Bounds on the Capacity of the Soft Handover Channel

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***Abstract*— In this paper, we introduce the model for a Gaussian soft handover channel (SHC) and investigate an achievable region for its capacity. The achievable region is given by the convex hull of the union of certain multiple access and interference channels. Some properties of the achievable region are studied.**

I. Introduction

Computing the capacity of wireless networks is an area with a rich history and many significant contributions have been made. Although the general problem remains unsolved, significant results have been obtained in many special cases *e.g.* relay channel, multiple access channel (MAC), and broadcast channel [1].

In this paper, we introduce the model for a soft handover channel (SHC), which can be considered as a modification of the well known interference channel (IC) [1]. The main difference between an IC and SHC is that in the SHC there is no designated receiver for each user. The transmit signals of all users are received at multiple distinct receivers and each of these transmit signals can be decoded from any one receiver or a combination of receivers. However, the received signals from multiple distinct receivers cannot be collated together for decoding. It is possible for one part of a message to be decoded at receiver 1 and another part of the message to be decoded at receiver 2. Further, since both receivers would forward decoded information to a backbone server, the complete information can be obtained at that server. Such a scheme only requires nominal coordination messages between the receivers as opposed to a case where entire received signals are exchanged.

A SHC naturally arises in many different scenarios [2], *e.g.* in a cellular system, due to user mobility the user's signal is received at more than one base station with varied gains. However, typically, data is decoded separately at each base station and if a valid cyclic redundancy check (CRC) is obtained at either of the base stations, that decoded signal is used.

The capacity region of the interference channel (IC) has been widely investigated (see [3] for a comprehensive survey on interference channels). The capacity region is completely known only in a few selected scenarios; for instance, with *strong* or *very strong* interference [4], [5], [6], [7], [8], [9], [10]. In general, only outer and inner bounds on the capacity region are known. The outer bound in [11] improves the outer bounds in [4], [10] and is currently the best known outer bound on the capacity of a Gaussian interference channel. In a recent work, Sason [12] provides a simpler way of calculating an achievable region for the Gaussian interference channel. Achievable regions for the Z-channel which is a special case of the IC is considered in [13], [14].

The main results of the paper can be summarized as follows. We compute an achievable region for the two user Gaussian soft handover channel. The achievable region is obtained as the convex hull of the union of the achievable regions of certain multiple access and interference channels. A simple MIMO capacity based outer bound on the SHC capacity is also given.

The interesting and crucial result is that the achievable region of the SHC is not dominated by the achievable region of any one of the component MAC/IC. The implication of this result is that to achieve all points on the boundary of the SHC achievable region it is not optimal to pick a unique location where each user is decoded. For instance, choosing the receiver where the signal-to-interference-plus-noise ratio (SINR) ratio is highest for each user does not achieve the boundary of the achievable region. Thus, one should investigate handover methods based on system requirements to optimally tradeoff the transmission rates for the two users.

In the special case of a symmetric SHC (formally defined in Section II), with equal transmission powers for the two users, the achievable region of the SHC is dominated by the achievable region of an interference channel. Also, for a Gaussian Z-soft handover channel, the achievable region is dominated by the achievable region of a MAC or IC depending on the cross channel gain.

We consider a time-invariant channel. The results are also given for an uplink system and can be easily extended to the downlink scenario.

The rest of this paper is organized as follows. We introduce the system notation in Section II. An achievable region and outer bound for SHC are given in Section III. We conclude in Section IV.

