Bounds on the Capacity of the Gaussian Soft Handover Channel

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***Abstract*— In this paper, we introduce the model for a Gaussian soft handover channel (SHC), which adds a new dimension of flexibility to the well known interference channel (IC). We provide a unified framework for computing achievable rate regions for the soft handover channel in both the uplink and downlink cases. This achievable rate region for SHC is given by the convex hull of the union of certain multiple access, interference, broadcast and $Z-$channels. Some properties of the achievable region are studied. Specifically, we show the following key results: i) In an uplink SHC, there are channel conditions under which decoding at a single receiver based, for example, on a maximum SNR condition, does not achieve the entire boundary of the achievable region. ii) In a downlink SHC, multiple base stations should transmit independent information to all users to achieve the boundary points of the achievable region and iii) When a mobile communicates with multiple base stations, the ratio of uplink rates with different base stations could be different from the ratio of the downlink rates with those base stations. A simple outer bound on SHC capacity based on the capacity of MIMO systems is also given.**

***Index Terms*— Interference Channel, Multiuser capacity, Soft Handover.**

I. INTRODUCTION

Computing the capacity of wireless networks is an area with a rich history and several outstanding contributions have been made. Although the general multiterminal capacity problem remains unsolved, significant results have been obtained in many special cases *e.g.* relay channel, multiple access channel (MAC), and broadcast channel [1].

In this paper, we introduce the model for a soft handover channel (SHC), which adds a new dimension of flexibility to the well known interference channel (IC) [1]. Further, we classify SHC's into two types: the uplink SHC and the downlink SHC (formally defined in Section II). The principal difference between an uplink SHC and an IC is that in the former there is no designated receiver for each user. In a SHC, the transmit signals of all users are received at multiple distinct receivers and each of these transmit signals can be decoded from any one receiver or a combination of receivers. However, the received signals from multiple distinct receivers cannot be collated together for joint decoding. Similarly, the main difference between an IC and a downlink SHC is that there is no designated transmitter for each user in the later case. In a SHC, more than one transmitter might send different parts of the information destined for a particular user. In a SHC, we do not allow distributed MIMO [2], signal level cooperative [3], or cognitive [4] transmission strategies.

A SHC naturally arises in many practical scenarios, *e.g.* in a cellular system, due to node mobility a user's signal is received at more than one base station with varied strengths [5]. However, typically, data is decoded separately at each of the base stations and if a valid CRC is obtained at any of the base stations, that decoded signal is used. Applying the decoding mechanism proposed in this paper to such a cellular network, both base stations 1 and 2 would forward their independently decoded information via a backbone network to the intended destination (see Figure 1). Such a scheme only requires nominal coordination messages between the receivers as opposed to a case where entire received signals are exchanged.

We consider a multiuser system, for simplicity, with two transmitters and two receivers. For this system, we first compute a quadruple of achievable rates, between each transmitter-receiver pair. This achievable region is obtained as the convex hull of the union of the achievable region of certain multiple access, broadcast, interference and Z-channels. We refer to these channels as the component channels of the SHC (details in Section III). We then apply two different linear mapping from this rate-quadruple to two dimensional rate pairs: one mapping provides an achievable region for the uplink SHC and the other mapping provides an achievable region for the downlink SHC.

The main contributions of this paper can be succinctly summarized as follows.

- We propose a unified framework for computing the achievable region for the uplink and downlink soft handover channels. In particular, this framework provides the network operator a method for computing/selecting an operational point on the boundary of the achievable region for both uplink and downlink communication.
- In an uplink SHC, allowing each user to connect to a single receiver *might not* achieve the boundary of the achievable region. In other words, the achievable region of the uplink SHC might not be dominated by the achievable region of any one of the component channels. Thus, choosing (based, for example, on maximizing the signal-to-interference-plus-noise ratio (SINR)) a single receiver to decode each user might not achieve the boundary of the achievable region. Hence, a practical implication is that soft handoff is a critical requirement for improving uplink system performance.

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- Similar to the uplink SHC, the entire achievable region of the downlink SHC is not obtained by transmitting to each user from one location. In other words, all the transmitters might have to use a fraction of their powers to send information to all users.
- In the special case of a symmetric uplink SHC, with equal transmission powers for the two users, the achievable region of the SHC is dominated by the achievable region of an interference channel. Hence, in this special case, it is sufficient to pick a unique location (base station) to decode for each user. In contrast to the uplink SHC, for a symmetric downlink SHC with equal transmission powers, the entire achievable region is not attained by transmitting to each user from a unique location.
- If one of the two cross over channel gains equals zero (*i.e.*, channel is actually a Z-channel) and the nonzero cross over channel gain satisfies certain conditions, the entire achievable region of the uplink SHC is obtained by picking a unique receiver to decode for each user. However, in the downlink of a Z-channel, both the base stations need to transmit information to one of the users to obtain the entire achievable region.[1]
- As mentioned before, the network operator can use the framework to determine an operating point that optimizes a desired metric, *e.g.* maximum sum rate, pricing based fairness, maximize the minimum rate. To illustrate this flexibility, we consider optimization of a linear combination of rates and show that the proportion of rates at which a user communicates with the multiple base stations is different for uplink and downlink data transmission. For instance, depending on the channel gains, a user might receive its entire data rate from one base station in the downlink while in the uplink, the user might send independent data with nonzero rates to more than one base stations.
- Simple outer bounds based on capacity of single user MIMO and broadcast MIMO channels are also given.

In this paper, we construct Gaussian codebooks at the transmitter to compute the achievable region for the SHC. It should be noted that such a restriction to Gaussian code books are strictly suboptimal in certain scenarios even in the presence of only Gaussian noise [6]. We consider a time-invariant channel with additive white Gaussian noise at the receivers.

The rest of this paper is organized as follows. The system notation is introduced in Section II. Achievable regions for the uplink and downlink SHCs given in Section III. Numerical evaluation and some properties of the achievable region are discussed in Section IV. Section V provides concluding remarks.

II. System and Problem Formulation

Consider a multiterminal system with two transmitters sending signals X_1 and X_2, respectively, which are received by two receivers. Received signals Y_1 and Y_2 at the two receivers are given by,

$$Y_1 = h_{11}X_1 + h_{21}X_2 + Z_1, \quad (1)$$
$$Y_2 = h_{12}X_1 + h_{22}X_2 + Z_2, \quad (2)$$

where, Z_i represents the additive Gaussian noise at the i^{th} receiver and h_{ij} is the channel gain between user i and receiver j. Let $\bar{P}_i$ be the power constraint on signal X_i and σ_i^2 represent the variance of Z_i. This interference channel in (1)-(2) can be converted into an equivalent interference channel in *standard* form [7] given by,

$$Y_1 = X_1 + c_{21}X_2 + \tilde{Z}_1, \quad (3)$$
$$Y_2 = c_{12}X_1 + X_2 + \tilde{Z}_2, \quad (4)$$

where, $c_{ij} = \sqrt{\frac{h_{ij}^2\sigma_i^2}{h_{ii}^2\sigma_j^2}}$ and $\tilde{Z}_i$ is unit variance Gaussian noise. Further, the power constraint is now modified as $\mathbb{E}[X_i^2] \leq \frac{h_{ii}^2\bar{P}_i}{\sigma_i^2} = P_i$. We focus on this modified channel model in the rest of the paper. Denote by $\mathbf{c}$ the matrix of channel gains, *i.e*, $\mathbf{c} = \begin{bmatrix} 1 & c_{21} \\ c_{12} & 1 \end{bmatrix}$. Define $\mathcal{R}_{2\times 2}(c_{11}, c_{12}, c_{21}, c_{22}, N_1, N_2, P_1, P_2)$ as the quadruple of rates R_{ij} that can be achieved between the i^{th} transmitter and j^{th} receiver under the given transmit power constraints, where N_i represents the variance of the noise at the i^{th} receiver. Let $C(x) = 0.5\log(1+x)$ and $\bar{x} = 1 - x$. All logarithms are to base 2 in this paper.

A. Prior Work

Computing the quadruple of achievable rates for the general channel in (3)-(4) is still an open problem. However, the solution is known is many special cases as discussed below.

1) Multiple access channel (MAC): For the MAC channel between the two transmitters and the first receiver, the capacity region is given by $\mathcal{R}_{MAC}(c_{11}, ., c_{21}, ., N_1, ., P_1, P_2) = \text{co} \cup$

[1]For the other user, data is only received from one base station since one of cross over channel gains equals 0.

