} \right), \max \left(R_{2,2r_2}^{R_x} \right) \right\}. \quad (18)$$

Node 4 can then decode v_s , if the rate of v_s satisfies

$$R_{2,s}^{R_x} \leq \lambda_3 C \left(c_{34}^2 P_3^{(1)} / (1 + c_{14}^2 P_1^{(3)} + c_{24}^2 P_2^{(3)}) \right). \quad (19)$$

After decoding v_{2r} and v_s , node 4 decodes w_{2r} if

$$R_{1,2r_1}^{R_x} \leq \lambda_3 C \left(c_{14}^2 P_1^{(3)} / (1 + c_{24}^2 P_2^{(3)}) \right). \quad (20)$$

Combining (13) and (20), node 3 can decode w_{2r} if

$$R_{1,r_2}^{R_x} \leq \min \left\{ \max \left(R_{1,2r_1}^{R_x} \right), \max \left(R_{1,2r_2}^{R_x} \right) \right\}. \quad (21)$$

³Note that w_{2r} is the message decoded at phase 3 of the previous block. It is re-encoded as $\mathbf{X}_2(w_{2r})$ and relayed to the intended receiver.

Finally, node 4 can decode v_d if the rate of v_d satisfies

$$R_{2,d}^{R_x} \leq \lambda_3 C \left(c_{24}^2 P_2^{(3)} \right). \quad (22)$$

We now consider the decoding of w_{1r} and v_{1r} . By decoding w_s and v_s , a compressed version of the signals received in phase 1 have been exchanged between the receivers. Let σ_1^2 and σ_2^2 be the compressing noise of the received signal at node 3 and 4 respectively. Using similar derivations as in [10], σ_1^2 and σ_2^2 are given by

$$\sigma_1^2 = \frac{(1 + \mathbf{g}_1 \Sigma_x \mathbf{g}_1^T) (1 + \mathbf{g}_2 \Sigma_x \mathbf{g}_2^T) - (\mathbf{g}_1 \Sigma_x \mathbf{g}_2^T)^2}{(2^{R_{2,s}^{R_x}} / \lambda_1 - 1) (1 + \mathbf{g}_2 \Sigma_x \mathbf{g}_2^T)} \quad (23)$$

$$\sigma_2^2 = \frac{(1 + \mathbf{g}_2 \Sigma_x \mathbf{g}_2^T) (1 + \mathbf{g}_1 \Sigma_x \mathbf{g}_1^T) - (\mathbf{g}_1 \Sigma_x \mathbf{g}_2^T)^2}{(2^{R_{1,s}^{R_x}} / \lambda_1 - 1) (1 + \mathbf{g}_1 \Sigma_x \mathbf{g}_1^T)} \quad (24)$$

where $\Sigma_x = \text{diag} \{ P_1^{(1)}, P_2^{(1)} \}$ is a 2×2 diagonal matrix.

As discussed in [10], since each receiver has a noisy version of the received signal at the other receiver, the network is equivalent to an IC with two receive antennas at each receiver. After normalizing the noise power to 1 for all receive ‘‘antennas’’, the equivalent channel gains between the transmit and receive node pairs are given as $\mathbf{c}_{13} = [c_{13} \sqrt{\zeta_2} c_{14}]^T$, $\mathbf{c}_{23} = [c_{23} \sqrt{\zeta_2} c_{24}]^T$, $\mathbf{c}_{14} = [\sqrt{\zeta_1} c_{13} \ c_{14}]^T$ and $\mathbf{c}_{24} = [\sqrt{\zeta_1} c_{23} \ c_{24}]^T$, where $\zeta_i = 1/(1 + \sigma_i^2)$. Let $\mathbf{SNR}_1 = \mathbf{c}_{13} \mathbf{c}_{13}^T P_1^{(1)}$, $\mathbf{INR}_1 = \mathbf{c}_{23} \mathbf{c}_{23}^T P_2^{(1)}$, $\mathbf{SNR}_2 = \mathbf{c}_{24} \mathbf{c}_{24}^T P_1^{(1)}$ and $\mathbf{INR}_2 = \mathbf{c}_{14} \mathbf{c}_{14}^T P_2^{(1)}$.

The capacity region of a 1-transmit-2-receive antennas IC is not known except for the strong interference case [16] ($\|\mathbf{c}_{14}\|^2 \geq \|\mathbf{c}_{13}\|^2$ and $\|\mathbf{c}_{23}\|^2 \geq \|\mathbf{c}_{24}\|^2$). In this case, the messages w_{1r} and v_{1r} can be decoded if their respective rate $R_{1,r_1}^{R_x}$ and $R_{2,r_1}^{R_x}$ satisfy [16]

$$R_{1,r_1}^{R_x} \leq \lambda_1 \mathbf{C}(\mathbf{SNR}_1) \quad (25)$$

$$R_{2,r_1}^{R_x} \leq \lambda_1 \mathbf{C}(\mathbf{SNR}_2) \quad (26)$$

$$R_{1,r_1}^{R_x} + R_{2,r_1}^{R_x} \leq \lambda_1 \min_{i=1,2} \{ \mathbf{C}(\mathbf{SNR}_i + \mathbf{INR}_i) \}. \quad (27)$$