II. System and Problem Formulation

Consider a system with two users transmitting signals X_1 and X_2, respectively, which are then received by two receivers. Received signals Y_1 and Y_2 at the two receivers are given by

$$Y_1 = h_{11}X_1 + h_{21}X_2 + Z_1, \quad (1)$$

$$Y_2 = h_{12}X_1 + h_{22}X_2 + Z_2, \quad (2)$$

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where, Z_i represents the additive Gaussian noise at the i^{th} receiver and h_{ij} is the channel gain between user i and receiver j. Let P_i be the power constraint on signal X_i and σ_i^2 represent the variance of Z_i. This interference channel can be converted into an equivalent interference channel in *standard* form [9] given by,

$$
\begin{aligned}
Y_1 &= X_1 + c_{21}X_2 + N_1, && (3)\\
Y_2 &= c_{12}X_1 + X_2 + N_2, && (4)
\end{aligned}
$$

where, $c_{ij} = \sqrt{\frac{h_{ij}^2\sigma_i^2}{h_{ii}^2\sigma_j^2}}$ and N_i is unit variance Gaussian noise. Further, the power constraint is now modified as $\mathbb{E}[X_i^2] \leq \frac{h_{ii}^2\bar{P}_i}{\sigma_i^2} = P_i$. We will focus on this modified channel model in the rest of the paper. Denote by $\mathbf{c}$ the matrix of channel gains, *i.e,* $\mathbf{c} = \begin{bmatrix} 1 & c_{21} \\ c_{12} & 1 \end{bmatrix}$. If $c_{12} = c_{21}$, we refer to the SHC as a symmetric SHC.

In the traditional interference channel model, the signal of user i is decoded at receiver i from received signal Y_i. Recognize that user i's signal can also be decoded from signal $Y_j, j \neq i$ when $c_{ij} \neq 0$. In practical systems, such situations arise when users are in soft-handover mode. In such a situation when the users are in soft-handover mode, each of their data are allowed to be decoded from either received signal. The maximum transmission rate for each user is defined as the maximum sum of the rates at which independent data from a user can be decoded at the various receivers.

Definition 1 (Soft Handover Channel): Consider a network consisting of M users transmitting signal that is received at R receivers. Each of the transmitted signals has to be decoded from at least one of the received signals; clearly the signals of two different users may be decoded from different receivers. The receivers are not allowed to combine their received signals to jointly decode the transmitted information. However, different parts of a users message may be decoded at different receivers.

For simplicity, we consider $M = R = 2$ in this paper.

Definition 2 (Achievable region for SHC): A rate pair (R_1, R_2) is achievable if there exists a sequence of $((2^{nr_{11}}, 2^{nr_{12}}, 2^{nr_{21}}, 2^{nr_{22}}), n)$ codes over a product probability distribution $p(x_{11})p(x_{12})p(x_{21})p(x_{22})$ with arbitrarily small average probability of error. Note that $R_1 = r_{11} + r_{12}$ and $R_2 = r_{21} + r_{22}$, where r_{ij} is the rate of transmission from user i to receiver j.

A $((2^{nr_{11}}, 2^{nr_{12}}, 2^{nr_{21}}, 2^{nr_{22}}), n)$ code consists of message sets $\{1, \ldots, 2^{nr_{ij}}\}$ encoding functions e_i and decoding functions d_j. The details are similar to multiuser codes defined in [1] and are not repeated here.

III. Achievable region for SHC

Let $R_{IC}(c_{12}, c_{21}, P_1, P_2)$ denote the capacity of the interference channel (3)-(4) with power constraints P_1, P_2. Similarly denote by $R_{MAC}(1, c_{21}, P_1, P_2)$ the capacity of the multiple access channel (3) under the same power constraints.

Theorem 1 (Achievable region for Gaussian SHC): An achievable region for the Gaussian SHC is given by the convex hull of the union of the following four achievable regions:

$$
\begin{aligned}
&R_{MAC}(1, c_{21}, P_1, P_2), && (5)\\
&R_{MAC}(c_{12}, 1, P_1, P_2), && (6)\\
&R_{IC}(c_{21}, c_{12}, P_1, P_2) && (7)\\
&R_{IC}(1/c_{21}, 1/c_{12}, c_{12}^2P_1, c_{21}^2P_2) && (8)
\end{aligned}
$$

Proof: In the interest of space, we only provide a sketch of the proof. The achievablity of each of the four regions mentioned is direct from prior results on capacity of MAC [1] and bounds on interference channel [3]. The remainder of the proof follows directly from standard time-sharing arguments. □

Note that a larger achievable region can be computed by allowing a generalized time and power sharing similar to [12]. However, the numerical computation of such an achievable region is cumbersome and is not considered in this paper.