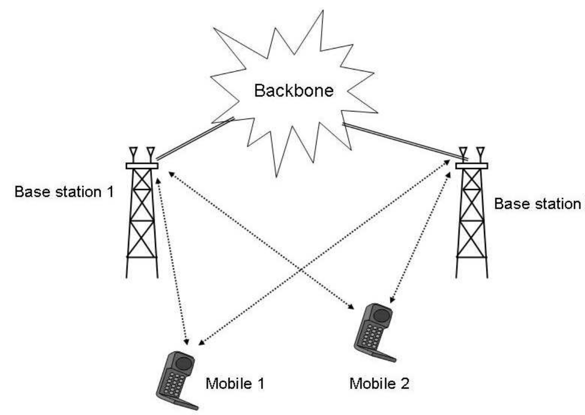


Fig. 1. Schematic of a Soft Handover Channel

$(R_{11}, 0, R_{21}, 0)$, where *co* represent the convex hull operation and R_{11} and R_{21} are such that

$$R_{11} \leq C\left(\frac{P_1 c_{11}^2}{N_1}\right) \quad (5)$$

$$R_{21} \leq C\left(\frac{P_2 c_{21}^2}{N_1}\right) \quad (6)$$

$$R_{11} + R_{21} \leq C\left(\frac{P_2 c_{21}^2 + P_1 c_{11}^2}{N_1}\right). \quad (7)$$

Since the capacity region does not depend on c_{12}, c_{22}, and N_2 these quantities are not explicitly shown as parameters in $\mathcal{R}_{MAC}$. Similar constraints can be written for the capacity of the MAC between the two transmitters and the second receiver.

2) Broadcast channel (BC): Although the capacity region for a broadcast channel is not completely known in the general case, the capacity region is completely specified for a degraded broadcast channel. The Gaussian broadcast channel is a degraded channel (with single transmit and receive antenna) and its capacity region is completely known [1]. For the broadcast channel from the first transmitter, the capacity region is given by $\mathcal{R}_{BC}(c_{11}, c_{12}, 0, 0, N_1, N_2, P_1, .) = \text{co} \cup (R_{11}, R_{12}, 0, 0)$ such that[2]

$$R_{11} \leq C\left(\frac{\alpha c_{11}^2 P_1}{N_1}\right) \quad (8)$$

$$R_{12} \leq C\left(\frac{\bar{\alpha} c_{12}^2 P_1}{N_2 + \alpha c_{12}^2 P_1}\right), \quad (9)$$

where, $0 \leq \alpha \leq 1$ and $\bar{\alpha} = 1 - \alpha$. In (8)-(9), it is assumed with out any loss in generality that $\frac{N_1}{c_{11}^2} \leq \frac{N_2}{c_{12}^2}$. Similar constraints can be written for the broadcast channel from the second transmitter.

3) Interference channel (IC): The capacity region of the interference channel (IC) has been widely investigated (see [8] for a comprehensive survey). The capacity region of an IC is completely known only in a few special scenarios; for instance, with *strong* or *very strong* interference [7], [9]–[14]. One surprising result is that the IC capacity region under *very strong* interference is the same as the capacity region when there is no interference. In general, only outer and inner bounds on the capacity region are known. The outer bound in [15] improves the outer bounds in [9], [14] and is currently the best known outer bound on the capacity of a Gaussian interference channel. The achievable region in [11] still remains the best known achievable region for an IC: however, computing that region is a prohibitively complex operation. In a recent work, Sason [16] provides a simple way of calculating an achievable region for the Gaussian interference channel. The achievable region in [16] is a combination of the time division/frequency division multiplexing region and time-sharing between two different rate-pairs similar to the two corner points in a MAC capacity region. We extend this achievable region ([16], Theorem 1) to include the rate pair given by single user decoding. The resulting achievable region,

[2]The capacity region can also be denoted as $\mathcal{R}_{BC}(c_{11}, c_{12}, ., ., N_1, N_2, P_1, 0)$.

$$\begin{aligned}
\mathcal{R}_{IC}&(c_{11}, c_{12}, c_{21}, c_{22}, N_1, N_2, P_1, P_2) = \text{co} \cup (R_{11}, 0, 0, R_{22}) \\
&= \text{co} \cup_{\alpha,\beta,\lambda \in [0,1]} \{(R_{11}, 0, 0, R_{22}) : \\
R_{11} &\leq \lambda C\left(\frac{\alpha c_{11}^2 P_1}{\lambda N_1}\right) + \bar{\lambda} \min\left\{C\left(\frac{c_{12}^2 \bar{\alpha} P_1}{N_2 \bar{\lambda} + c_{22}^2 \bar{\beta} P_2}\right),\right. \\
&\qquad \left. C\left(\frac{c_{11}^2 \bar{\alpha} P_1}{N_1 \bar{\lambda} + c_{21}^2 \bar{\beta} P_2}\right)\right\}, \\
R_{22} &\leq \bar{\lambda} C\left(\frac{c_{22}^2 \bar{\beta} P_2}{N_2 \bar{\lambda}}\right) + \lambda \min\left\{C\left(\frac{c_{21}^2 \beta P_2}{N_1 \lambda + c_{11}^2 \alpha P_1}\right),\right. \\
&\qquad \left. C\left(\frac{c_{22}^2 \beta P_2}{N_2 \lambda + c_{12}^2 \alpha P_1}\right)\right\}\Bigg\} \cup \{(R_{11}\ , 0, 0, R_{22}) : \\
R_{11} &\leq C\left(\frac{c_{11}^2 P_1}{N_1 + c_{21}^2 P_2}\right), R_{22} \leq C\left(\frac{c_{22}^2 P_2}{N_2 + c_{12}^2 P_1}\right)\Bigg\} \quad (10)
\end{aligned}$$

where $\bar{x} = 1 - x$ and α, β, and λ are equivalent time-sharing parameters. Several variations of IC capacity have been recently studied including the cognitive interference channel [4], the Gaussian Zig-zag channel [17], interference channel with common information [18] and game theoretic approaches to IC [19], [20].

4) Z-channel: Achievable regions for the Z-channel, which is a special case of the IC, is considered in [21], [22]. For the Gaussian Z-channel, the achievable region is given by $\mathcal{R}_Z(c_{11}, 0, c_{21}, c_{22}, N_1, N_2, P_1, P_2) = \text{co} \cup (R_{11}, 0, R_{21}, R_{22})$ such that

$$R_{11} \leq C\left(\frac{P_1}{(N_1 + c_{21}^2 \beta P_2)}\right) \quad (11)$$

$$R_{21} \leq C\left(\frac{c_{21}^2 (1 - \beta) P_2}{(N_1 + c_{21}^2 \beta P_2)}\right) \quad (12)$$

$$R_{11} + R_{21} \leq C\left(\frac{P_1 + c_{21}^2 (1 - \beta) P_2}{N_1 + c_{21}^2 \beta P_2}\right) \quad (13)$$

$$R_{22} \leq C\left(\frac{\beta P_2}{N_2 + (1 - \beta) P_2}\right), \quad (14)$$

where $0 \leq \beta \leq 1$ represents the fraction of power of transmitter 2 that is used to send information to receiver 2. The achievable region in (11)-(14) is under the condition $\frac{N_1}{N_2} < c_{21}^2 < \frac{N_1 + P_1}{N_2}$. If $c_{21}^2 \leq \frac{N_1}{N_2}$, the achievable region [21] is very similar to the region above with the difference that the bound in (14) is replaced by

$$R_{22} \leq C\left(\frac{\beta P_2}{N_2}\right). \quad (15)$$

Similarly, for $c_{21}^2 > \frac{N_1 + P_1}{N_2}$, the achievable region [22] is given by,

$$R_{11} \leq C\left(\frac{P_1}{N_1}\right) \quad (16)$$

$$R_{21} \leq C\left(\frac{c_{21}^2 (1 - \beta P_2)}{N_1}\right) \quad (17)$$

$$R_{11} + R_{21} \leq C\left(\frac{P_1 + c_{21}^2 (1 - \beta) P_2}{N_1}\right) \quad (18)$$

$$R_{22} \leq C\left(\frac{\beta P_2}{N_2 + (1 - \beta) P_2}\right). \quad (19)$$

In the special case of all three channel gains being unity, the capacity region for the Z-channel is completely specified in [22].

III. Soft Handover Channel

In this section, we formally define the uplink and downlink soft handover channels and provide achievable regions for the same.