When $\|\mathbf{c}_{14}\|^2 \geq \|\mathbf{c}_{13}\|^2$, $\|\mathbf{c}_{23}\|^2 < \|\mathbf{c}_{24}\|^2$, node 4 can eliminate the interference by completely decoding the message transmitted from node 1. Node 3 can decode by treating the interference from node 2 as noise. Thus, the achievable rates of w_{1r} and v_{1r} are respectively

$$R_{1,r_1}^{R_x} \leq \lambda_1 \log(|\mathbf{I} + \mathbf{SNR}_1 + \mathbf{INR}_1| / |\mathbf{I} + \mathbf{INR}_1|) \quad (28)$$

$$R_{2,r_1}^{R_x} \leq \lambda_1 \mathbf{C}(\mathbf{SNR}_2). \quad (29)$$

Similarly, when $\|\mathbf{c}_{14}\|^2 < \|\mathbf{c}_{13}\|^2$, $\|\mathbf{c}_{23}\|^2 \geq \|\mathbf{c}_{24}\|^2$, the achievable rates of w_{1r} and v_{1r} are respectively

$$R_{1,r_1}^{R_x} \leq \lambda_1 \mathbf{C}(\mathbf{SNR}_1) \quad (30)$$

$$R_{2,r_1}^{R_x} \leq \lambda_1 \log(|\mathbf{I} + \mathbf{SNR}_2 + \mathbf{INR}_2| / |\mathbf{I} + \mathbf{INR}_2|). \quad (31)$$

When $\|\mathbf{c}_{14}\|^2 < \|\mathbf{c}_{13}\|^2$ and $\|\mathbf{c}_{23}\|^2 < \|\mathbf{c}_{24}\|^2$, node 3 and node 4 can decode w_{1r} and v_{1r} respectively by treating interference as noise. The achievable rates of w_{1r} and v_{1r} are then given by (28) and (31).

B. Outer Bound

The single user upper bounds in (9) are also upper bounds under RC. Further, if we let $c_{34} = \infty$, the channel becomes a two user 1-transmit-2-receive antenna MIMO MAC. Thus, the achievable region is also bounded by this MIMO MAC capacity, which is given by [13]

$$R_1^+ + R_2^+ \leq \mathbf{C}(\mathbf{h}_1^T P_1 \mathbf{h}_1 + \mathbf{h}_2^T P_2 \mathbf{h}_2). \quad (32)$$

V. NUMERICAL EXAMPLES

We compare our achievable region to some known results through numerical examples. We focus on the symmetric channel case (similar results can be shown for the asymmetric case). We set the direct channel gains as $c_{13} = c_{24} = 1$, the cross channel gains as $c_{14} = c_{23} = \sqrt{2}$ and the average power constraints $P_i = 5$, $i = 1, 2, 3, 4$.

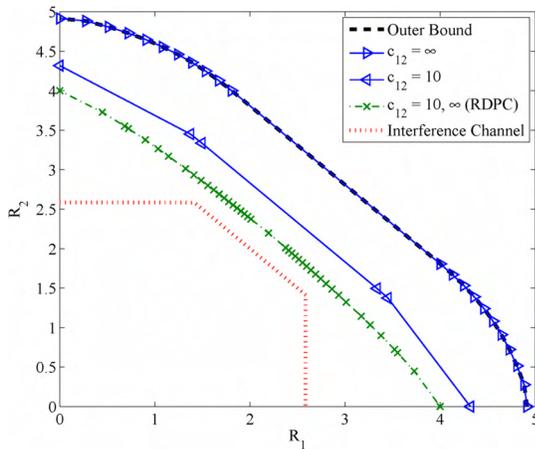


Fig. 2. Achievable regions for transmitter cooperation and RDPC.

Fig. 2 compares the achievable region from the proposed TC scheme with RDPC [1], [2]. It is shown that the achievable region using our TC scheme is significantly larger than using RDPC. Further, the capacity gain of the proposed TC scheme increases with the cooperation channel gain: As we increase the cooperation channel gain from $c_{12} = 10$ to ∞ , the achievable region meets the outer bound. On the other hand, the achievable region of RDPC does not increase as long as the cooperation channel is not a capacity threshold (see equations (8) and (9) in [1]). The achievable regions are also compared to the capacity of a standard strong IC (without node cooperation). It is clear that by allowing node cooperation, the achievable region increases significantly.

Fig. 3 shows the achievable regions for both TC and RC. Similar to TC, the achievable region of RC also increases with cooperation channel gain. When $c_{34} = \infty$, the achievable region of RC overlaps with the outer bound. The RC achievable region is also compared with TC. When $c_{12} = c_{34} = 10$, the achievable region of TC is strictly larger than RC. When $c_{12} = c_{34} = \infty$, both schemes meet their respective outer bound. However, due to the single user half-duplex relay

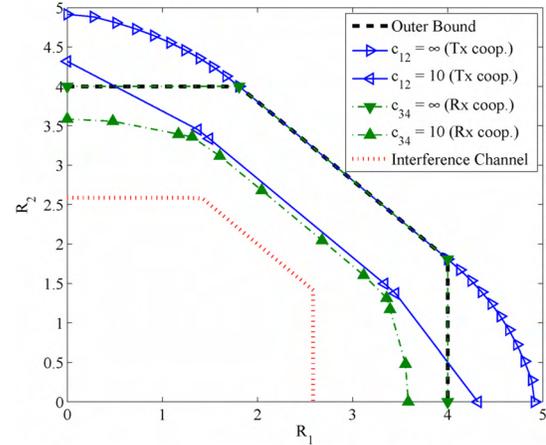


Fig. 3. Achievable regions for transmitter and receiver cooperation.

channel capacity constraints (see (9)), RC achieves less single user rates under the assumed channel conditions.

Bridging the gap between the outer bound and the achievable region for finite cooperative channel gains should be considered in future works.

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