Denote by R_{ij} the rate at which user i can be decoded from received signal Y_j. We will consider the following special cases depending on the value of $\mathbf{c}$. In each case, there are four possible decoding scenarios.

<u>**Case 1:**</u> $c_{12} \geq 1$ and $c_{21} \geq 1$

1) *Both users decoded from signal Y_1:* In this case, the effective channel of interest is a multiple access channel (MAC) and the capacity region is given by,

$$
\begin{aligned}
R_{11} &\leq 0.5\log(1 + P_1), && (9)\\
R_{21} &\leq 0.5\log(1 + c_{21}^2P_2), && (10)\\
R_{11} + R_{21} &\leq 0.5\log(1 + P_1 + c_{21}^2P_2). && (11)
\end{aligned}
$$

All logarithms are to base 2 in this paper.

2) *Both users decoded from signal Y_2:* Again, the effective channel is a MAC and the capacity region is given by,

$$
\begin{aligned}
R_{12} &\leq 0.5\log(1 + c_{12}^2P_1), && (12)\\
R_{22} &\leq 0.5\log(1 + P_2), && (13)\\
R_{12} + R_{22} &\leq 0.5\log(1 + c_{12}^2P_1 + P_2). && (14)
\end{aligned}
$$

3) *User 1 decoded from Y_1 and User 2 decoded from Y_2:* In this case, the effective channel behaves like a *strong* interference channel and the capacity region is given by

$$
\begin{aligned}
R_{11} &\leq 0.5\log(1 + P_1), && (15)\\
R_{22} &\leq 0.5\log(1 + P_2), && (16)\\
R_{11} + R_{22} &\leq 0.5\log(1 + c_{12}^2P_1 + P_2), && (17)\\
R_{11} + R_{22} &\leq 0.5\log(1 + P_1 + c_{21}^2P_2). && (18)
\end{aligned}
$$

Clearly, this capacity region (15)-(18) is contained within the intersection of the capacity regions of the two MAC channels above (9)-(14).

4) *User 1 decoded from Y_2 and User 2 decoded from Y_1:* In this case, the effective channel is still an interference channel with channel gain matrix given by $c' = \begin{bmatrix} c_{12} & 1 \\ 1 & c_{21} \end{bmatrix}$. This interference channel can be converted into an equivalent interference channel [9]

with $\tilde{c} = \begin{bmatrix} 1 & 1/c_{21} \\ 1/c_{12} & 1 \end{bmatrix}$ and new power constraints $\tilde{P}_1 = c_{12}^2 P_1$ and $\tilde{P}_2 = c_{21}^2 P_2$. The capacity of this equivalent interference channel is not known in general. In a recent paper by Sason [12] an achievable region for the SHC is given as a combination of the time division/frequency division multiplexing region and time-sharing between two different points similar to the two corner points in a MAC capacity region. We extend this achievable region in ([12], Theorem 1) to include the rate pair given by single user decoding (22)-(23). The resulting achievable region, D, is given by,