A. Uplink Soft Handover Channel

In the traditional interference channel model, the signal of user i is decoded at receiver i from received signal Y_i. Recognize that user i's signal can also be decoded from signal $Y_j, j \neq i$ when $c_{ij} \neq 0$. In cellular systems, such situations arise when users are in soft-handover mode. In soft-handover mode, each of the mobile users data are allowed to be decoded from either received signal. Traditionally, the maximum transmission rate for each user is defined as the maximum rate at which the user's data can be decoded at any one of the receivers.

Definition 1 (Uplink SHC): Consider a network consisting of M users, each transmitting a signal that is received at N receivers. Each of the transmitted signals has to be decoded by at least one of the received signals; clearly the signals of two different users may be decoded from different receivers. The receivers are not allowed to combine their received signals to jointly decode the transmitted information. However, different receivers are allowed to decode different parts of the message sent by any user.
For simplicity, we consider $M = N = 2$ in this paper.

Definition 2 (Achievable Rates: uplink SHC): The achievable rate, R_i, for user i in an uplink SHC is the sum of the rates at which independent information transmitted from user i is decoded by all the receivers.

Given a 4-dimensional achievable region for the general 2×2 channel, an achievable region for the uplink SHC is given by the following mapping from 4-D space to 2-D space,

$$\phi_u(R_{11}, R_{12}, R_{21}, R_{22}) = (R_{11} + R_{12}, R_{21} + R_{22}). \quad (20)$$

B. Downlink Soft Handover Channel

Definition 3 (Downlink SHC): Consider a network consisting of M base stations transmitting signals that are received at N receivers. The message for each receiver can be sent from one transmitter or a combination of different transmitters. However, the transmitters are not allowed to "cooperatively" transmit the signals; the different transmitters may transmit different parts of the message for each user. Clearly, the signals of two different users may be transmitted from different base stations.

Definition 4 (Achievable Rates: downlink SHC): The achievable rate, R_i, for user i in an downlink SHC is the sum of the rates at which independent information from the different transmitters is decoded at user i.

Given a 4-dimensional achievable region for the general 2×2 channel, an achievable region for the downlink SHC is given by the following mapping from the $4-D$ space to $2-D$ space,

$$\phi_d(R_{11}, R_{12}, R_{21}, R_{22}) = (R_{11} + R_{21}, R_{12} + R_{22}) \quad (21)$$

C. Achievable region

Definition 5: (Achievable region for general multiterminal networks) A rate quadruple $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable if there exists a sequence of $((2^{nR_{11}}, 2^{nR_{12}}, 2^{nR_{21}}, 2^{nR_{22}}), n)$ codes with arbitrarily small average probability of error.

A $((2^{nR_{11}}, 2^{nR_{12}}, 2^{nR_{21}}, 2^{nR_{22}}), n)$ code and the error probability of a code are defined identical to [1] and are not repeated here.

*Theorem 6: (Achievable region for Gaussian multiterminal system with 2 transmitters and 2 receivers)*An achievable region, $\mathcal{R}_{2\times 2}(c_{11}, c_{12}, c_{21}, c_{22}, 1, 1, \bar{P}_1, \bar{P}_2)$, in $4-D$ space for the $M = N = 2$ network is given by

$$\begin{aligned}
\mathcal{R}_{2\times 2}(c_{11}, c_{12}, c_{21}, c_{22}, 1, 1, \bar{P}_1, \bar{P}_2) \quad &= \\
\alpha_1 R_{MAC}\left(c_{11}, 0, c_{21}, 0, 1, ., P_1^{(1)}, P_2^{(1)}\right) \quad &+ \\
\alpha_2 R_{MAC}\left(0, c_{12}, 0, c_{22}, ., 1, P_1^{(2)}, P_2^{(2)}\right) \quad &+ \\
+\alpha_3 R_{IC}\left(c_{11}, c_{12}, c_{21}, c_{22}, 1, 1, P_1^{(3)}, P_2^{(3)}\right) \quad &+ \\
\alpha_4 R_{IC}\left(c_{12}, c_{11}, c_{22}, c_{21}, 1, 1, P_1^{(4)}, P_2^{(4)}\right) \quad &+ \\
+\alpha_5 R_Z\left(c_{11}, 0, c_{21}, c_{22}, 1, 1 + P_1^{(5)} c_{12}^2, P_1^{(5)}, P_2^{(5)}\right) \quad &+ \\
\alpha_6 R_Z\left(c_{11}, c_{12}, 0, c_{22}, 1 + P_2^{(6)} c_{21}^2, 1, P_1^{(6)}, P_2^{(6)}\right) \quad &+ \\
+\alpha_7 R_{BC}\left(c_{11}, c_{12}, 0, 0, 1, 1, P_1^{(7)}, .\right) \quad &+ \\
\alpha_8 R_{BC}\left(0, 0, c_{21}, c_{22}, 1, 1, ., P_2^{(8)}\right) \quad & \quad (22)
\end{aligned}$$

where $\alpha_i > 0, P_j^{(i)} > 0, \sum_{i=1}^{8} \alpha_i = 1$ and $\sum_{i=1}^{8} \alpha_i P_j^{(i)} \leq \bar{P}_j,\ j = 1, 2$.

Proof: The MAC, IC, BC, and Z-channels are referred to as the constituent components of the SHC. The achievability of each of the individual constituent regions in (6) is direct from prior results on capacity of MAC, BC, IC, and $Z-$channel. Note that in the constituent $Z-$channels, the noise at one of the receivers is increased to include the effect of the interference from the appropriate user. This increase in the effective noise variance is required since the channel may not be a true $Z-$channel but is being modeled by an equivalent *imposed* $Z-$channel. The remainder of the proof follows directly from standard time-sharing arguments. □

Next, we evaluate the achievable rates using simple transmission schemes like FDMA/TDMA and using a single user decoder which treats the interference as noise.

D. FDMA/TDMA achievable region

Using only FDMA/TDMA the achievable rate pairs, (R_1, R_2), in an uplink SHC is easily computed as,

$$\begin{aligned} R_1 &\leq \alpha \max\left\{C\left(\frac{P_1}{\alpha}\right), C\left(\frac{c_{12}^2 P_1}{\alpha}\right)\right\} \\ &= \alpha C\left(\frac{P_1 \max\{1, c_{12}^2\}}{\alpha}\right), \quad (23) \\ R_2 &\leq (1-\alpha)\max\left\{C\left(\frac{P_2}{(1-\alpha)}\right), C\left(\frac{c_{21}^2 P_2}{(1-\alpha)}\right)\right\} \\ &= (1-\alpha) C\left(\frac{P_2 \max\{1, c_{21}^2\}}{(1-\alpha)}\right) \quad (24) \end{aligned}$$

where, $0 \leq \alpha \leq 1$, represents the fraction of bandwidth/time-slot allocated to user 1. The FDMA region is obtained by decoding each user at the receiver where its signal is received with highest SNR[3] and allowing a dynamic sharing of the available spectrum/time-slots.

It can be easily shown that the maximum sum rate using FDMA is attained by allocating the total bandwidth to the two users proportional to the received powers. Consequently the individual transmission rates $R^*_{1,FDMA}, R^*_{2,FDMA}$ that maximize the sum rate are given by,

$$\begin{aligned} R^*_{1:FDMA} &= \frac{P_1 x_1}{2(P_1 x_1 + P_2 x_2)} C(P_1 x_1 + P_2 x_2), \quad (25) \\ R^*_{2:FDMA} &= \frac{P_2 x_2}{2(P_1 x_1 + P_2 x_2)} C(P_1 x_1 + P_2 x_2), \quad (26) \end{aligned}$$

where, $x_1 = \max(c_{12}^2, 1)$ and $x_2 = \max(c_{21}^2, 1)$ and the maximum sum rate using FDMA equals $C(P_1 x_1 + P_2 x_2)$. One interpretation of this FDMA region is that the rates are equivalent to those achieved in a single user, single transmit, two receiver antennas system and performing receive antenna selection diversity.

For the downlink SHC, the achievable rates using a generalized FDMA/TDMA with power scaling is given by

$$\begin{aligned} R_1 &\leq \alpha C\left(\frac{\beta_1 P_1 + \beta_2 c_{21}^2 P_2}{\alpha}\right), \quad (27) \\ R_2 &\leq (1-\alpha) C\left(\frac{(1-\beta_2)P_2 + (1-\beta_1)c_{12}^2 P_1}{(1-\alpha)}\right), \quad (28) \end{aligned}$$

where $0 \leq \alpha, \beta_1, \beta_2 \leq 1$ represent the time sharing and power sharing parameters. Essentially, (27) represents the maximum sum rate of a MAC that is used for α fraction of time, where the transmitters have powers $\beta_1 P_1/\alpha$ and $\beta_2 P_2/\alpha$ and the corresponding channel gains are 1 and c_{21}^2. The maximum sum rate using FDMA is given by $C(\max(P_1 + c_{21}^2 P_2, P_2 + c_{12}^2 P_1))$: This maximum is attained when the rate of one user decreases to 0 and the entire power and bandwidth is used to send information to the other user.

[3] since there is no interference.

E. Single user decoding achievable region

Using single user decoding (SUD), the achievable rates for the uplink SHC are given by

$$\begin{aligned} R_1 &\leq \max\left\{C\left(\frac{P_1}{(P_2 c_{21}^2 + 1)}\right), C\left(\frac{c_{12}^2 P_1}{(P_2 + 1)}\right)\right\} \\ &= C\left(P_1 \max\left\{\frac{1}{(P_2 c_{21}^2 + 1)}, \frac{c_{12}^2}{(P_2 + 1)}\right\}\right), \quad (29) \\ R_2 &\leq \max\left\{C\left(\frac{P_2}{(P_1 c_{12}^2 + 1)}\right), C\left(\frac{c_{21}^2 P_2}{(P_1 + 1)}\right)\right\} \\ &= C\left(P_2 \max\left\{\frac{1}{(P_1 c_{12}^2 + 1)}, \frac{c_{21}^2}{(P_1 + 1)}\right\}\right). \quad (30) \end{aligned}$$

In this case, each user is decoded at the receiver where its SINR is maximized.

For the downlink SHC, by single user decoding we imply that each user only decodes information meant for itself. Further, if the data for a particular user is transmitted from two different transmitters, then we do not consider successive decoding of those data at the receiver. However, we do allow each transmitter to use a dirty paper code (DPC) [23] to send the data to the two users. In this case, a transmitter sending information to the two receivers can create a DPC in two ways: i) treating the data for user 1 as interference and ii) treating the data for user 2 as interference. Consequently, the achievable region of the downlink SHC under these conditions is given by $(R_1, R_2) = \text{co}(R_{11} + R_{21}, R_{12} + R_{22})$ such that

$$\begin{aligned} &\left(R_{11} \leq C\left(\frac{\alpha P_1}{1 + c_{21}^2 P_2}\right), R_{12} \leq C\left(\frac{\bar{\alpha} P_1 c_{12}^2}{1 + P_2 + c_{12}^2 P_1 \alpha}\right)\right) \cup \\ &\quad \left(R_{11} \leq C\left(\frac{\alpha P_1}{1 + c_{21}^2 P_2 + \bar{\alpha} P_1}\right), R_{12} \leq C\left(\frac{\bar{\alpha} P_1 c_{12}^2}{1 + P_2}\right)\right) \\ &\left(R_{21} \leq C\left(\frac{\beta P_2}{1 + c_{12}^2 P_1}\right), R_{22} \leq C\left(\frac{\bar{\beta} P_2 c_{21}^2}{1 + P_1 + c_{21}^2 P_2 \beta}\right)\right) \cup \\ &\quad \left(R_{21} \leq C\left(\frac{\beta P_2}{1 + c_{12}^2 P_1 + \bar{\beta} P_2}\right), R_{22} \leq C\left(\frac{\bar{\beta} P_2 c_{21}^2}{1 + P_1}\right)\right), \quad (31) \end{aligned}$$

where $0 \leq \alpha, \beta \leq 1$ and $\bar{x} = 1 - x$.

F. MIMO system outer bound

In a SHC, we do not allow centralized processing (decoding) of the received signals from the multiple receivers. With a central decoder, the SHC simplifies to a standard multiple input multiple output (MIMO) channel, whose capacity is known in many settings [24], [25]. This MIMO channel capacity provides an outer bound on the capacity of the uplink SHC and is computed as follows:

$$R_{i_u:MIMO} \leq \log(\mu_i \lambda_i), \;\; i = 1, 2, \quad (32)$$

where, λ_i is the square root of the singular value of $[c(i,1)\; c(i,2)]$ and $\mu_i = P_i + 1/\lambda_i$. Further, a bound on the sum rate is obtained as

$$R_{1_u:MIMO} + R_{2_u:MIMO} \leq \sum_{i=1}^{2} \log(\mu \gamma_i)^+, \quad (33)$$

where, μ is selected using water-filling as $P_1+P_2=\sum_{i=1}^{2}(\mu-1/\gamma_i)^+$, γ_i is the eigenvalue of $\mathbf{c}^T\mathbf{c}$ and $(x)^+=\max(x,0)$. It turns out that in many scenarios, the outer bounds on $R_{i_u:MIMO}$ are loose since we do not allow the receivers to jointly decode the signal. These outer bounds are plotted in Figures 2a-4a and discussed in Sections IV-B and IV-D.

G. MIMO Broadcast Channel Outer bound

The capacity of the MIMO broadcast channel has been recently computed in [26]. We make use of that capacity result to compute an outer bound on the capacity of the downlink SHC.[4] To compute the outer bound, we assume that both the transmitters are present at one location with two effective transmit antennas. Instead of a joint total power constraint of P_1+P_2 on the two antennas, we impose individual power constraints of P_1 and P_2, on the two antennas. Let $H_1=[c_{11}\ c_{21}]$ and $H_2=[c_{12}\ c_{22}]$ and let Q_1, S, Q_2 be positive semidefinite matrices such that S is of the form

$$S=\begin{pmatrix} P_1 & \mathrm{s} \\ \mathrm{s} & P_2 \end{pmatrix}, \tag{34}$$

where $-\sqrt{P_1P_2}\leq s\leq\sqrt{P_1P_2}$ (to ensure positive semi-definiteness) and $Q_1+Q_2\preceq S$. Then the following two rate pairs are achievable,

$$R_1 \leq C(H_1BH_1^t), R_2\leq C\left(\frac{H_2DH_2^t}{H_1BH_1^t+1}\right) \tag{35}$$

$$R_1 \leq C\left(\frac{H_1BH_1^t}{H_2DH_2^t+1}\right), R_2\leq C(H_2DH_2^t). \tag{36}$$

The convex hull of the union of these pairs over all possible S, Q_1, and Q_2 matrices yields the capacity region for the MIMO broadcast channel. These outer bounds are plotted in Figures 2b-4b and discussed in Sections IV-B and IV-D.

In the next section, we evaluate the achievable regions for the uplink and downlink SHCs in a few special cases and study their properties.

IV. SHC: Achievable region and properties

A. Computing the uplink and downlink SHC achievable region for 2×2 system

Consider both uplink and downlink transmissions for a two-user system in soft handover with two base stations. We assume that the channel gain between the i^{th} base station and j^{th} mobile is the same in both the uplink and downlink *e.g.* a TDD system. With this assumption, to find the achievable regions for the uplink and downlink SHC's we need to compute the $\mathcal{R}_{2\times 2}$ regions for $\mathbf{c}$ and $\mathbf{c}^t$ respectively, where subscript t represents matrix transpose. Mappings (20) and (21) are then applied to the respective $\mathcal{R}_{2\times 2}$ regions to find the uplink and downlink SHC achievable regions.

Numerically evaluating the achievable region for the uplink and downlink SHC using Theorem 6 is computationally prohibitive. We use the following optimization framework to compute an *approximation* to the achievable region. Recognize that the achievable region given in Theorem 6 is convex. Thus, a family of lines of the form R_1+wR_2 for different values of w will touch (be tangential to the region) the "face" of the achievable region at different points on the boundary. Considering many different values of w and finding the intersection point with the achievable region will enable us to compute different points on the boundary. Connecting these points by straight lines provides an approximation of the achievable region. The more the number of w values considered, the closer is the approximation to the true achievable region.