$$
\begin{aligned}
D \quad &= \quad \mathrm{co} \cup_{\alpha,\beta,\lambda\in[0,1]} \{(R_1, R_2) : \qquad (19) \\
R_1 \leq \lambda \quad &\log \left(1 + \frac{\alpha P_1}{\lambda}\right) + \bar{\lambda} \log \left(1 + \frac{c_{12}^2 \bar{\alpha} P_1}{\lambda + \beta P_2}\right), \\
R_2 \leq \bar{\lambda} \quad &\log \left(1 + \frac{\bar{\beta} P_2}{\lambda}\right) + \lambda \log \left(1 + \frac{c_{21}^2 \beta P_2}{\lambda + \alpha P_1}\right) \Bigg\} \\
\cup \{(R_1, \quad R_2 \quad &) : R_1 \leq 0.5 \log \left(1 + \frac{c_{12}^2 P_1}{1 + P_2}\right), \\
R_2 \quad &\leq \quad 0.5 \log \left(1 + \frac{c_{21}^2 P_2}{1 + P_1}\right) \Bigg\}
\end{aligned}
$$

where $\bar{x} = 1 - x$, α, β, λ are equivalent time-sharing parameters and *co* represent the convex hull operation.

Case 2: $c_{12} \leq 1$ and $c_{21} \geq 1$. As before, there are four possible decoding scenarios.

1) *Both users decoded from signal Y_1* In this case, the effective channel is a MAC with capacity given by (9)-(11).
2) *Both users decoded from signal Y_2* Again, the effective channel is a MAC with capacity given by (12)-(14).
3) *User 1 decoded from Y_1 and User 2 decoded from Y_2* The capacity region is not completely known in this case. The best known achievable region is again given by (19).
4) *User 1 decoded from Y_2 and User 2 decoded from Y_1* The capacity region is not completely known in this case. The best known achievable region is again given by (19).

Case 3: $c_{12} \leq 1$ and $c_{21} \leq 1$. This case is similar to case 1 and is not discussed further in this paper.

Case 4: $c_{12} \geq 1$ and $c_{21} \leq 1$. This case is similar to case 2 and is not discussed further.

Next, we evaluate the achievable rates using simple transmission schemes like FDMA/TDMA and using a simple single user receiver which treats the interference as noise.

A. FDMA/TDMA achievable region

Using only FDMA/TDMA the achievable rate region in a SHC is easily computed as,

$$
\begin{aligned}
R_1 &\leq \max\left\{0.5\alpha \log\left(1 + \frac{P_1}{\alpha}\right), 0.5\alpha \log\left(1 + \frac{c_{12}^2 P_1}{\alpha}\right)\right\} \\
&= 0.5\alpha \log\left(1 + \frac{P_1 \max\{1, c_{12}^2\}}{\alpha}\right), \qquad (20) \\
R_2 &\leq \max\left\{0.5(1-\alpha) \log\left(1 + \frac{P_2}{(1-\alpha)}\right),\right. \\
&\qquad \left. 0.5(1-\alpha) \log\left(1 + \frac{c_{21}^2 P_2}{(1-\alpha)}\right)\right\} \\
&= 0.5(1-\alpha) \log\left(1 + \frac{P_2 \max\{1, c_{21}^2\}}{(1-\alpha)}\right) \qquad (21)
\end{aligned}
$$

where, $0 \leq \alpha \leq 1$, represents the fraction of bandwidth/time-slot allocated to user 1. Essentially, the FDMA region is obtained by decoding each user at the receiver where its signal is received with highest SNR [1] and allowing a dynamic sharing of the available spectrum/time-slots.

It can be easily shown that the maximum sum rate using FDMA is attained by allocating the total bandwidth to the two users proportional to the received powers. Consequently the individual transmission rates $R^*_{1,FDMA}, R^*_{2,FDMA}$ that maximize the sum rate are given by,

$$
\begin{aligned}
R^*_{1:FDMA} &= \frac{P_1 x_1}{2(P_1 x_1 + P_2 x_2)} \log(1 + P_1 x_1 + P_2 x_2), \\
R^*_{2:FDMA} &= \frac{P_2 x_2}{2(P_1 x_1 + P_2 x_2)} \log(1 + P_1 x_1 + P_2 x_2),
\end{aligned}
$$

where, $x_1 = \max(c_{12}^2, 1)$ and $x_2 = \max(c_{21}^2, 1)$ and the maximum sum rate using FDMA equals $0.5 \log(1 + P_1 x_1 + P_2 x_2)$.

B. Single user decoding (SUD) achievable region

Using SUD, the achievable rates for a SHC are given by

$$
\begin{aligned}
R_1 &\leq \max\left\{0.5 \log\left(1 + \frac{P_1}{(P_2 c_{21}^2 + 1)}\right),\right. \\
&\qquad \left. 0.5 \log\left(1 + \frac{c_{12}^2 P_1}{(P_2 + 1)}\right)\right\} \qquad (22) \\
&= 0.5 \log\left(1 + P_1 \max\left\{\frac{1}{(P_2 c_{21}^2 + 1)}, \frac{c_{12}^2}{(P_2 + 1)}\right\}\right), \\
R_2 &\leq \max\left\{0.5 \log\left(1 + \frac{P_2}{(P_1 c_{12}^2 + 1)}\right),\right. \\
&\qquad \left. 0.5 \log\left(1 + \frac{c_{21}^2 P_2}{(P_1 + 1)}\right)\right\} \qquad (23) \\
&= 0.5 \log\left(1 + P_2 \max\left\{\frac{1}{(P_1 c_{12}^2 + 1)}, \frac{c_{21}^2}{(P_1 + 1)}\right\}\right).
\end{aligned}
$$

In this case, each user is decoded at the receiver where its SINR is maximized.

Next, we evaluate the SHC capacity bounds in a few special cases.

[1] since there is no interference.

C. Symmetric SHC

We now consider a symmetric SHC, *i.e.*. $c_{12} = c_{21} = c$ and further let the transmit powers P_1 and P_2 be equal. In this case, it turns out that the achievable region of the SHC is dominated by the achievable region of an interference channel.

Theorem 2 (Symmetric Gaussian SHC achievable region): For a symmetric Gaussian SHC, the achievable region of one of the interference channels dominates the achievable region of the SHC.

Proof: Again, due to space limitations, we only provide a sketch of the proof. Without loss in generality we assume $c > 1$. It is sufficient to show that the two significant corners of each of the possible MAC (for instance points A,B,C and D in Figure 1) are contained within the achievable region of one of the interference channels (19). Recognize that two of the corner points are already included in (19). Clearly, the convex hull in (19) includes the straight line segment between these two corner points. By writing the equation for this straight line, it can be easily verified that the two other MAC corner points lie on this line. □

The implication of this theorem is that in the symmetric Gaussian SHC, it is sufficient to select one unique location for decoding each user to achieve the boundary of the achievable region. The plot of the achievable region for a symmetric Gaussian SHC is given in Figure 1. In this case, the maximum sum-rate is attained using SUD. The figure also clearly shows that the achievable region of the two MAC's are contained inside the achievable region of the IC.

D. Asymmetric SHC

In this case, it turns out that neither of the four possible achievable regions (2 MAC and 2 IC regions) dominate the achievable region for the SHC. Although we have not yet analytically proved this statement, numerical examples illustrate this result.

Consider a scenario with cross over channel gains $c_{12}^2 = 0.1$ and $c_{21}^2 = 0.9$. Such cross over gains occur frequently in practical cellular systems when one user is midway between two base stations and the other user is very close to one base station and far from the second base station. Let $P_1 = P_2 = 6$. The achievable region for SHC is given in Figure 2. It can be clearly seen that point C, which is one of the corners of the MAC capacity region, is not contained in the achievable region for IC.

E. Sum-rate in SHC

For a MAC, the sum rate is always attained by FDMA.[2] For a general MAC, the achievable sum rate using SUD is strictly lesser than the sum-rate using FDMA.[3]

For an interference channel, however, the maximum sum rate is not always attained by either of FDMA or SUD. For very low values of interference, the sum rate using SUD is greater than FDMA and for higher interference levels, the reverse is true.