For a fixed w, to find the point where the family of lines R_1+wR_2 is tangential to the uplink SHC achievable region, we solve the following optimization problem:

$$\max R_{11}+R_{12}+w(R_{21}+R_{22}) \tag{37}$$
$$\text{s.t.}(R_{11},R_{12},R_{21},R_{22})\in\mathcal{R}_{2\times 2}(1,c_{12},c_{21},1,1,1,P_1,P_2)$$

The variables of optimization in (37) are $\alpha_i, P_1^i, P_2^i, i=1\ldots 8$, with the constraint that $\sum_i\alpha_i=1, \sum_i\alpha_iP_j^i=1, j=1,2$. Clearly, (37) is a convex optimization problem (since the achievable region is convex) and hence, the local minimum is a global minimum. The objective function in (37) is, however, not strictly convex[5] and thus, there could be multiple optima. The existence of multiple optima indicate the possibility that various coding and decoding schemes could achieve the boundary of the SHC. Representative numerical results of such optimization are given in Examples 7 and 8.

For the downlink SHC, the achievable region is computed by solving the following equation:

$$\max R_{11}+R_{21}+w(R_{12}+R_{22}) \tag{38}$$
$$\text{s.t.}(R_{11},R_{12},R_{21},R_{22})\in\mathcal{R}_{2\times 2}(1,c_{21},c_{12},1,1,1,P_1,P_2)$$

The variables of optimization in (38) are $\alpha_i, P_1^i, P_2^i, i=1\ldots 8$, with the constraint that $\sum_i\alpha_i=1, \sum_i\alpha_iP_j^i=1, j=1,2$. The properties of this optimization problem are similar to the uplink optimization problem. In the following examples, we discuss some representative numerical results of the optimization.

Example 7: Consider uplink SHC optimization problem with $c=\begin{bmatrix}1 & \sqrt{0.4}\\ 0 & 1\end{bmatrix}$ and weight $w=2$. The result of the optimization gives $R_{11}=0.7336, R_{12}=0, R_{21}=0, R_{22}=1.404$. For the same channel conditions, the downlink optimization results in the following values for the rates $R_{11}=0.399, R_{12}=0.188, R_{21}=0, R_{22}=1.273$. In the uplink, both users are decoded at only one location each, whereas for the downlink, independent data is sent from both base stations to user 2 and only from one base station to the user 1. In this example, 12.8% of rate for user 2 is sent from base station 1 and the rest from base station 2. Since $w=2$, user 2 has higher preference in selecting the operational point, which is reflected in the fact that user 2 has higher rate in both the uplink and downlink than user 1.

[4] A similar outer bound is calculated in [4] for the cognitive interference channel.

[5] It is easy to see that the objective function is linear in the α variables.

Example 8: In this case, let $c = \begin{bmatrix} 1 & \sqrt{0.1} \\ \sqrt{0.9} & 1 \end{bmatrix}$ and weight $w = 0.95$. The result of the optimization gives $R_{11} = 0.577, R_{12} = 0, R_{21} = 0.062, R_{22} = 1.23$ for the uplink case. For the same channel conditions, the downlink optimization results in the following values for the rates $R_{11} = 1.37, R_{12} = 0, R_{21} = 0, R_{22} = 0.508$. In this case, user 2 has higher rate in the downlink and user 1 has higher rate in the uplink for the same channel. Even though $w = 0.95$ results in slightly higher importance to the rate of user 1 in the optimization function, it turns out that in the downlink, user 2 has a higher rate since its channel gains from both base stations are higher than the channel gains experienced by user 1. The data of user 1 is decoded at only one base station, whereas part of the data of user 2 is decoded at both base stations (4.8% of rate in one base station and the rest in the other). In the downlink, only base station 1 transmits to user 1 and base station 2 transmits data to user 2.

These examples indicate that a network provider could use a similar framework to compute the system operational point. For example, the provider could maximize a linear/nonlinear combination of rates for each user to ensure fairness or provide pricing based service differentiation using utility functions [27].

These examples also illustrate that if a network provider chooses to optimize a weighted linear function of the rates, then it may not be optimal for each mobile to transmit and receive the same percentage of its rate from a particular base station. This observation has an enormous implication on the encoding and decoding method to be used is in the network and also on the data routing in the backbone network.[6]

B. Symmetric SHC

Consider a symmetric SHC, *i.e.*. $c_{12} = c_{21} = c$ and further let the transmit powers P_1 and P_2 be equal. In this case, we show that the achievable region of the SHC is dominated by the achievable region of one of the constituent interference channels. We make use of the following 3 lemmas to prove this result.

We first show that in any uplink SHC (not necessarily symmetric), any rate pair (R_1, R_2) that can be achieved using one of the constituent broadcast channel, can also be achieved using an appropriate MAC channel.

Lemma 9 (BC and MAC): After mapping the quadruple of rates to the 2-dimensional rate pairs using (20), the achievable region of a BC is contained within the achievable region of an appropriate MAC, *i.e.*, $\phi_u(\mathcal{R}_{BC}(1, c, ., ., 1, 1, P_1, 0)) \subseteq \phi_u(\mathcal{R}_{MAC}(1, ., c, ., 1, ., P_1, 0))$.

Proof: Without loss in generality we assume $c \leq 1$. The proof is identical for the case of $c > 1$.

The intuition for this result comes from realizing that in the uplink SHC achievable region computation, we are only interested in the sum rates achieved using each constituent BC. The sum rate in a Gaussian BC is maximized by transmitting all information to only one receiver: due to the degraded nature of the Gaussian BC, one of the receivers is *better* than the other and can decode all the information that is transmitted. The same sum rate can be obtained by considering a MAC with one of the transmit powers equal to zero. Thus,

$$\max_{(R_1,0)=\phi_u(\mathcal{R}_{BC}(1,c,.,.,1,1,P_1,0))} R_1 = \max_{(R_1,0)=\phi_u(\mathcal{R}_{MAC}(1,.,c,.,1,.,P_1,0))} R_1 = C(P_1). \quad (39)$$

□

Next, we show that in the symmetric case, the MAC capacity region is contained inside the IC achievable region after mapping to $2-D$ regions for uplink SHC.

Lemma 10 (MAC and IC: symmetric case): Consider a symmetric channel with 2 nodes transmitting to 2 base stations, *i.e.* $c_{12} = c_{21} = c$ and without loss in generality let $c < 1$. Further, let the transmit powers be equal, $P_1 = P_2$. The rate pairs of both the constituent MAC capacity regions are contained within one of the IC after mapping to $2-D$ regions using (20), *i.e.* $\phi_u(\mathcal{R}_{MAC}(1, ., c, ., 1, ., P_1, P_2)) \subseteq \phi_u(\mathcal{R}_{IC}(1, 1, c, c, 1, 1, P_1, P_2))$ and $\phi_u(\mathcal{R}_{MAC}(., 1, ., c, ., 1, P_1, P_2)) \subseteq \phi_u(\mathcal{R}_{IC}(1, 1, c, c, 1, 1, P_1, P_2))$.

Proof: It is sufficient to show that the two significant corners of both MACs (for instance points A,B,E and D in Figure 2a) are contained within the achievable region (10) of one of the interference channels. The (R_1, R_2) coordinates of the four corner points are: $A : \left(C(P_1), C\left(\frac{P_2c^2}{1+P_1}\right)\right)$, $B : \left(C\left(\frac{P_1}{1+c^2P_2}\right), C(P_2c^2)\right)$, $E : \left(C(P_1c^2), C\left(\frac{P_2}{1+P_1c^2}\right)\right)$, $D : \left(C\left(\frac{P_1c^2}{1+P_2}\right), C(P_2)\right)$. Recognize that two of these corner points, namely, A and D, are already included in (10). Clearly, the convex hull in (10) includes the straight line segment between these two corner points. The equation for the line joining these points is of the form $R_1 + R_2 = C(P_1 + P_2c^2)$. Now, setting $P_1 = P_2$, we find that all four corner points lie on the same straight line. To complete the proof, we need to verify that points B and E lie between points A and D. It is straightforward to verify that the R_1 coordinates of points A and D are the largest and smallest, respectively, of the four corner points. Similarly, the R_2 coordinates of A and D are the smallest and largest, respectively, of the four corner points. Hence, points B and E lie between A and D. □

Now, we show that with our modelling of the 2×2 channel as an *imposed* Z-channel, the achievable region of the imposed $Z-$channel is a subset of the achievable region of the IC, after using the mapping (20).

Lemma 11 (IC and imposed Z-channel: symmetric case): After using the mapping (20), the achievable rates with the *imposed* Z-channel is also achieved using the IC, with equal transmit powers *i.e.* $\phi_u(\mathcal{R}_Z(1, 0, c, 1, 1, 1, 1+P_1c^2, P_1, P_2)) \subseteq \phi_u(\mathcal{R}_{IC}(1, c, c, 1, 1, 1, P_1, P_2))$, where $P_1 = P_2$.

Proof: Without loss in generality we assume $c \leq 1$. The proof is identical for the case of $c > 1$.

Consider the following two cases: i) $c^2 \leq \frac{1}{1+P_1c^2}$ and ii) $\frac{1}{1+P_1c^2} \leq c^2 \leq 1$. In the first case, the achievable region of the *imposed* Z-channel is given by (11)-(13) and (15), with $N_1 = 1$ and $N_2 = 1 + P_1c^2$. For the uplink SHC, recall

[6] These implications are beyond the scope of this paper and should be studied in future work.

that using mapping (20), $R_2 = R_{21} + R_{22}$. Now, consider the rate pair $\phi_u(R_{IC}(1, c, c, 1, 1, 1, P_1, P_2))$. It is clear that R_1 achieved in this IC is the same as that using the imposed Z-channel. Further, notice that the bound (15) inherently assumes that onion peeling decoding of information corresponding to R_{21} is carried out at decoder 2 and then information corresponding to R_{22} is decoded. Hence, the same decoding method can be applied in the IC and both R_{21} and R_{22} can be decoded at receiver 2. Thus, any rate $R_2 = R_{21} + R_{22}$ that can be achieved using the imposed Z-channel can also be achieved using the IC.

Now, in the second case, we show that the achievable region of the imposed Z-channel is contained in the MAC, which in turn is contained in the achievable region of the IC. The maximum sum rate using the MAC is given by $C(P_1 + c^2P_2)$. For the imposed Z-channel, the rate region is given by (11)-(14), with $N_1 = 1$ and $N_2 = 1+P_1c^2$. Using mapping (20), the sum rate is bounded by $C\left(\frac{P_1+c^2(1-\beta)P_2}{1+c^2\beta P_2}\right)+C\left(\frac{\beta P_2}{1+c^2P_1+(1-\beta)P_2}\right)$. It is sufficient to show that this sum rate is smaller than the sum rate of one of the MAC's. The difference in sum rate of the imposed Z-channel and MAC is given by,

$$\frac{1}{2}\log\left(\frac{(1+P_1+c^2P_2)(1+c^2P_1+P_2)}{(1+c^2\beta P_2)(1+c^2P_1+(1-\beta)P_2)}\right) - \frac{1}{2}\log(1 + P_1+ c^2P_2) = \frac{1}{2}\log\left(\frac{(1+P_1+c^2P_2)}{(1+c^2\beta P_2)(1+c^2P_1+(1-\beta)P_2)}\right) \quad (40)$$

To show that the term inside the logarithm on RHS of (40) is no greater than one, we expand the denominator of that term as follows:

$$\begin{aligned}(1 + c^2\beta P_2)(1 + c^2P_1 + (1-\beta)P_2) &= \\ 1 + c^2P_1 + P_2(1 + \beta(-1 + c^2 + c^4P_1 + c^2(1-\beta)P_2)) &= \\ 1 + c^2P_1 + P_2 + P_2\beta(-1 + c^2(1 + c^2P_1) + c^2(1-\beta)P_2) & \quad (41)\end{aligned}$$

Since $P_1 = P_2$ and $c^2(1+c^2P_1) \geq 1$, the term in (41) is greater than or equal to the term in the numerator of the logarithm term in the RHS of (40). Hence, the achievable region of the Z-channel is contained in the MAC channel, which in turn is contained in the achievable region of the IC after applying mapping (20).

A similar analysis can be used to show that the achievable region of the other imposed Z-channel is also contained in the IC.[7] □

Theorem 12: (Symmetric Gaussian uplink SHC achievable region with equal power) For a symmetric Gaussian uplink SHC, with equal transmit powers at the two transmitters, the achievable region of one of the interference channels dominates the achievable region of the SHC.

Proof: Without loss in generality we assume $c < 1$. The proof is identical for the case of $c \geq 1$. In this case, we need to show that $\phi_u(R_{IC}(1, 1, c, c, 1, 1, P, P))$ equals $\phi_u(R_{2\times2}(1, 1, c, c, 1, 1, P, P))$.

Applying Lemmas 9, 10 and 11, we know that the achievable region of the constituent MAC, BC and imposed Z-channels are contained in $\phi_u(R_{IC}(1, 1, c, c, 1, 1, P, P))$ after mapping to 2-D region using (20). It is also easy to verify that the $R_{IC}(c, c, 1, 1, 1, 1, P, P)$ is a strong interference channel and its capacity is contained in the intersection of capacity of the two constituent MAC regions. Hence, the achievable region of the uplink SHC equals the achievable region of one of the constituent IC. □

The implication of this theorem is that to achieve the boundary of the achievable region in a symmetric Gaussian uplink SHC, it is sufficient to select one unique location for decoding each user. Different points on the boundary are achieved by varying the coding rates but keeping the decoding location fixed for each user. Note that even though one unique decoding location is sufficient for each user, *onion peeling* decoding might be required.[8]

Next we show that unlike the symmetric uplink SHC, for the symmetric downlink SHC, transmitting to each user from a fixed base station does not achieve the entire boundary of the achievable region.

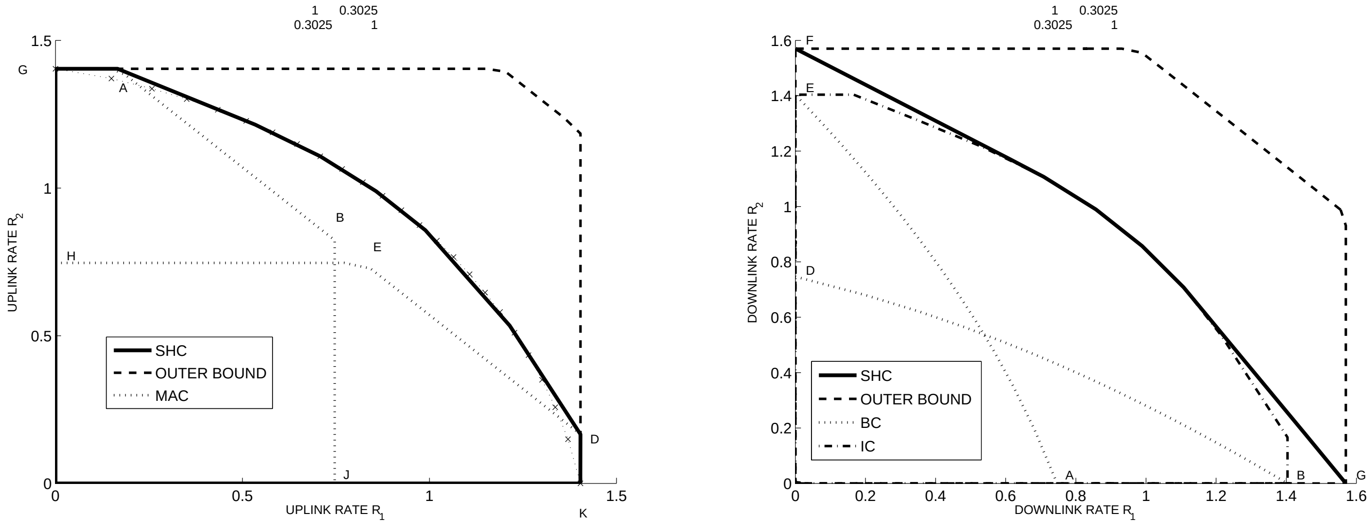
Proposition 13 (Gaussian downlink SHC achievable region): All points on the boundary of the achievable region for Gaussian downlink SHC are *not* achieved by transmitting to each user from a single fixed transmitter location.

Proof: The proof is straightforward if one considers the two extreme points of the boundary of the achievable region. The boundary intersects the R_2 axis when both the transmitters use all their power to send information to the second receiver: the maximum value of R_2 so attained is given by the maximum sum rate of the corresponding MAC, which equals $C(P_1 + c^2P_2)$. This point is also achieved by considering a Z-channel. Similarly, the boundary intersects the R_1 axis when both transmitters use all their power to send information to the first receiver and the corresponding maximum rate equals $C(c^2P_1 + P_2)$. □

Numerical results - Symmetric SHC: The plot of the achievable region for a symmetric SHC is given in Figures 2a and b for the uplink and downlink, respectively. The transmit powers are $P_1 = P_2 = 6$ and the channel cross over matrix equals $\mathbf{c} = \begin{bmatrix} 1 & 0.3025 \\ 0.3025 & 1 \end{bmatrix}$. In Figure 2a, the capacity region of the two constituent MAC regions (with powers P_1 and P_2) are given by pentagons [0 G A B J 0] and [0 H E D K 0]. The achievable region for one of the constituent IC is the same as the SHC achievable region. The outer bound on capacity is calculated using (33) and is clearly not a very tight outer bound. In Figure 2b, the capacity region of the two constituent BC are given by [0 E A 0] and [0 D B 0]. It can be seen that the achievable region of the SHC is larger than the achievable region of either BC or IC. The outer bound on capacity is plotted using (35)-(36) and as in the uplink case is not a tight bound.