It turns out that for the SHC, not surprisingly, the sum rate behavior using FDMA and SUD is similar to the interference channel. For a symmetric $\mathbf{c}$, the maximum rate achieved using SUD is greater than the FDMA rate if i) $P \geq \frac{1-2c^2}{2c^4}$ and $c^2 \leq 1$ or ii) $P \geq \frac{c^2-2}{2}$ and $c^2 \geq 1$. Similar conditions can be obtained for an asymmetric channel. The region where sum-rate of SUD is greater than FDMA is referred to as *weak* interference and the region where sum-rate using FDMA is greater than SUD is referred to as *moderate* interference [5]. The plot of the sum-rate is given in Figure 3. The behavior of sum-rate of SHC is similar to sum-rate behavior in an IC [5].

F. Special case $c_{12} = 0$: Z-channel under soft handover

In the special case of $c_{12} = 0$, there are only two possibilities for decoding the users; user 1 has to be decoded from Y_1 whereas user 2 can be decoded from either Y_1 or Y_2.

If $c_{21} \geq 1$ then decoding user 2 at receiver 1 is better than decoding user 2 at receiver 2. In this case, the MAC capacity region dominates the SHC achievable region. If $c_{21} \leq 1$ the MAC region does not contain the rate pair obtained by single user decoding of both users at the respective receivers. In this case, the interference channel achievable region dominates the SHC achievable region. Note that improved bounds on the SHC capacity may be obtained in this special case by utilizing the bounds in [14].

G. MIMO system outer bound

In this paper, we do not allow a centralized processing (decoding) of the received signals from the multiple receivers. Such an approach would lead to the channel reducing to a standard multiple input multiple output (MIMO) channel, the capacity region for which is known in many settings [15]. However, this MIMO channel capacity provide an outer bound on the capacity of the SHC as follows:

$$R_{i:MIMO} \leq \log(\mu_i \lambda_i), \; i = 1, 2, \tag{24}$$

where, λ_i are the square root of the singular values of $[c(i,1)\, c(i,2)]$ and $\mu_i = P_i + 1/\lambda_i$. Further a bound on the sum rate is obtained as

$$R_{1:MIMO} + R_{2:MIMO} \leq \sum_{i=1}^{2} \log(\mu \gamma_i)^+, \tag{25}$$

where, μ is selected using water-filling as $P_1 + P_2 = \sum_{i=1}^{2} (\mu - 1/\gamma_i)^+$, γ_i is the eigenvalue of $\mathbf{c}^T \mathbf{c}$ and $(x)^+ = \max(x, 0)$.

It turns out that for the scenarios we considered, these outer bounds are quite loose on SHC capacity and hence are not shown in the figures.

IV. Conclusions

In this paper, we introduced the model for a soft handover channel and computed bounds on its capacity. The results in this paper should be extended to include arbitrary number of users and receivers. Considering non-Gaussian fading channels would be another way to extend the current work.

[2]Recognize that the FDMA region touches the MAC capacity region, which is a pentagon, at one point along the *significant* edge of the pentagon.

[3]Except in the trivial case where MAC capacity pentagon reduces to a rectangle.

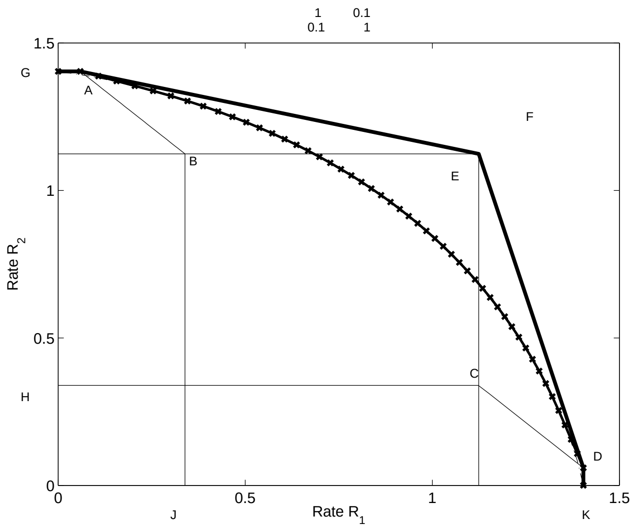

Fig. 1. An achievable region for a symmetric Gaussian SHC. The capacity region of the two MAC regions are given by pentagons [0 G A B J 0] and [0 H C D K 0], respectively. Point E gives the achievable rate using SUD and the solid line passing through points A, E and D is the achievable region for the SHC and one of the constituent IC. The line with $\times$ markers is the achievable region using FDMA for the SHC. Transmit powers $P_1 = P_2 = 6$.

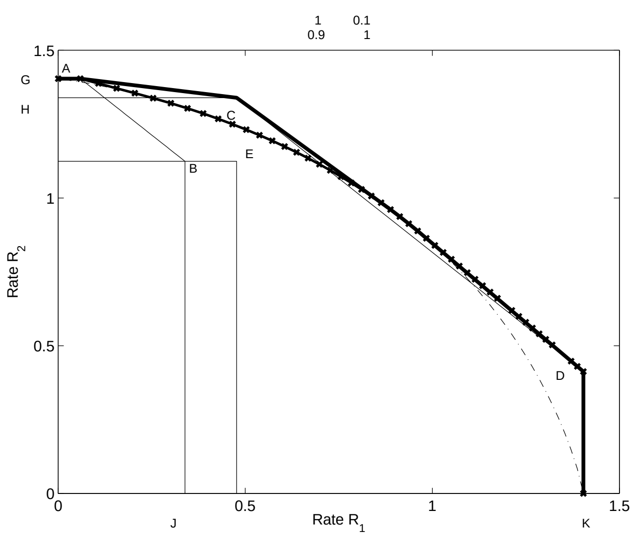

Fig. 2. An achievable region for a symmetric Gaussian SHC. The capacity region of the two MAC regions are given by pentagons, [0 G A B J] and [0 H C D K] respectively. Point E gives the achievable rate using SUD and the line passing through points A, C and D is the achievable region for the SHC. The achievable region for one of the IC is given by the line with $\times$ markers. The dash-dot line is the achievable region using FDMA and partially overlaps with the boundary of the achievable region of IC. Transmit power $P_1 = P_2 = 6$.

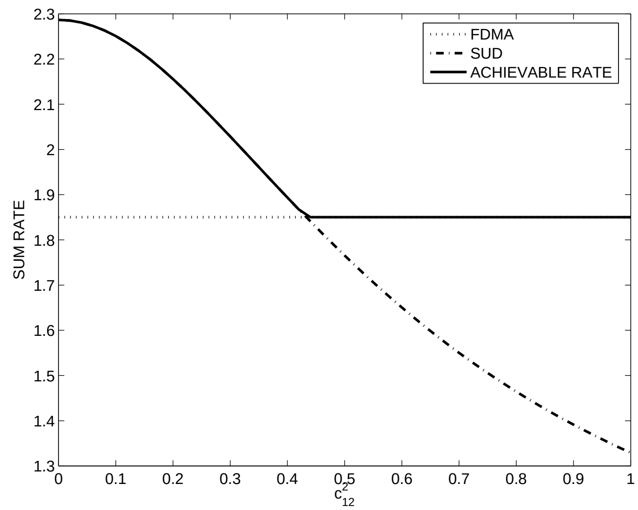

Fig. 3. The sum-rate in a SHC for FDMA, SUD along with achievable and outer bound. The transmit powers $P_1 = P_2 = 6$ and $c_{21}^2 = 0.5$. The sum rate of the SHC achievable region overlaps with the sum rate using SUD for small c_{12} and with the sum-rate using FDMA for large c_{12}. The sum rate with FDMA does not change since $c_{12}^2 \leq 1$ (20).

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