[7]It seems counter intuitive that using (20), the achievable region of a Z-channel is contained inside the achievable region of an appropriate IC. The reason for this behavior is that since the channel is not a true Z-channel, modeling it as an IC allows the possibility of onion peeling decoding at both receivers. In our imposed Z-channel model, onion peeling decoding is not allowed at one of the receivers; all the interference is forcibly treated as noise, *e.g.*, $N_2 = 1 + P_1c^2$.

[8]If the transmitters are allowed to cooperate, then it would be possible to achieve the same rates with just SUD at the receiver by using dirty paper coding.



(a) (b)

Fig. 2. The achievable region and outer bound on capacity of a symmetric Gaussian SHC in (a) uplink scenario and (b) downlink scenario. In (a), the capacity region of the two constituent MAC regions are given by pentagons [0 G A B J 0] and [0 H C D K 0]. The achievable region for one of the constituent IC is the same as the SHC achievable region. The line with $\times$ markers is the achievable region using FDMA for the SHC. In (b), the capacity region of the two constituent BC are given by [0 E A 0] and [0 D B 0]. Transmit powers $P_1 = P_2 = 6$ and the channel matrix $\mathbf{c}$ is as indicated on top of each figure.

C. Z-SHC

In a SHC, if one of the cross over channel gains equals 0, we refer to the channel as a Z-SHC. The following proposition gives conditions on the nonzero cross over channel gain that ensures that it is sufficient to decode each user at only a single receiver.

Proposition 14: Consider the uplink SHC with channel gains $\mathbf{c} = \begin{bmatrix} 1 & c_{21} \\ 0 & 1 \end{bmatrix}$. The entire achievable region of the SHC is obtained by decoding user 2 at a single receiver if $c_{21}^2 \leq 1$ or if $c_{21}^2 \geq 1 + P_1$.

Proof: For the given channel, the data of user 1 can only be decoded from receiver 1. The data of user 2 can be decoded from either receivers.

First, consider $c_{21}^2 \leq 1$. In this case, it is easy to verify from (11)-(13), (15) that any data that is sent from user 2 to receiver 1 can also be decoded at receiver 2.[9] Similarly, when $c_{21}^2 \geq 1 + P_1$, the capacity of the $Z-$channel is given by (16)-(19). In this case, receiver 1 uses an onion peeling decoding and any data sent from user 2 to receiver 2 can also be decoded at receiver 1. Thus, in both cases, decoding user 1 at a unique location is sufficient to achieve all points in the achievable region. □

In contrast to the uplink Z-SHC, in the downlink Z-SHC, both the base stations need to transmit information to one of the users to obtain the entire achievable region.[10] For the other user, since one of the cross over channel gain equals 0, data is only sent from one base station.

<u>*Numerical results - Z-SHC:*</u> The plot of the achievable region for a Z-channel under soft handover is given in Figures 4a and b for the uplink and downlink, respectively. The transmit powers are $P_1 = P_2 = 6$ and the channel cross over matrix equals $\mathbf{c} = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$ for the uplink case and the transpose of that matrix for the downlink case. In both cases, it turns out that the achievable region of one of the component Z-channels equals the achievable region of the SHC. It can also be seen that the outer bounds are not very tight.

D. Asymmetric SHC

For a general asymmetric SHC, there exist channel conditions when none of the component achievable regions dominate the achievable region for the SHC. We present numerical examples to establish this existence result. Finding analytic conditions on the cross over channel gains, under which no single component achievable region dominates the SHC achievable region is still an open problem and should be investigated in future work.

<u>*Numerical results - Asymmetric SHC:*</u> The plot of the achievable region for an asymmetric SHC is given in Figures 3a and b for the uplink and downlink, respectively. The transmit powers are $P_1 = P_2 = 6$ and the matrix of channel gains equals $\mathbf{c} = \begin{bmatrix} 1 & 0.1 \\ 0.9 & 1 \end{bmatrix}$. Such cross over gains occur frequently in practical cellular systems when one user is midway between two base stations and the other user is very close to one base station and far from the second base station. In Figure 3a, the capacity region of the two constituent MAC regions are given by pentagons [0 G A B J 0] and [0 H C D K 0]. It can be seen that the achievable region of the SHC is strictly larger than the achievable region of any of the constituent MAC and IC's. Thus, it is not sufficient to pick a unique location to decode for each user to operate on the

[9]Recall that receiver 2 uses an onion peeling decoder.
[10]The proof is similar to Proposition 13.

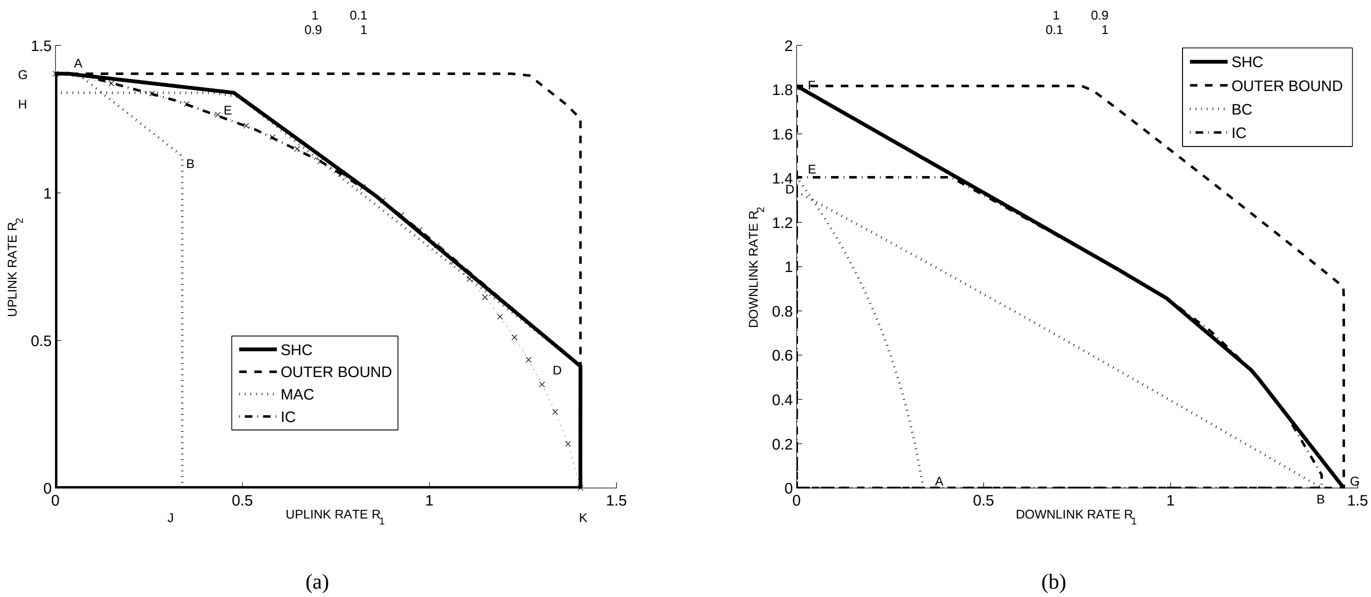


Fig. 3. The achievable region and outer bound on capacity of an asymmetric Gaussian SHC in (a) uplink and (b) downlink. In (a), the capacity region of the two MAC regions are given by pentagons [0 G A B J 0] and [0 H C D K 0], respectively. The line with $\times$ markers is the achievable region using FDMA for the SHC. In (b), the capacity region of the two constituent BC's are given by [0 E A 0] and [0 D B 0]. Transmit powers $P_1 = P_2 = 6$ and the channel matrix $\mathbf{c}$ is as indicated on top of each figure.

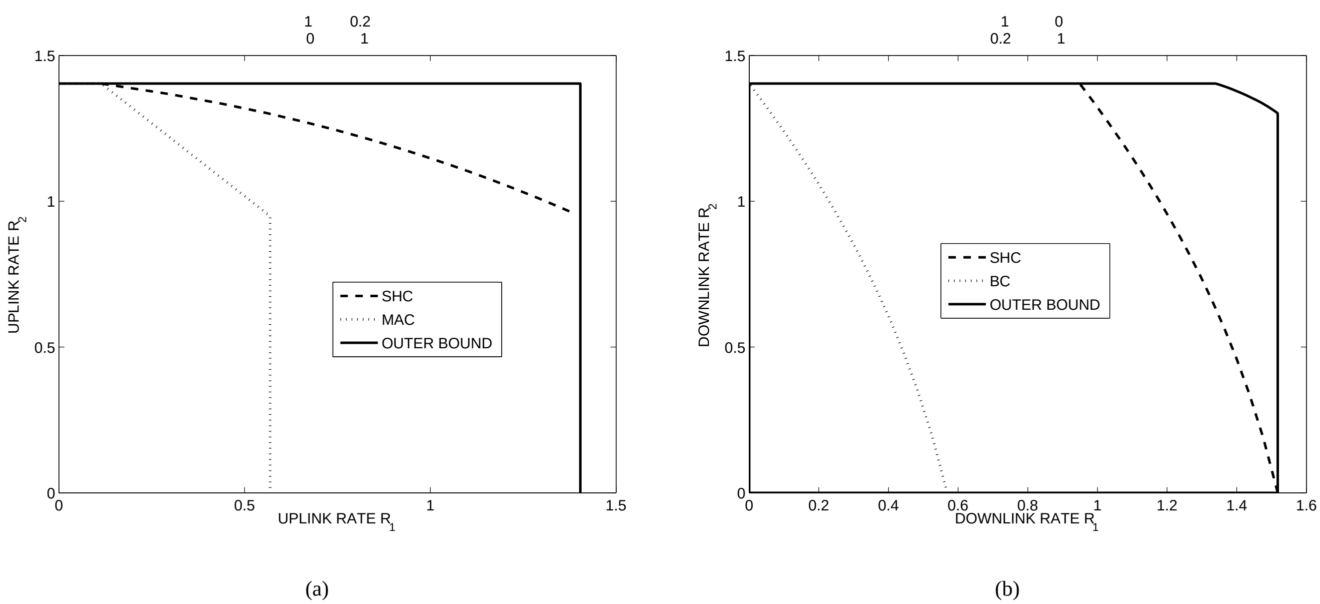


Fig. 4. The achievable region and outer bound on capacity of a Gaussian Z-channel under soft handover in (a) uplink and (b) downlink. Transmit powers $P_1 = P_2 = 6$ and the channel matrix $\mathbf{c}$ is as indicated on top of each figure.

boundary of the achievable region. As in the symmetric case, the outer bound on SHC capacity is loose. The achievable region for the corresponding downlink is given in Figure 3 and exhibits a similar trend. The SHC achievable region is larger than the achievable region of all its constituent channels. Thus, transmitting to each user from a fixed location does not achieve the boundary of the achievable region. The implication of this result is that once the network operator selects a desired metric to optimize performance, the rate of transmission between each transmitter-receive pair would be determined using the proposed optimization. Purely, SNR or SINR based decision on which base station to decode data or transmit data from could be suboptimal.

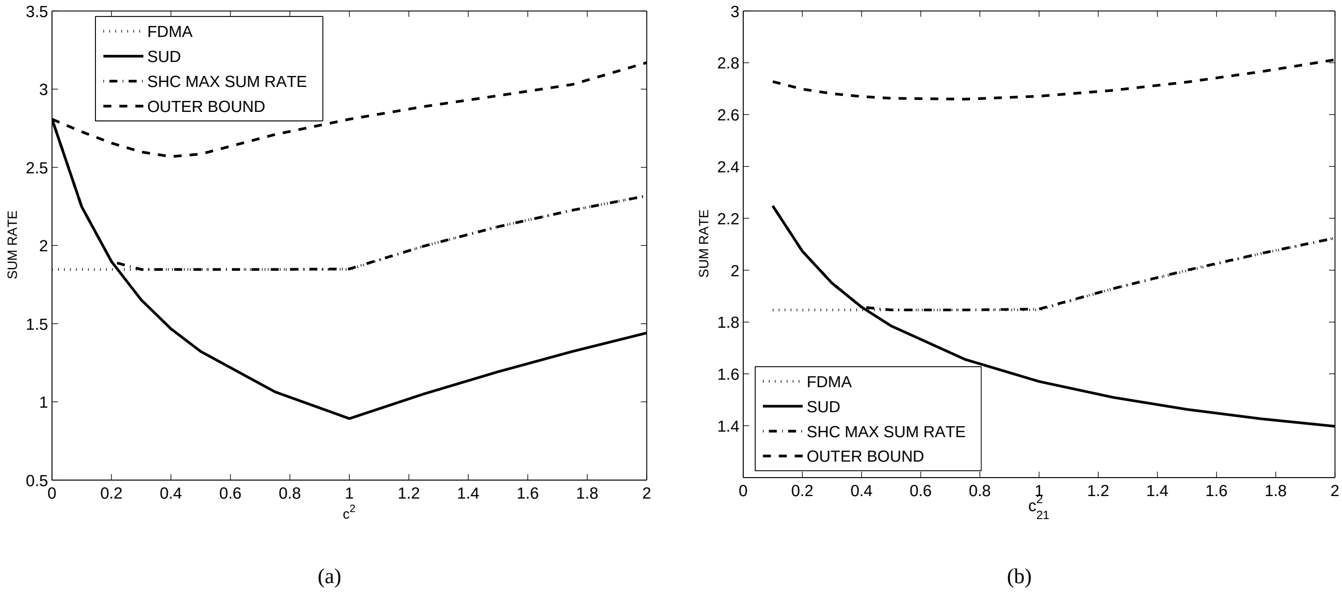


Fig. 5. The sum rate of SHC using FDMA, SUD, maximum sum rate of SHC and outer bound on sum rate for (a) symmetric SHC and (b) asymmetric SHC with $c_{12} = 0.1$ and c_{21} values as indicated on the X-axis. Transmit powers $P_1 = P_2 = 6$.

E. Sum-rate in SHC

For a MAC, the sum rate is always attained by FDMA/TDMA region[11] and the achievable sum rate using SUD is strictly lesser than the sum-rate using FDMA.[12] For an interference channel, however, the maximum sum rate is not always attained by either of FDMA or SUD. For very low values of interference, the sum rate using SUD is greater than FDMA and for higher interference levels, the reverse is true. It turns out that for the SHC, not surprisingly, the sum rate behavior using FDMA and SUD is similar to the interference channel. For a symmetric **c**, comparing (26) with (29),(30) we can easily find conditions when SUD is better than FDMA. The maximum rate achieved using SUD is greater than the FDMA rate under the following scenarios:

$$\begin{cases} P \geq \frac{1-2c^2}{2c^4} & \text{if } c^2 \leq 1 \\ P \geq \frac{c^2-2}{2} & \text{if } c^2 \geq 1 \end{cases} \tag{42}$$

Similar conditions can be obtained for an asymmetric channel. The region where sum-rate of SUD is greater than FDMA is referred to as *weak* interference and the region where sum-rate using FDMA is greater than SUD is referred to as *moderate* interference [10].

The plot of the sum-rate is given in Figures 5a and b for the symmetric and asymmetric uplink SHC. The behavior of sum-rate of SHC is similar to sum-rate behavior in an IC [10]. As expected, for small c, SUD has higher sum rate than FDMA and for large c, the trend is reversed.

[11]Recognize that the FDMA region touches the MAC capacity region, which is a pentagon, at one point along the *significant* edge of the pentagon.

[12]Except in the trivial case where MAC capacity pentagon reduces to a rectangle.

V. Conclusions

In this paper, we introduced the model for a Gaussian soft handover channel and computed bounds on its capacity. The main implications of the results are: i) It is not always sufficient to communicate with one transceiver to achieve the boundary of the achievable region and ii) the fractions of data being transfered between a mobile node to two different base stations are different in the uplink and downlink scenarios. The results in this paper could be extended in several directions, *e.g.* considering arbitrary number of users and receivers, considering fading channels and studying the impact of multiple antennas at both the transmitters and receivers. Tighter outer bounds should also be investigated in future work for both uplink and downlink soft handover channels. Conditions on the channel gains and powers when one of the constituent channels does not dominate the SHC achievable region should be derived in future work.

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